Action

The boson action:

$$
\begin{aligned}
S &= \int dtdx \sum_{\nu=\rho,\sigma} \frac{1}{2\pi K_\nu}\left[\frac{1}{u_\nu}\left(\partial_t\phi_\nu\right)^2 - u_\nu\left(\partial_x\phi_\nu\right)^2\right] + \frac{2g_{1\perp}}{(2\pi\alpha)^2}\cos\sqrt{8}\phi_\sigma + \\
&+ \frac{\sqrt{2}}{\pi}\eta(x)\partial_x\phi_\rho - \frac{1}{\pi\alpha}\left(\xi(x)e^{i(\sqrt{2}\phi_\rho+2k_F x)}\cos\sqrt{2}\phi_\sigma + h.c\right)
\end{aligned}
$$

By making the shift $\phi_\rho \to \phi_\rho + \tilde{\eta}$ where $\tilde{\eta}(x) = \frac{\sqrt{2}K_\rho}{u_\rho}\int^x \eta(z)dz$ we eliminate the forward term. By rescaling $\phi_{\rho,\sigma} \to \frac{\phi_{\rho,\sigma}}{\sqrt{K_{\rho,\sigma}}}$ we get a free term and an interaction term:

$$
\begin{aligned}
S_0 &= \int dtdx \sum_{\nu=\rho,\sigma} \frac{1}{2\pi}\left[\frac{1}{u_\nu}\left(\partial_t\phi_\nu\right)^2 - u_\nu\left(\partial_x\phi_\nu\right)^2\right] \\
S_I &= \int dtdx \left[\frac{2g_{1\perp}}{(2\pi\alpha)^2}\cos\sqrt{8K_\sigma}\phi_\sigma - \frac{1}{\pi\alpha}\left(\xi(x)e^{i(\sqrt{2K_\rho}\phi_\rho+2k_F x)}\cos\sqrt{2K_\sigma}\phi_\sigma + h.c\right)\right]
\end{aligned}
$$

Making a Wick rotation: $y = iu_\sigma t$

$$
\begin{aligned}
S_0 &= -\frac{1}{2\pi i}\int dydx \left[\left(\nabla\phi_\sigma\right)^2 + \left(\tilde{\nabla}\phi_\rho\right)^2\right] \\
S_I &= \frac{1}{i}\int dydx \left[\frac{g_{1\perp}}{2\pi^2 u_\sigma\alpha^2}\cos\sqrt{8K_\sigma}\phi_\sigma - \frac{1}{\pi u_\sigma\alpha}\left(\xi(x)e^{i(\sqrt{2K_\rho}\phi_\rho+2k_F x)}\cos\sqrt{2K_\sigma}\phi_\sigma + h.c\right)\right]
\end{aligned}
$$

with $\nabla = (\partial_y, \partial_x)$ $\quad \tilde{\nabla} = (\sqrt{\frac{u_\sigma}{u_\rho}}\partial_y, \sqrt{\frac{u_\rho}{u_\sigma}}\partial_x)$

The backward disorder potential $\xi(x)$ has a gaussian distributon $P_\xi = exp\left(-D_\xi^{-1}\int \xi^*(x)\xi(x)dx\right)$. Averging the disorder

$$
\begin{aligned}
\langle Z\rangle_\xi &= \int D\phi \int D\xi(x)D\xi^*(x)e^{-iS} \\
\langle Re\xi(x)Re\xi(y)\rangle &= \frac{D_\xi}{2}\delta(x-y)
\end{aligned}
$$

finally

$$\begin{aligned}
S_0 &= -\frac{1}{2\pi i}\int d^2x\left[(\nabla\phi_\sigma)^2 + \left(\tilde{\nabla}\phi_\rho\right)^2\right] \\
S_I &= \frac{1}{i}\int d^2x\left[g\cos\sqrt{8K_\sigma}\phi_\sigma - \right. \\
& \left. -D\int d^2y\delta^{(1)}(x_1-y_1)\cos\sqrt{2K_\sigma}\phi_\sigma(x)\cos\sqrt{2K_\sigma}\phi_\sigma(y)\cos\sqrt{2K_\rho}\left(\phi_\rho(x)-\phi_\rho(y)\right)\right]
\end{aligned}$$

