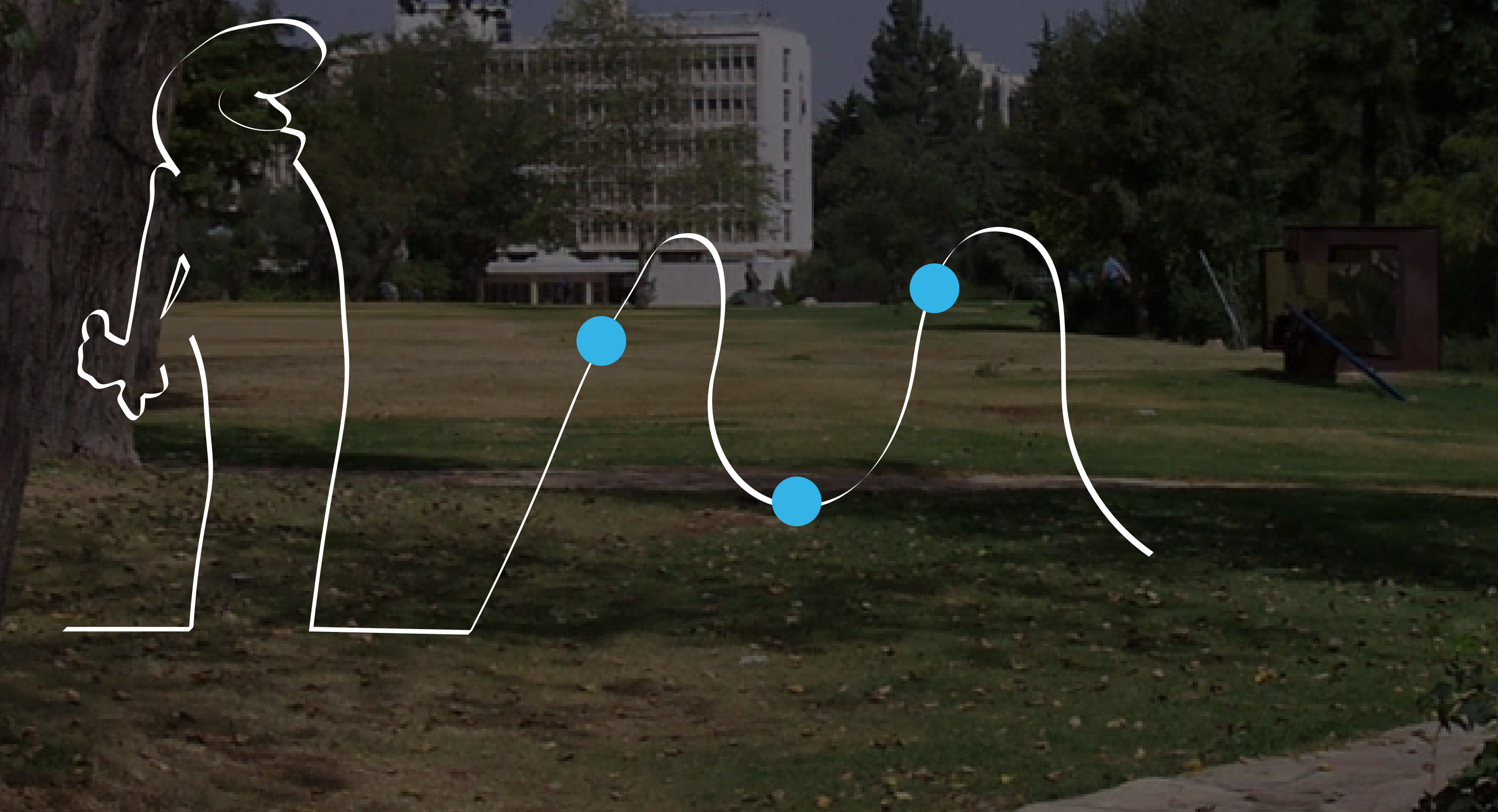
Effects of disorder in fluctuating one-dimensional interacting systems

Additional Information



Contents

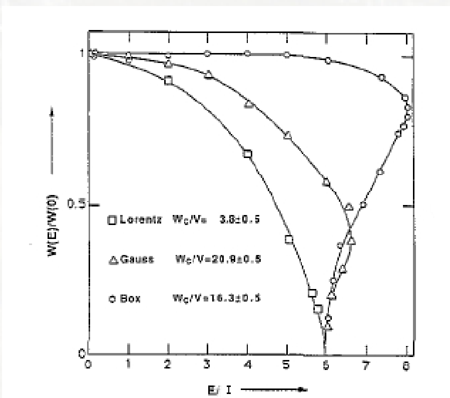
Anderson Model

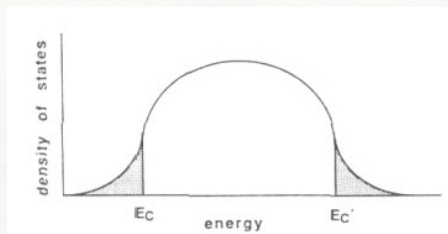
$$H = \sum_i W_i c_i^\dagger c_i - \sum_{i,j} I_{ij} c_i^\dagger c_j + h.c$$

