ADVANCED ASTROPHYSICS (77938) - PROBLEM SET NO. 1
Due Date: March 29, 2010

1. Stellar Parallax

In the 16^{th} century, the danish astronomer Tycho Brahe failed to find stellar parallax, since the nearest star Proxima Centauri (P-Cen) is at a distance of $1.3\mathrm{pc}$ and the maximal resolution of Brahe's telescope was $\Delta = 4''$. At the time, the most distant planet known was Saturn, at distance of $9.54\mathrm{AU}$ from the Sun. If Tycho Brahe could travel to Saturn, how long would he need to stay there to measure the parallax of P-cen? Assume the orbit of Saturn is circular.

2. Magnitude of Objects

Calculate the apparent magnitude of:

a. A sun-like star at 50 pc away.

b. A 100 Watt light bulb, 10 km away.

c. A galaxy containing $\sim 10^{11}$ stars with an average luminosity of $0.5 L_{\odot}$, 20 Mpc away (i.e., a bright galaxy in the Virgo cluster).

d. A quasar with luminosity $L_Q = 10^{46}$ erg/s, 1 Gpc away (similar to the first quasar discovered).

3. Eclipsing Binaries

The mass of a distant star can be accurately measured if the star is part of a binary system. By using Kepler's laws and the observed dynamical properties of the system, the masses of the stars can be deduced.

a. Assume a binary system in which the two stars are in circular orbits around the center of mass. Even if the system cannot be resolved into two object, it is still possible to identify the two stars by their spectra (these cases are known as spectroscopic binaries). By measuring the Doppler shift of the spectral lines the velocities along the line of sight (LOS) of the stars can be measured, v_1, v_2. The period of the rotation P is also easily measured. Assume the plane of rotation is parallel to the line of sight and find the expression for the masses M_1, M_2 as a function of the observables v_1, v_2, P.

b. In most cases, the plane of rotation will *not* be parallel to the LOS. How will the results change due to an inclination angle of i between the plane of rotation and the LOS? Use the convention that $i = 90°$ corresponds to a rotation curve parallel to the LOS.

c. There is no observable way to know the value of i, and the usual practice is to collect a large sample of stars and use statistical approximations. Assuming i is a completely random variable and thus uniformly distributed between 0° and 90°, what is the mean value of the mass function found earlier? Can you think of a selection effect in observations which will cause this result to be different?

4. Central Pressure Limit

Use the equation for Hydrostatic Equilibrium and the relationship between mass and density and show that

$$P_c > \frac{GM_{tot}^2}{8\pi R^4}$$

What is the value of this lower limits in terms of the solar parameters $M_\odot, R_\odot$?

5. Simple Star

A Star of mass M, radius R and central density ρ_c is described by the density profile

$$\rho(r) = \rho_c \left(1 - \left(\frac{r}{R}\right)^2\right)$$

a. Find the mass profile $M(< r)$.

b. What is the relation between R and M for a given ρ_c? Show that the mean density $\bar{\rho}$ is given by $\bar{\rho} = 0.4\rho_c$.

c. Find the central density of the star P_c. Show that it is indeed above the lower limit found in the previous question.

6. Thermal Pressure in the Sun

Assume that the conditions in the center of the sun are $\rho = 150\text{gr}/\text{cm}^3, T = 1.58 \times 10^7\text{K}, X = 0.34$ and $Y = 0.64$.

a. What is the thermal pressure of the gas, assuming full ionization? To obtain an exact number, assume that all other metals in the sun are ^{16}O.

b. What is the radiation pressure in this case? Compare to the thermal pressure.