with

$$\begin{aligned}
g &= \frac{g_{1\perp}}{2\pi^2 u_\sigma \alpha^2} = \frac{y}{2\pi\alpha^2} \\
D &= \frac{D_\xi}{(\pi u_\sigma \alpha)^2} = \frac{1}{2\pi\alpha^3}\left(\frac{u_\rho}{u_\sigma}\right)^{K_\rho}\mathcal{D}
\end{aligned}$$

Free propogator

the free prpogator is

$$G_{\sigma,\rho}(x-y) = \int D\phi_{\rho,\sigma}\phi_{\rho,\sigma}(x)\phi_{\rho,\sigma}(y)e^{iS_0}$$

since ϕ is periodic in the length of the sample and in a time period of one over the temperature

$$S_0 = -\frac{1}{2\pi i}\int d^2x\nabla\phi^2 = \frac{1}{2\pi i}\int d^2x\phi\nabla^2\phi$$

thus we get a gaussian integral

$$\begin{aligned}
G(x-y) &= \int D\phi\phi(x)\phi(y)e^{-\frac{1}{2}\int d^2x d^2y\phi(x)G^{-1}(x,y)\phi(y)} \\
G^{-1}(x-y) &= -\frac{1}{\pi}\delta(x-y)\nabla_x^2
\end{aligned}$$

In momentum space the propogator reads

$$G(k_1, k_2) = \pi \frac{(2\pi)^2 \delta(k_1 + k_2)}{k_1^2}$$

The RG procedure involves cutting momentum

$$G(x, y) = \int \frac{d^2k_1}{(2\pi)^2} \frac{d^2k_2}{(2\pi)^2} e^{i(k_1 \cdot x + k_2 \cdot y)} G(k_1, k_2) = \pi \int_{\Lambda - d\Lambda < k < \Lambda} \frac{d^2k}{(2\pi)^2} \frac{e^{ik \cdot (x-y)}}{k^2} = \frac{J_0(\Lambda(x-y))}{2\Lambda} d\Lambda$$

Define

$$A_{\sigma,\rho} = e^{-4K_{\sigma,\rho} G(x)}$$

Cumulant Expansion

The procedure is to take the partiton function

$$Z = \int D\phi e^{iS} = \int D\phi_\Lambda e^{iS_0} \left\langle e^{iS_I} \right\rangle_h = \int D\phi_\Lambda exp \left[-\nabla\phi^2 + \langle g \rangle - \langle D \rangle + \frac{\left\langle g^2 \right\rangle - \langle g \rangle^2}{2} - \frac{\langle gD \rangle - \langle g \rangle \langle D \rangle}{2} \right]$$

Thus we look at the moments of iS_I in the exponent.

C.E to 1st order in g

$$i\delta S^{(1g)} = g \int d^2x \left\langle \cos \sqrt{8K_\sigma} \left(\phi_\sigma(x) + h_\sigma(x)\right) \right\rangle_{h_\sigma}$$

$$\left\langle \cos \sqrt{8K_\sigma} \left(\phi_\sigma(x) + h_\sigma(x)\right) \right\rangle_{h_\sigma} = \frac{1}{2} \left(\left\langle e^{i\sqrt{8K_\sigma} h_\sigma(x)} \right\rangle e^{i\sqrt{8K_\sigma} \phi_\sigma(x)} + h.c \right)$$

since

$$\left\langle e^{i\sqrt{8K_\sigma}h_\sigma(x)}\right\rangle = e^{-4K_\sigma\langle h_\sigma(x)h_\sigma(x)\rangle} = A_\sigma(0)$$