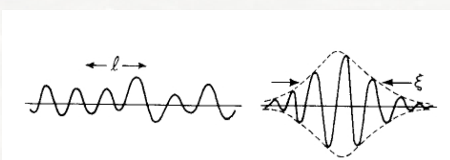
$$A = |W_i| \leq \frac{W}{2}$$

$$I_{ij} = \begin{cases} I & \text{j=i}\pm 1 \\ 0 & \text{otherwise} \end{cases}$$

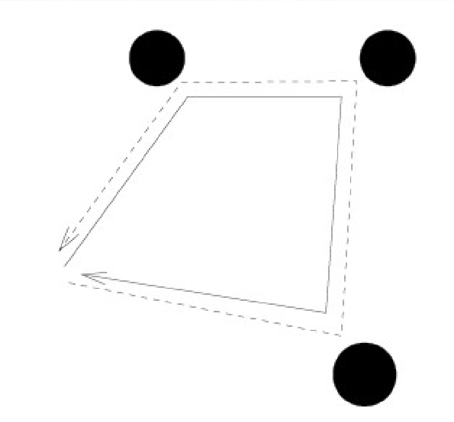
$$p\left(W_i\right) = \begin{cases} \frac{1}{W} & |W_i| \leq \frac{W}{2} \\ 0 & \text{otherwise} \end{cases}$$







CuO
MIT
1D
GS
Fluct
R
C
toc



Scaling Theory

one parameter scaling $g(L) = \frac{E_c(L)}{\Delta_L}$

Thouless energy $E_c(L) = \frac{\hbar D(L)}{L^2}$

mean level spacing $\Delta_L = \frac{1}{\nu_L L^d}$

scaling hypothesis: the conductance of a sample built of little blocks is a function of the conductance od a single block $g(L)$

Gellman-Low eq:

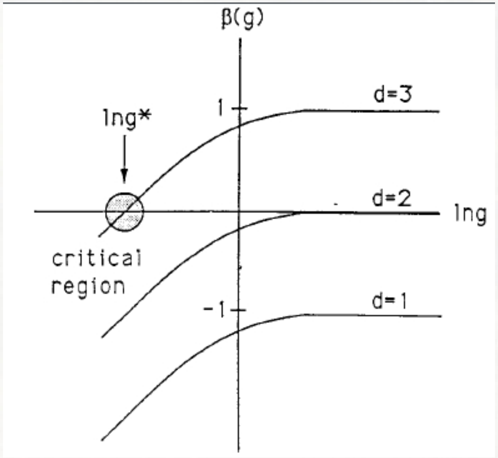
$$\frac{d \ln g}{d \ln L} = \beta(g)$$

metal - Ohm's law

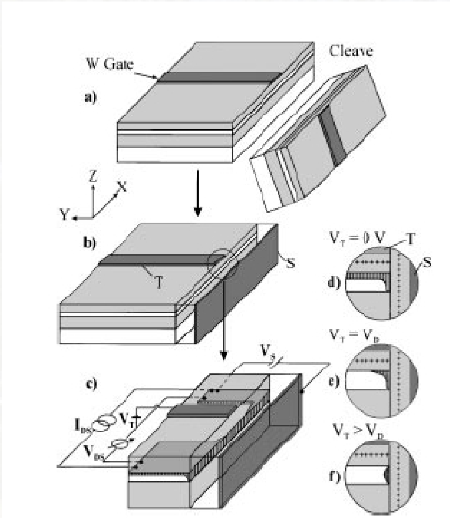
$$g(L) = \frac{\hbar}{e^2}\sigma L^{d-2}$$
$$\beta(g) = d - 2$$

insulator - hopping transport

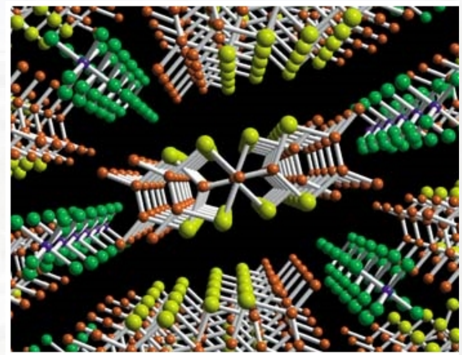
$$g(L) \sim e^{-L/\xi}$$
$$\beta(g) = \ln\left(g/g_c\right)$$



