# Effects of disorder in fluctuating one-dimensional interacting systems by Uri London supervisor: Dr. Dror Orgad



### Metal-Insulator Transition

Metal Insulator  $\sigma(T=0) > 0$   $\sigma(T=0) = 0$ 

- Anderson transition localization induced by disordered.
- Mott transition localization induced by interactions.

#### 1D non-interacting systems:

- exact solution.
- always insulating.
- localization length  $\sim$  mean free path.

2D non-interacting systems:

- scaling theory.
- always insulating.
- long localization length .

3D non-interacting systems:

- scaling theory.
- Metal-Insulator transition depends on disorder strength

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• Luttinger Model: low energy physics

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• weak random potential  $\xi(x)$  uncorrelated in space

$$\overline{\xi^*(x)\xi(x')} = D\delta(x - x')$$

#### Renormalization

perturbation in g (e-e backward spin-flip scattering) and D (backward disorder) real-space renormalization group

> electron interactions: attractive non-interactive repulsive K > 1K = 1K < 10.12 0.10 0.08 D Delocalized 0.06 0.04 Localized 0.02 0 1.8 2.41.6 2.02.2K

> > T. Giamarchi, H. J. Schulz, PRB 37:325, 1988



# Fluctuations

Three models of string fluctuations

Rigid string



Elastic string



Floppy string



$$L_u = \frac{M}{2} (\partial_\tau u)^2 + \frac{M\omega_0^2}{2} u^2$$
$$\langle u^2 \rangle = \lambda^2$$
$$\lambda = \frac{1}{\sqrt{2M\omega_0}}$$

$$L_{u} = \int_{0}^{L_{x}} dx \left[ \frac{\rho}{2} \left( \partial_{\tau} u \right)^{2} + \frac{\sigma}{2} \left( \partial_{x} u \right)^{2} \right]$$
$$\left\langle \left( \frac{\partial u}{\partial x} \right)^{2} \right\rangle = \frac{\lambda}{2\pi\alpha}$$
$$\lambda = \frac{1}{\left(\sigma\rho\right)^{\frac{1}{4}}}$$

$$L_{u} = \int_{0}^{L} ds \left[ \frac{\rho}{2} \left( \partial_{\tau} u \right)^{2} + \frac{\gamma}{2} \left( \partial_{s}^{2} u \right)^{2} \right]$$
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The critical point can be shifted to  $K^* \leq 2$  for different types of disorder average.

- Floppy string the critical point is shifted to  $K^* = 7/4$ .
- Extension of the floppy model: Elastic Energy  $= \int ds \gamma \partial_s^n u$ , results in shift of the critical point to  $K^* = 3/2$  when  $n \to \infty$ .

# Conclusions

• Fluctuations tend to delocalize, as expected.

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- Further investigation coupled fluctuating strings (imitating better quasi-one dimensionality)