then

$$\delta S^{(1g)} = gA_\sigma(0)\int d^2x \cos\sqrt{8K_\sigma}\phi_\sigma(x)$$

C.E to 2nd order in g

$$\begin{aligned} i\delta S^{(2g)} &= \frac{g^2}{2}\int d^2x d^2y \left[\left\langle \cos\sqrt{8K_\sigma}\left(\phi_\sigma(x)+h_\sigma(x)\right)\cos\sqrt{8K_\sigma}\left(\phi_\sigma(y)+h_\sigma(y)\right)\right\rangle_{h_\sigma} - \right. \\ & \left. - \left\langle \cos\sqrt{8K_\sigma}\left(\phi_\sigma(x)+h_\sigma(x)\right)\right\rangle_{h_\sigma}\left\langle \cos\sqrt{8K_\sigma}\left(\phi_\sigma(y)+h_\sigma(y)\right)\right\rangle_{h_\sigma}\right] \end{aligned}$$

$$\begin{aligned} \langle cos\cdot cos\rangle &= \frac{1}{4}\sum_{\epsilon_1,\epsilon_1=\pm 1} e^{-4K_\sigma\left\langle(\epsilon_1 h_\sigma(x)+\epsilon_2 h_\sigma(y))^2\right\rangle} e^{i\sqrt{8K_\sigma}(\epsilon_1\phi_\sigma(x)+\epsilon_2\phi_\sigma(y))} \\ &= \frac{A_\sigma^2(0)}{2}\left[A_\sigma^2(x-y)\cos\sqrt{8K_\sigma}\left(\phi_\sigma(x)+\phi_\sigma(y)\right) + A_\sigma^{-2}(x-y)\cos\sqrt{8K_\sigma}\left(\phi_\sigma(x)-\phi_\sigma(y)\right)\right] \\ \langle cos\rangle\langle cos\rangle &= A_\sigma^2(0)\cos\sqrt{8K_\sigma}\phi_\sigma(x)\cos\sqrt{8K_\sigma}\phi_\sigma(y) = \\ &= \frac{A_\sigma^2(0)}{2}\left[\cos\sqrt{8K_\sigma}\left(\phi_\sigma(x)+\phi_\sigma(y)\right) + \cos\sqrt{8K_\sigma}\left(\phi_\sigma(x)-\phi_\sigma(y)\right)\right] \end{aligned}$$

By changing coordinates

$$z = \frac{1}{2}(x+y) \quad \xi = x-y$$

we get

$$i\delta S^{(2g)} = \frac{1}{4}g^2A_\sigma^2(0)\int d^2z d^2\xi\left[\left(A_\sigma^2(\xi)-1\right)\cos 2\sqrt{8K_\sigma}\phi_\sigma(z) + \left(A_\sigma^{-2}(\xi)-1\right)\cos\sqrt{8K_\sigma}\xi\nabla_z\phi_\sigma(z)\right]$$

Deine

$$
\begin{aligned}
a_1 &= \int d^2x \left(A_\sigma^2(x) - 1\right) \\
a_2 &= \int d^2x x^2 \left(A_\sigma^{-2}(x) - 1\right) \\
a_3 &= \int d^2x \left(A_\sigma^{-2}(x) - 1\right)
\end{aligned}
$$

and write the cosine to second order we have

$$
i\delta S^{(2g)} = \frac{1}{4} g^2 A_\sigma^2(0) \int d^2x \left[a_3 - 4a_2 K_\sigma \left(\nabla\phi_\sigma(x)\right)^2 + a_1 \cos 2\sqrt{8K_\sigma}\phi_\sigma(x)\right]
$$

C.E to 1st order in D

$$
\begin{aligned}
i\delta S^{(1D)} &= -D \int d^2x d^2x' \delta(x-x') \Big\langle \cos\sqrt{2K_\sigma}\left(\phi_\sigma(x) + h_\sigma(x)\right) \cos\sqrt{2K_\sigma}\left(\phi_\sigma(x') + h_\sigma(x')\right) \times \\
&\quad \times \cos\sqrt{2K_\rho}\left(\phi_\rho(x) - \phi_\rho(x') + h_\rho(x) - h_\rho(x')\right) \Big\rangle_h
\end{aligned}
$$