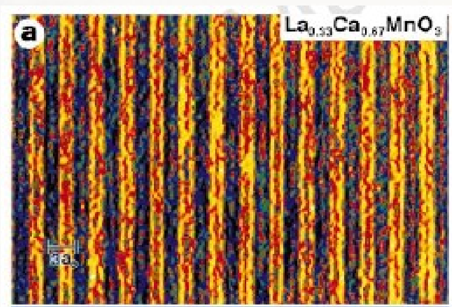
One dimensional systems



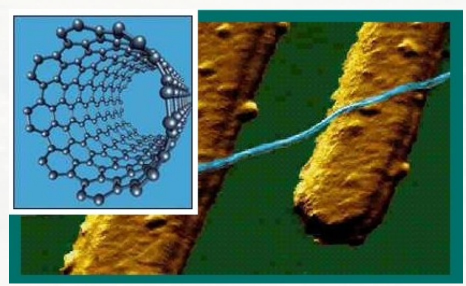
Quantum Wires



Organic Conductors



Stripes

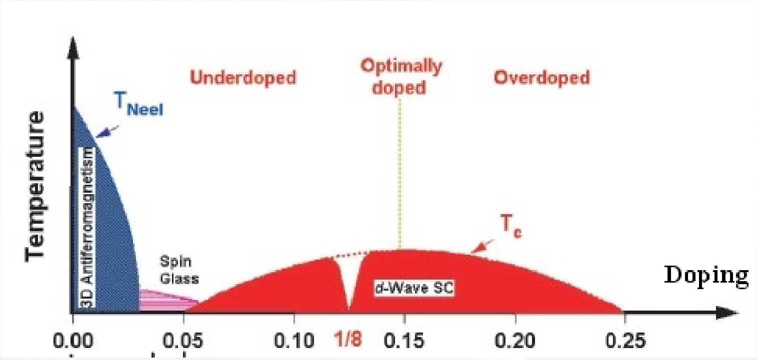


Nano-tubes

Quantum Hall edge states,

Cuprates

Stripes

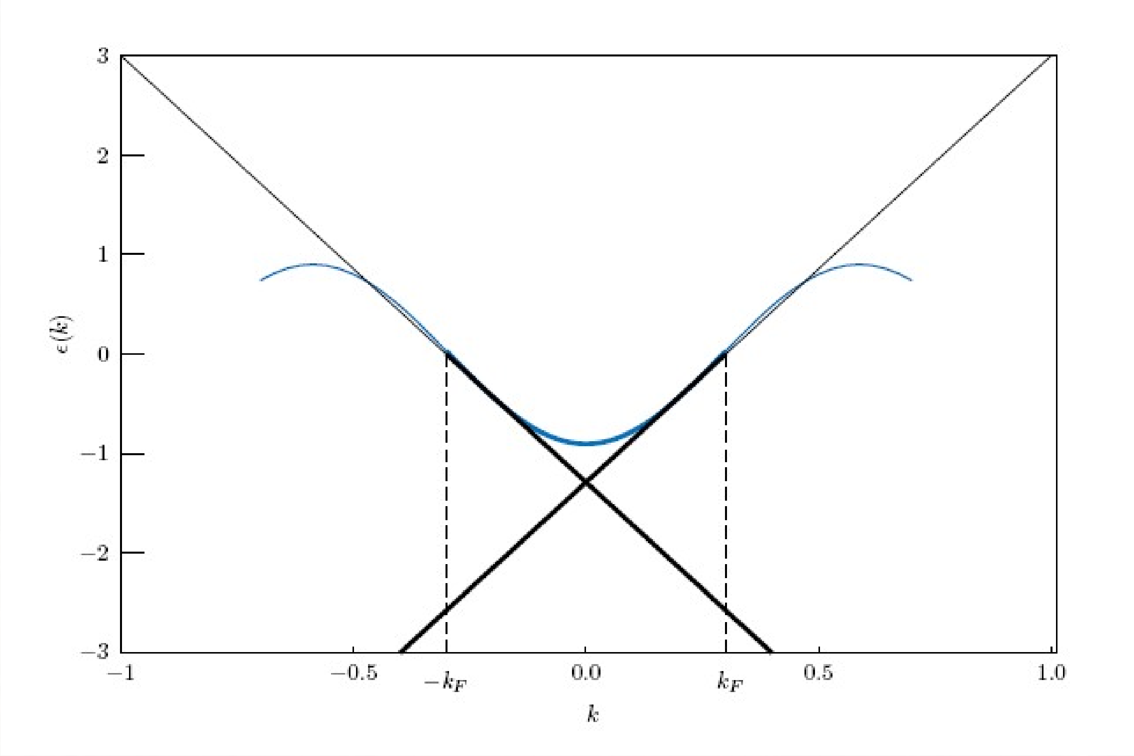


Temperature
Underdoped
Optimally doped
Overdoped
T_{Neel}
3D Antiferromagnetism
Spin Glass
T_c
d-Wave SC
Doping
0.00
0.05
0.10
1/8
0.15
0.20
0.25

