

ADVANCED ASTROPHYSICS (77938) - FINAL EXAM

This is a take home exam. You are allowed to use any non-living material you wish (e.g., notes, the internet, books, fortune cookies, but not other humans). If you have any self respect, it will remain this way. You should answer 4 out of 6 questions. As is the case in real life, not all information is necessarily given. In such a case, you should make reasonable assumptions. Note, I will be abroad between 12/7/2012 and 2/8/2012, but will be available by mail or skype (nirshaviv) for questions¹.

Due Date²: Aug 5th, 2012

1. Evolution of Very Massive Stars

Let us consider the evolution of a very massive star in the Hydrogen burning phase. Let us also assume that it can be approximated as an $n = 3$ polytrope with a spatially independent gas to radiation pressure ratio, that it is well mixed all the time, and that the opacity is that of Thomson scattering. As it burns hydrogen, it will change its molecular weight, and with it, shine *closer* to the Eddington luminosity.

Moreover, let us also assume that above a given fraction of the Eddington luminosity (say, $\Gamma_{crit} = 0.5$), the star sheds mass very rapidly, but below this value it doesn't. This implies that we can consider two "phases" for Hydrogen burning:

- (i) Initially, the molecular weight is low, and it sheds no mass. Evolution takes place while keeping a fixed mass.
- (ii) Once $\Gamma = \Gamma_{crit}$, the star evolves while shedding mass, but keeping Γ fixed at the critical value.

Given the above, answer the following.

- a. Assuming that initially $X = 0.7$ and $Z = 0.02$, what is the range of masses for which the star passes through both phases? (i.e., too low of a mass, and the star is never above Γ_{crit} , too high a mass, and the star begins above Γ_{crit}).
- b. Plot the track of a star within the above range on the HR diagram (i.e., choose some specific mass and plot the track with actual numbers).

¹Don't wake me up. I will be 7 hrs behind Israel, in the US east coast.

²Any exam submitted by this date will be graded and the grade passed within a week, anything submitted later might be graded only during the holidays, and submitted as a "moed beit".

2. An unnamed bound to the size of stars

Stars with a very large radius have a low escape velocity. This means that the thermal wind mass loss is going to be relatively high. Let us consider a star with given M , R and L .

- Calculate the mass loss rate that this star will have.
- Assuming it burns some advanced fuel (e.g., triple α , which releases 7 MeV per carbon formed), and that it can at most burn perhaps 10% of its mass this way. What is (approximately) the largest radius it can have before thermal wind mass loss becomes important?
- Plot a line in the HR diagram above which one doesn't expect to find stars with $1M_{\odot}$ and with $10M_{\odot}$.

3. Why is Earth blue? What is the color of a D_2O planet?

If you google the first question, you will find that it is blue because water absorbs the red spectrum more than it does the blue³. However, this is only part of the answer. The other part is scattering (without it, the oceans would have been black!)

- Consider a semi-infinite medium having given absorption and scattering coefficients. If the boundary conditions is some radiative flux entering the medium on one side, but none from the other side (at infinity), calculate how much of the radiation is emitted back.⁴
- Using the absorption and scattering coefficients of water, plot the reflectivity of a deep ocean as a function of wavelength. For convenience, the coefficients can be found here:
<http://www.wetlabs.com/iopdescript/Table%201.pdf>.
So, why are the oceans blue?
- Suppose a low mass planet lost all of its hydrogen, and has a deep D_2O ocean⁵. Because D_2O is heavier, the absorption band at 750nm is at around 1000nm, and the visible spectrum absorption coefficient is relatively flat, at around the level of 0.01 m^{-1} (i.e., from red to blue). What would the color of the ocean be then? How will the bond albedo change? (i.e., if you average over the solar spectrum, by how much will the energy reflectivity change?)
- The absorption spectrum of Hemoglobin is found at
<http://omlc.ogi.edu/spectra/hemoglobin/summary.html>,
see also graph at:
<http://omlc.ogi.edu/spectra/hemoglobin/>.
Estimate at which depth, the blood of a diver looks green. (From experience, it does!) Will it look green at any depth when diving on the D_2O planet?

³You will also find that it is because it reflects the sky, but this is wrong.

⁴Following the cloud problem we solved, it is equivalent to asking how much is emitted by the clouds back to space, but for a cloud system which is semi-infinite and includes absorption too.

⁵Actually, if Earth would have lost its hydrogen, the amount of heavy water left would be less than a meter deep, so it is unrealistic to get a deep D_2O ocean.

4. Spectrum of a thin-disk accretion system

Let us consider a binary system which includes a WD with a mass of $1 M_{\odot}$, a companion having a mass of $0.5 M_{\odot}$, and a separation of $a = 10^{12} \text{cm}$. The companion transfers mass onto the WD at a rate of $\dot{M} = 10^{-9} M_{\odot}/\text{yr}$. Calculate the emitted spectrum (namely, plot a graph of νF_{ν} , which is the energy per logarithmic interval. F_{ν} is of course the specific intensity in units of $\text{erg sec}^{-1} \text{Hz}^{-1}$). Note that there are *several* components to this system. Some of the parameters should be approximated, e.g., the outer radius of the accretion disk. It is of course highly recommended to solve the relevant integrals numerically.

5. Spectrum of a jet

We now consider a jet formed during the accretion onto a black hole, and we'll try to model (very roughly!!) the emitted radiation. We shall assume the following approximations. The jet has a fixed opening angle $\Delta\theta$, and a fixed bulk Lorentz factor Γ . We assume that electrons are accelerated at the base of jet, and have a power law distribution $dN/dE \propto E^{-p}$ at relativistic energies, in the *rest frame* of the jet. We shall also assume that there is a disordered magnetic field, which is decreasing with radius because of the jet opening angle. Last, we shall assume that the electrons do not cool, that is, that the energy that they emit is negligible compared to the total energy they have.

- The electrons will emit synchrotron radiation. Estimate the form that the spectrum will have in the jet's frame of reference. How will this spectrum look to an observer located at an angle α which satisfies: $1/\Gamma > \alpha \sim \Delta\theta$? What happens for a high electron density and low frequencies?
- A second important component will be inverse-Compton scattering of the synchrotron photons by the electrons. How will that spectrum look like in the jet's frame of reference, and to the above observer?

6. Age of cosmic rays

Let us now consider the behavior of ^{10}Be , which is a secondary radioactive isotope (i.e., it is formed together with the stable isotope ^9Be , as primary nuclei propagate from the source and undergo spallation. We assume that there is a typical time τ_{esc} for cosmic rays to escape the galaxy, and that there is a typical time τ_{ps} for primaries to undergo spallation into the secondary (say, half to ^9Be and half to ^{10}Be), and τ_x for either primaries or secondaries to be destroyed by spallation (and disappear).

- In a steady state production of primary particles (i.e., the standard leaky box model), calculate the ratio between ^{10}Be and ^9Be . Note that this relation can be inverted. i.e., one can define the age of the cosmic rays as $a \equiv 1/(\tau_{esc}^{-1} + \tau_x^{-1})$ and write it as a function of the Beryllium isotope ratio, which is measurable.
- In a second model, one assumes that all cosmic rays were formed in a burst (e.g., a nearby cluster) at a time T . Calculate the ratio between ^{10}Be and ^9Be , and the relation between the above age a (wrongly) derived while assuming it was a steady state, and the actual age T of the cosmic rays.