Since spin and charge are independed variables we can averege independedly. The spin averege is like 2nd order in g only half the argument and the charge avrege is like first order with $h \to \Delta h$:

$$
\begin{aligned}
\langle cos \cdot cos \rangle_\sigma &= \frac{A_\sigma^{\frac{1}{2}}(0)}{2} \left[A_\sigma^{\frac{1}{2}}(x-y) \cos\sqrt{2K_\sigma}\left(\phi_\sigma(x) + \phi_\sigma(x')\right) + A_\sigma^{-\frac{1}{2}}(x-y) \cos\sqrt{2K_\sigma}\left(\phi_\sigma(x) - \phi_\sigma(x')\right)\right] \\
\langle cos(\Delta) \rangle_\rho &= A_\rho^{\frac{1}{2}}(0) A_\rho^{-\frac{1}{2}}(x-y) \cos\sqrt{2K_\rho}\left(\phi_\rho(x) - \phi_\rho(x')\right)
\end{aligned}
$$

Thus

$$
\begin{aligned}
i\delta S^{(1D)} &= \frac{1}{2}A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)D\int d^2xd^2x'\delta(x-x')\left[A_\sigma^{\frac{1}{2}}(x-x')A_\rho^{-\frac{1}{2}}(x-x')\cos\sqrt{2K_\sigma}\left(\phi_\sigma(x)+\phi_\sigma(x')\right)+\right.\\
&\left.+A_\sigma^{-\frac{1}{2}}(x-x')A_\rho^{-\frac{1}{2}}(x-x')\cos\sqrt{2K_\sigma}\left(\phi_\sigma(x)-\phi_\sigma(x')\right)\right]\cos\sqrt{2K_\rho}\left(\phi_\rho(x)-\phi_\rho(x')\right)=\\
&= \frac{1}{2}A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)D\int d^2xd^2x'\delta(x-x')\left[\left(A_\sigma^{\frac{1}{2}}(x-x')A_\rho^{-\frac{1}{2}}(x-x')-1\right)\cos\sqrt{2K_\sigma}\left(\phi_\sigma(x)+\phi_\sigma(x')\right)+\right.\\
&\left.+\left(A_\sigma^{-\frac{1}{2}}(x-x')A_\rho^{-\frac{1}{2}}(x-x')-1\right)\cos\sqrt{2K_\sigma}\left(\phi_\sigma(x)-\phi_\sigma(x')\right)\right]\cos\sqrt{2K_\rho}\left(\phi_\rho(x)-\phi_\rho(x')\right)+\\
&+A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)D\int d^2xd^2x'\delta(x-x')\cos\sqrt{2K_\sigma}\phi_\sigma(x)\cos\sqrt{2K_\sigma}\phi_\sigma(y)\cos\sqrt{2K_\rho}\left(\phi_\rho(x)-\phi_\rho(x')\right)
\end{aligned}
$$

The first integral becomes with the change of coordinates

$$
\begin{aligned}
&\frac{1}{2}A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)D\int d^2zd^2\xi\delta(\xi_1)\left[\left(A_\sigma^{\frac{1}{2}}(\xi)A_\rho^{-\frac{1}{2}}(\xi)-1\right)\cos 2\sqrt{2K_\sigma}\phi_\sigma(z)+\right.\\
&\qquad\left.+\left(A_\sigma^{-\frac{1}{2}}(\xi)A_\rho^{-\frac{1}{2}}(\xi)-1\right)\cos\sqrt{2K_\sigma}\xi\nabla_z\phi_\sigma(z)\right]\cos\sqrt{2K_\rho}\xi\nabla_z\phi_\rho(z)
\end{aligned}
$$