Luttinger Liquid

- Luttinger Model: low energy physics

$$\psi_\sigma(x) = e^{ik_F x}\psi_{1,\sigma}(x) + e^{-ik_F x}\psi_{-1,\sigma}(x)$$



One-Dimensional Interacting Systems

- Luttinger Model: low energy physics

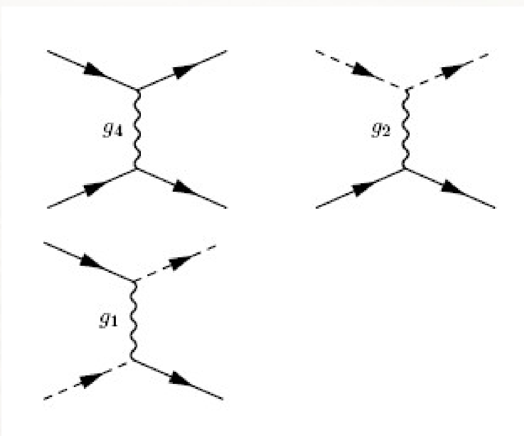
$$\psi_\sigma(x) = e^{ik_F x}\psi_{1,\sigma}(x) + e^{-ik_F x}\psi_{-1,\sigma}(x)$$

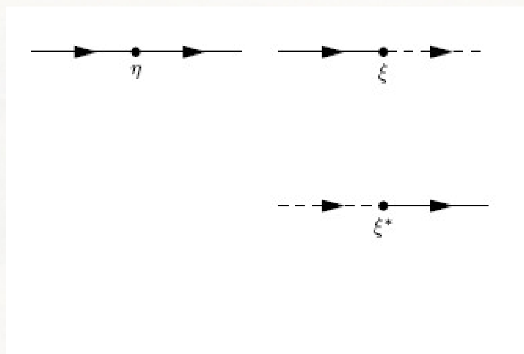
$$H = \int dx \sum_{\nu,\sigma}\Big[i v_F \nu \psi^\dagger_{\nu\sigma}\partial_x \psi_{\nu\sigma} \longleftarrow \text{kinetic term}$$

$$\text{e-e forward scattering} \longrightarrow + \frac{1}{2}\sum_{\nu'}\left[g_4 \delta_{\nu,\nu'} + (g_{2||} - g_{1||})\delta_{\nu,-\nu'}\delta_{\sigma,\sigma'} + g_{2\perp}\delta_{\nu,-\nu'}\delta_{\sigma,-\sigma'}\right]\rho_{\nu\sigma}\rho_{\nu'\sigma'}$$

$$+ \frac{g_{1\perp}}{2}\psi^\dagger_{\nu\sigma}\psi^\dagger_{-\nu,-\sigma}\psi_{\nu,-\sigma}\psi_{-\nu,\sigma} \longleftarrow \text{e-e backward spin-flip scattering}$$

$$\text{impurity forward scattering} \longrightarrow + \eta(x)\rho_{\nu\sigma}\Big] + \sum_\sigma \left(\xi(x)\psi^\dagger_{1,\sigma}\psi_{-1,\sigma} + h.c\right) \longleftarrow \text{impurity backward scattering}$$





CuO
MIT
1D
GS
Fluct
R
C
toc

Bosonization

Renormalization Group

One-Dimensional Interacting Systems

- Luttinger Model: low energy physics

$$\psi_\sigma(x) = e^{ik_F x}\psi_{1,\sigma}(x) + e^{-ik_F x}\psi_{-1,\sigma}(x)$$

$$H = \int dx \sum_{\nu,\sigma}\Big[i v_F \psi^\dagger_{\nu\sigma}\partial_x\psi_{\nu\sigma} \longleftarrow \text{kinetic term}$$

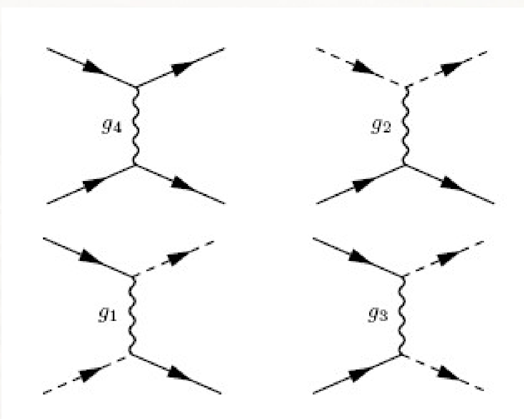
e-e forward scattering $\longrightarrow$

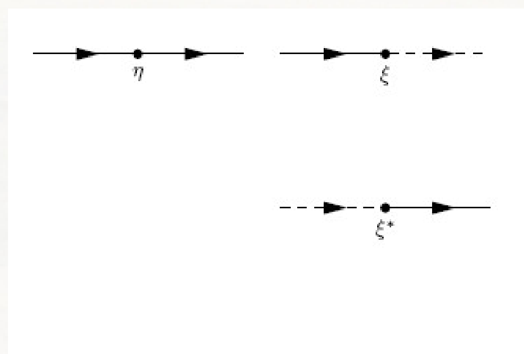
$$+ \frac{1}{2}\sum_{\nu'}\left[g_4\delta_{\nu,\nu'} + (g_{2||} - g_{1||})\delta_{\nu,-\nu'}\delta_{\sigma,\sigma'} + g_{2\perp}\delta_{\nu,-\nu'}\delta_{\sigma,-\sigma'}\right]\rho_{\nu\sigma}\rho_{\nu'\sigma'}$$

$$+ \frac{g_{1\perp}}{2}\psi^\dagger_{\nu\sigma}\psi^\dagger_{-\nu,-\sigma}\psi_{\nu,-\sigma}\psi_{-\nu,\sigma} \longleftarrow$$ e-e backward spin-flip scattering

impurity forward scattering $\longrightarrow$

$$+ \eta(x)\rho_{\nu\sigma}\Big] + \sum_\sigma \left(\xi(x)\psi^\dagger_{1,\sigma}\psi_{-1,\sigma} + h.c\right) \longleftarrow$$ impurity backward scattering





RG equations for Static String

$$\frac{dK_c}{dl} = -\frac{1}{2}\frac{v_c}{v_s}K_c^2 D_b \left[1 - \frac{1}{2}\left(\frac{v_s}{v_c}\right)^{2-K_c}\left(1+\frac{1}{K_c^2}\right)(3-K_c-K_s-y)\right]$$

$$\frac{dv_c}{dl} = -\frac{1}{2}\frac{v_c^2}{v_s}K_c D_b \left[1 + \frac{1}{2}\left(\frac{v_s}{v_c}\right)^{2-K_c}\left(1-\frac{1}{K_c^2}\right)(3-K_c-K_s-y)\right]$$

$$\frac{dK_s}{dl} = -\frac{K_s^2}{2}\left[y^2 + D_b\left(1 - \frac{1}{2}\left(\frac{v_s}{v_c}\right)^{-K_c}\left(1+\frac{1}{K_s^2}\right)(3-K_c-K_s-y) - 2y\left(\frac{v_s}{v_c}\right)^{-K_c}\right)\right]$$

$$\frac{dv_s}{dl} = -\frac{1}{2}v_s K_s D_b \left[1 + \frac{1}{2}\left(\frac{v_s}{v_c}\right)^{-K_c}\left(1-\frac{1}{K_s^2}\right)(3-K_c-K_s-y)\right]$$

$$\frac{dy}{dl} = (2-2K_s)y - D_b\left[1 - \left(\frac{v_s}{v_c}\right)^{-K_c}\left(1 + \frac{y}{4}\left(K_s - \frac{1}{K_s}\right)(3-K_c-K_s-y) + \frac{y}{2}(K_s^2+1) - (K_c-K_s+y)\right)\right]$$

$$\frac{dD_b}{dl} = (3-K_c-K_s-y)D_b$$

$$\frac{dD_f}{dl} = D_f$$

CuO
MIT
1D
GS
Fluct
R
C
toc

RG equations for Static String - Spin Symmetry

$$\frac{dK_c}{dl} = -\frac{1}{2}\frac{v_c}{v_F}K_c^2 D_b \left[1 - \frac{1}{2}\left(\frac{v_F}{v_c}\right)^{2-K_c}\left(1+\frac{1}{K_c^2}\right)\left(2-K_c-\frac{3}{2}y\right)\right]$$

$$\frac{dv_c}{dl} = -\frac{1}{2}\frac{v_c^2}{v_F}K_c D_b \left[1 + \frac{1}{2}\left(\frac{v_F}{v_c}\right)^{2-K_c}\left(1-\frac{1}{K_c^2}\right)\left(2-K_c-\frac{3}{2}y\right)\right]$$