Define

$$
\begin{aligned}
a_4 &= \int d^2x\delta(x_1)\left(A_\sigma^{\frac{1}{2}}(x)A_\rho^{-\frac{1}{2}}(x)-1\right)\\
a_5 &= \int d^2x\delta(x_1)x^2\left(A_\sigma^{\frac{1}{2}}(x)A_\rho^{-\frac{1}{2}}(x)-1\right)\\
a_6 &= \int d^2x\delta(x_1)\left(A_\sigma^{-\frac{1}{2}}(x)A_\rho^{-\frac{1}{2}}(x)-1\right)\\
a_7 &= \int d^2x\delta(x_1)x^2\left(A_\sigma^{-\frac{1}{2}}(x)A_\rho^{-\frac{1}{2}}(x)-1\right)\\
a_8 &= \int d^2x\delta(x_1)x^4\left(A_\sigma^{-\frac{1}{2}}(x)A_\rho^{-\frac{1}{2}}(x)-1\right)
\end{aligned}
$$

and write the cosines to 1st order

$$
\begin{aligned}
i\delta S^{(1D)} &= -\frac{1}{2}A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)D\int d^2x\left[a_6-a_7K_\sigma\left(\nabla\phi_\sigma\right)^2-a_7K_\rho\left(\nabla\phi_\rho\right)^2+a_4\cos\sqrt{8K_\sigma}\phi_\sigma+\right.\\
&+2\int d^2x'\delta(x-x')\cos\sqrt{2K_\sigma}\phi_\sigma(x)\cos\sqrt{2K_\sigma}\phi_\sigma(x')\cos\sqrt{2K_\rho}\left(\phi_\rho(x)-\phi_\rho(x')\right)-\\
&\left.-a_5K_\rho\left(\nabla\phi_\rho\right)^2\cos\sqrt{8K_\sigma}\phi_\sigma+a_8K_\sigma K_\rho\left(\nabla\phi_\sigma\right)^2\left(\nabla\phi_\rho\right)^2\right]
\end{aligned}
$$

C.E to 2nd order in gD

$$
\begin{aligned}
i\delta S^{(2gD)} &= gD\int d^2xd^2x'd^2x''\delta(x'-x'')\left[\left\langle\cos\sqrt{8K_\sigma}\left(\phi_\sigma+h_\sigma\right)\cos\sqrt{2K_\sigma}\left(\phi_\sigma+h_\sigma\right)\cos\sqrt{2K_\sigma}\left(\phi_\sigma+h_\sigma\right)\times\right.\right.\\
&\times\cos\sqrt{2K_\rho}\left(\Delta\phi_\rho+\Delta h_\rho\right)\Big\rangle-\left\langle\cos\sqrt{8K_\sigma}\left(\phi_\sigma+h_\sigma\right)\right\rangle\Big\langle\cos\sqrt{2K_\sigma}\left(\phi_\sigma+h_\sigma\right)\cos\sqrt{2K_\sigma}\left(\phi_\sigma+h_\sigma\right)\\
&\times\cos\sqrt{2K_\sigma}\left(\phi_\sigma+h_\sigma\right)\cos\sqrt{2K_\rho}\left(\Delta\phi_\rho+\Delta h_\rho\right)\Big\rangle
\end{aligned}
$$

This term is equivalent to 3rd order of g, but we can extract a 2nd order term using $\cos\sqrt{8K_\sigma}\phi_\sigma=2\cos^2\sqrt{2K_\sigma}\phi_\sigma-1$

$$
\begin{aligned}
i\delta S^{(2gD)} &= \langle gD\rangle-\langle g\rangle\langle D\rangle=\langle gD\rangle-2gA_\sigma(0)\int\cos^2\sqrt{2K_\sigma}\phi_\sigma(\ldots)+\\
&+A_\sigma^{\frac{3}{2}}(0)A_\rho^{\frac{1}{2}}(0)gD\int d^2xd^2x'\delta(x-x')\cos\sqrt{2K_\sigma}\phi_\sigma(x)\cos\sqrt{2K_\sigma}\phi_\sigma(y)\cos\sqrt{2K_\rho}\Delta\phi_\rho
\end{aligned}
$$