$$\frac{dy}{dl} = -y^2 - D_b\left[1 - \left(\frac{v_s}{v_c}\right)^{-K_c}\left(2-K_c+\frac{y}{2}\right)\right]$$

$$\frac{dD_b}{dl} = \left(2-K_c-\frac{3}{2}y\right)D_b$$

$$\frac{dD_f}{dl} = D_f$$

RG equations for Rigid String - Spin Symmetry

$$\frac{dK_c}{dl} = -\frac{1}{2}\frac{v_c}{v_F}K_c^2\frac{D_b}{\sqrt{1-e^{-\varpi}}}\left[1-\frac{1}{2}\left(\frac{v_F}{v_c}\right)^{2-K_c}\left(1+\frac{1}{K_c^2}\right)\right]$$

$$+\frac{v_F}{4v_c}D_b(K_c^2+1)\left(1-K_c-\frac{3}{2}y\right)\left(\frac{v_F}{v_c}\right)^{-K_c}\left[1+\frac{2}{\varpi}\ln\left(1+\sqrt{1-e^{-\varpi}}\right)\right]$$

$$\frac{dv_c}{dl} = -\frac{1}{2}\frac{v_c^2}{v_F}K_c\frac{D_b}{\sqrt{1-e^{-\varpi}}}\left[1+\frac{1}{2}\left(\frac{v_F}{v_c}\right)^{2-K_c}\left(1-\frac{1}{K_c^2}\right)\right]$$

$$-\frac{v_F}{4}D_b\left(K_c-\frac{1}{K_c}\right)\left(1-K_c-\frac{3}{2}y\right)\left(\frac{v_F}{v_c}\right)^{-K_c}\left[1+\frac{2}{\varpi}\ln\left(1+\sqrt{1-e^{-\varpi}}\right)\right]$$

$$\frac{dy}{dl} = -y^2-\frac{D_b}{\sqrt{1-e^{-\varpi}}}\left[1-\left(\frac{v_F}{v_c}\right)^{-K_c}\right]$$

$$+D_b\left(1-K_c+\frac{y}{2}\right)\left(\frac{v_F}{v_c}\right)^{-K_c}\left[1+\frac{2}{\varpi}\ln\left(1+\sqrt{1-e^{-\varpi}}\right)\right]$$

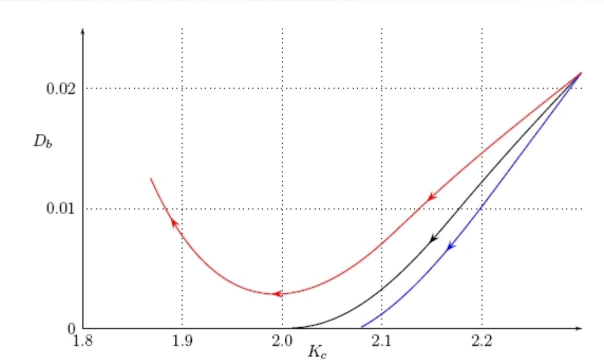
$$\frac{dD_b}{dl} = \left(2-K_c-\frac{3}{2}y\right)D_b$$

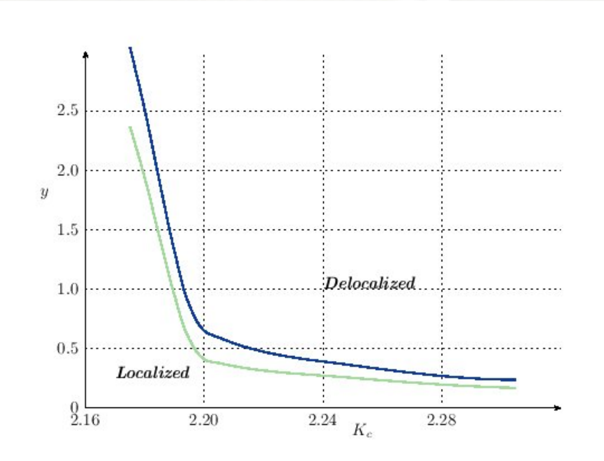
$$\frac{dD_f}{dl} = D_f$$

RG equations for Rigid String - Spin Symmetry

$$
\begin{aligned}
\frac{d\varpi}{dl} &= \varpi + \mathcal{L}D_b \frac{1-\cosh\varpi}{\left(1-e^{-\varpi}\right)^{\frac{3}{2}}} \left[1 - \frac{1}{4}\left(\frac{v_F}{v_c}\right)^{-K_c} \left(\left(\frac{v_F}{v_c}\right)^2 \left(\frac{1}{K_c} - K_c\right) - y\right)\right] \\
&\quad - \frac{\mathcal{L}D_b}{4}(1 - K_c - \frac{3}{2}y)\left(\frac{v_F}{v_c}\right)^{-K_c} \left[\left(\frac{v_F}{v_c}\right)^2 \left(\frac{1}{K_c} - K_c\right) - y\right] \int_{e^{-l}}^{1} dz \frac{1-\cosh(\varpi z)}{z^2\left(1-e^{-\varpi z}\right)^{3/2}} \\
\frac{d\lambda}{dl} &= -\frac{\lambda \mathcal{L}D_b}{\left(1-e^{-\varpi}\right)^{\frac{3}{2}}} \left(\frac{1-\cosh\varpi}{\varpi} + \sinh\varpi\right) \left[1 - \frac{1}{4}\left(\frac{v_F}{v_c}\right)^{-K_c} \left(\frac{v_F}{v_c}\right)^2 \left(\frac{1}{K_c} - K_c\right) - y\right] \\
&\quad + \frac{\lambda\mathcal{L}D_b}{4}(1 - K_c - \frac{3}{2}y)\left(\frac{v_F}{v_c}\right)^{-K_c} \left[\left(\frac{v_F}{v_c}\right)^2 \left(\frac{1}{K_c} - K_c\right) - y\right] \\
&\quad \times \int_{e^{-l}}^{1} dz \left[\frac{1-\cosh(\varpi z)}{\varpi} + z\sinh(\varpi z)\right] \frac{1}{z^2\left(1-e^{-\varpi z}\right)^{3/2}} \\
\frac{d\mathcal{L}}{dl} &= -\mathcal{L}
\end{aligned}
$$

RG Flow for Rigid String





RG equations for Elastic String - Spin Symmetry

$$\frac{dK_c}{dl} = -\frac{1}{2}\frac{v_c}{v_F}K_c^2\lambda \int \frac{dk}{2\pi} D_b(k) \left[1 - \frac{1}{2}\left(\frac{v_F}{v_c}\right)^{2-K_c}\left(1 + \frac{1}{K_c^2}\right)\frac{2 - K_c - \frac{3}{2}y - \frac{k^2\lambda^2}{2\pi}}{1 - \frac{k^2\lambda^2}{2\pi}}\right]$$

$$\frac{dv_c}{dl} = -\frac{1}{2}\frac{v_c^2}{v_F}K_c\lambda \int \frac{dk}{2\pi} D_b(k) \left[1 + \frac{1}{2}\left(\frac{v_F}{v_c}\right)^{2-K_c}\left(1 - \frac{1}{K_c^2}\right)\frac{2 - K_c - \frac{3}{2}y - \frac{k^2\lambda^2}{2\pi}}{1 - \frac{k^2\lambda^2}{2\pi}}\right]$$

$$\frac{dy}{dl} = -y^2 - D_b\left[1 - \left(\frac{v_s}{v_c}\right)^{-K_c}\frac{2 - K_c + \frac{y}{2} - \frac{k^2\lambda^2}{2\pi}}{1 - \frac{k^2\lambda^2}{2\pi}}\right]$$

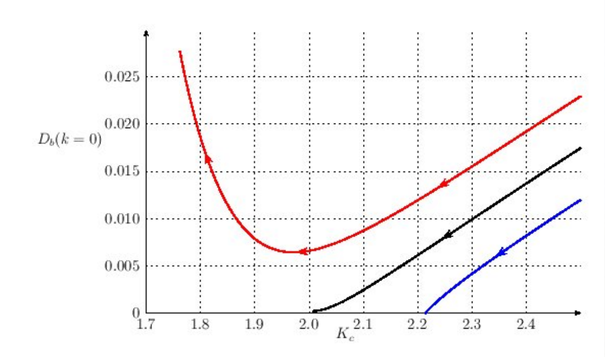
$$\frac{dD_b(k)}{dl} = \left(2 - K_c - \frac{3}{2}y - \frac{k^2\lambda^2}{2\pi}\right) D_b(k)$$