Scaling transformations

$$
\begin{aligned}
iS+i\delta S &= (low\,modes)-\frac{1}{2\pi}\int d^2x\left(1+2\pi a_2A_\sigma^2(0)g^2K_\sigma-\pi a_7A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)DK_\sigma\right)\left(\nabla\phi_\sigma\right)^2-\\
&-\frac{1}{2\pi}\int d^2x\left(\left(\tilde{\nabla}\phi_\rho\right)^2-\pi a_7A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)DK_\rho\left(\nabla\phi_\rho\right)^2\right)+\\
&+\left(A_\sigma(0)g-\frac{1}{2}a_4A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)D\right)\int d^2x\cos\sqrt{8K_\sigma}\phi_\sigma(x)-\\
&-A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)D\int d^2x'\delta(x-x')\cos\sqrt{2K_\sigma}\phi_\sigma(x)\cos\sqrt{2K_\sigma}\phi_\sigma(x')\cos\sqrt{2K_\rho}\left(\phi_\rho(x)-\phi_\rho(x')\right)+\\
&+(high\,modes)
\end{aligned}
$$

A closer look on the charge term:

$$
\begin{aligned}
\left(\tilde{\nabla}\phi_\rho\right)^2-sDK_\rho\left(\nabla\phi_\rho\right)^2 &= \left(\frac{u_\sigma}{u_\rho}\right)\left(1-sDK_\rho\frac{u_\rho}{u_\sigma}\right)\left(\partial_y\phi_\rho\right)^2+\left(\frac{u_\rho}{u_\sigma}\right)\left(1-sDK_\rho\frac{u_\sigma}{u_\rho}\right)\left(\partial_x\phi_\rho\right)^2=\\
&= \frac{u_\sigma}{\left(1-sDK_\rho\frac{u_\sigma}{u_\rho}\right)}\frac{\left(1-sDK_\rho\frac{u_\sigma}{u_\rho}\right)\left(1-sDK_\rho\frac{u_\rho}{u_\sigma}\right)}{u_\rho}\left(\partial_y\phi_\rho\right)^2+\\
&+\frac{\left(1-sDK_\rho\frac{u_\sigma}{u_\rho}\right)}{u_\sigma}u_\rho\left(\partial_x\phi_\rho\right)^2
\end{aligned}
$$

We recover the original action with the following scale transformations:

$$
\begin{aligned}
g &\rightarrow A_\sigma(0)g - \frac{1}{2}a_4 A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)D \\
D &\rightarrow A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)D \\
\phi_\sigma &\rightarrow \left(1 + 2\pi a_2 A_\sigma^2(0)g^2 K_\sigma - \pi a_7 A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)DK_\sigma\right)^{\frac{1}{2}} \phi_\sigma \\
K_\sigma &\rightarrow \left(1 + 2\pi a_2 A_\sigma^2(0)g^2 K_\sigma - \pi a_7 A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)DK_\sigma\right)^{-1} K_\sigma \\
u_\sigma &\rightarrow \left(1 - \pi a_7 A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)DK_\rho \frac{u_\sigma}{u_\rho}\right)^{-1} u_\sigma \\
u_\rho &\rightarrow \left[\left(1 - \pi a_7 A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)DK_\rho \frac{u_\sigma}{u_\rho}\right)\left(1 - \pi a_7 A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)DK_\rho \frac{u_\rho}{u_\sigma}\right)\right]^{-\frac{1}{2}} u_\rho \\
\phi_\rho &\rightarrow \left[\left(1 - \pi a_7 A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)DK_\rho \frac{u_\sigma}{u_\rho}\right)\left(1 - \pi a_7 A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)DK_\rho \frac{u_\rho}{u_\sigma}\right)\right]^{\frac{1}{4}} \phi_\rho \\
K_\rho &\rightarrow \left[\left(1 - \pi a_7 A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)DK_\rho \frac{u_\sigma}{u_\rho}\right)\left(1 - \pi a_7 A_\sigma^{\frac{1}{2}}(0)A_\rho^{\frac{1}{2}}(0)DK_\rho \frac{u_\rho}{u_\sigma}\right)\right]^{-\frac{1}{2}} K_\rho
\end{aligned}
$$