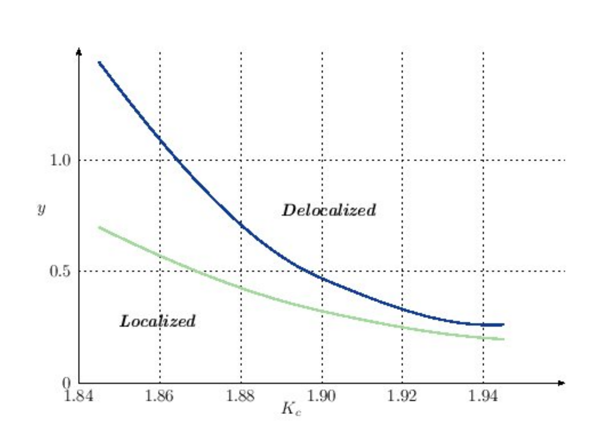
$$\frac{dD_f(k)}{dl} = \left(1 - \frac{k^2\lambda^2}{2\pi}\right) D_f(k)$$

$$\frac{d\lambda}{dl} = -\frac{\lambda^4\eta}{8\pi}\int \frac{dk}{2\pi}k^2 D_b(k)\left[1 - \frac{1}{4}\left(\frac{v_F}{v_c}\right)^{-K_c}\left(\left(\frac{v_F}{v_c}\right)^2\left(\frac{1}{K_c} - K_c\right) - y\right)\right.$$
$$\left.\times\left(1 + \frac{\left(1 - K_c - \frac{3}{2}y - \frac{k^2\lambda^2}{2\pi}\right)\left(1 - e^{-l\left(1 - k^2\lambda^2/2\pi\right)}\right)}{1 - \frac{k^2\lambda^2}{2\pi}}\right)\right]$$

$$\frac{d\eta}{dl} = -\frac{\lambda^3\eta^2}{4\pi}\int \frac{dk}{2\pi}k^2 D_b(k)\left[1 - \frac{1}{4}\left(\frac{v_F}{v_c}\right)^{-K_c}\left(\left(\frac{v_F}{v_c}\right)^2\left(\frac{1}{K_c} - K_c\right) - y\right)\right.$$
$$\left.\times\left(1 + \frac{\left(1 - K_c - \frac{3}{2}y - \frac{k^2\lambda^2}{2\pi}\right)\left(1 - e^{-l\left(1 - k^2\lambda^2/2\pi\right)}\right)}{1 - \frac{k^2\lambda^2}{2\pi}}\right)\right]$$

RG Flow for Elastic String





RG equations for Floppy String - Spin Symmetry

$$\frac{dK_c}{dl} = -\frac{1}{2}\frac{v_c}{v_F}K_c^2 D_b\left[1-\frac{1}{2}\left(\frac{v_F}{v_c}\right)^{2-K_c}\left(1+\frac{1}{K_c^2}\right)\frac{1+3n-2n\left(K_c+\frac{3}{2}y\right)}{1+n}\right]$$

$$\frac{dv_c}{dl} = -\frac{1}{2}\frac{v_c^2}{v_F}K_c D_b\left[1+\frac{1}{2}\left(\frac{v_F}{v_c}\right)^{2-K_c}\left(1-\frac{1}{K_c^2}\right)\frac{1+3n-2n\left(K_c+\frac{3}{2}y\right)}{1+n}\right]$$

$$\frac{dy}{dl} = -y^2 - D_b\left[1-\left(\frac{v_s}{v_c}\right)^{-K_c}\frac{1+3n-2n\left(K_c+\frac{y}{2}\right)}{1+n}\right]$$

$$\frac{dD_b}{dl} = \left(\frac{3n+1}{2n} - K_c - \frac{3}{2}y\right)D_b$$

$$\frac{dD_f}{dl} = \frac{n+1}{2n}D_f$$

$$\frac{ds}{dl} = (1-n)\,s - \frac{n-1}{4\Gamma\left(\frac{1}{n}\right)}D_b s^{\frac{1}{n}+1}\left[1-\frac{1}{4}\left(\frac{v_F}{v_c}\right)^{-K_c}\left(\left(\frac{v_F}{v_c}\right)^2\left(\frac{1}{K_c}-K_c\right)-y\right)\right.$$
$$\left.\times\left(1+\frac{2n(1-K_c-(3/2)y)\left(1-e^{-l(3+2n)/2n}\right)}{3+2n}\right)\right]$$

$$\frac{d\lambda}{dl} = \frac{n-1}{4\Gamma\left(\frac{1}{n}\right)(n-3)}D_b s^{\frac{1}{n}}\lambda\left[1-\frac{1}{4}\left(\frac{v_F}{v_c}\right)^{-K_c}\left(\left(\frac{v_F}{v_c}\right)^2\left(\frac{1}{K_c}-K_c\right)-y\right)\right.$$
$$\left.\times\left(1+\frac{2n(1-K_c-(3/2)y)\left(1-e^{-l(3+2n)/2n}\right)}{3+2n}\right)\right]$$

CuO
MIT
1D
GS
Fluct
R
C
toc

RG Flow for Floppy String