

In spherical polar coordinates r, ϕ, θ we have for the stress tensor

$$\sigma_{rr} = -p + 2\eta \frac{\partial v_r}{\partial r},$$

$$\sigma_{\phi\phi} = -p + 2\eta \left(\frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot \theta}{r} \right),$$

$$\sigma_{\theta\theta} = -p + 2\eta \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right),$$

$$\sigma_{r\theta} = \eta \left(\frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right),$$

$$\sigma_{\theta\phi} = \eta \left(\frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{1}{r} \frac{\partial v_\phi}{\partial \theta} - \frac{v_\phi \cot \theta}{r} \right),$$

$$\sigma_{\phi r} = \eta \left(\frac{\partial v_\phi}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\phi}{r} \right),$$

while the Navier–Stokes equations are

$$\begin{aligned} \frac{\partial v_r}{\partial t} + (\mathbf{v} \cdot \mathbf{grad})v_r - \frac{v_\theta^2 + v_\phi^2}{r} \\ = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[\Delta v_r - \frac{2}{r^2 \sin^2 \theta} \frac{\partial(v_\theta \sin \theta)}{\partial \theta} - \frac{2}{r^2 \sin \theta} \frac{\partial v_\phi}{\partial \phi} - \frac{2v_r}{r^2} \right], \end{aligned}$$

$$\begin{aligned} \frac{\partial v_\theta}{\partial t} + (\mathbf{v} \cdot \mathbf{grad})v_\theta + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot \theta}{r} \\ = -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} + \nu \left[\Delta v_\theta - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2 \sin^2 \theta} \right], \end{aligned}$$

$$\begin{aligned} \frac{\partial v_\phi}{\partial t} + (\mathbf{v} \cdot \mathbf{grad})v_\phi + \frac{v_r v_\phi}{r} + \frac{v_\theta v_\phi \cot \theta}{r} \\ = -\frac{1}{\rho r \sin \theta} \frac{\partial p}{\partial \phi} + \nu \left[\Delta v_\phi + \frac{2}{r^2 \sin \theta} \frac{\partial v_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\theta}{\partial \phi} - \frac{v_\phi}{r^2 \sin^2 \theta} \right], \end{aligned}$$

where

$$(\mathbf{v} \cdot \mathbf{grad})f = v_r \frac{\partial f}{\partial r} + \frac{v_\theta}{r} \frac{\partial f}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial f}{\partial \phi},$$

$$\Delta f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2} = 0.$$

The equation of continuity is

$$\frac{1}{r^2} \frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(v_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} = 0.$$

In cylindrical polar coordinates r, ϕ, z components of the stress tensor are

$$\begin{aligned}\sigma_{rr} &= -p + 2\eta \frac{\partial v_r}{\partial r}, & \sigma_{r\phi} &= \eta \left(\frac{1}{r} \frac{\partial v_r}{\partial \phi} + \frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r} \right), \\ \sigma_{\phi\phi} &= -p + 2\eta \left(\frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} \right), & \sigma_{\phi z} &= \eta \left(\frac{\partial v_\phi}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \phi} \right), \\ \sigma_{zz} &= -p + 2\eta \frac{\partial v_z}{\partial z}, & \sigma_{zr} &= \eta \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right).\end{aligned}$$

The three components of the Navier–Stokes equation are

$$\begin{aligned}\frac{\partial v_r}{\partial t} + (\mathbf{v} \cdot \mathbf{grad})v_r - \frac{v_\phi^2}{r} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\Delta v_r - \frac{2}{r^2} \frac{\partial v_\phi}{\partial \phi} - \frac{v_r}{r^2} \right), \\ \frac{\partial v_\phi}{\partial t} + (\mathbf{v} \cdot \mathbf{grad})v_\phi + \frac{v_r v_\phi}{r} &= -\frac{1}{\rho r} \frac{\partial p}{\partial \phi} + \nu \left(\Delta v_\phi + \frac{2}{r^2} \frac{\partial v_r}{\partial \phi} - \frac{v_\phi}{r^2} \right), \\ \frac{\partial v_z}{\partial t} + (\mathbf{v} \cdot \mathbf{grad})v_z &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \Delta v_z,\end{aligned}$$

where

$$\begin{aligned}(\mathbf{v} \cdot \mathbf{grad})f &= v_r \frac{\partial f}{\partial r} + \frac{v_\phi}{r} \frac{\partial f}{\partial \phi} + v_z \frac{\partial f}{\partial z}, \\ \Delta f &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}.\end{aligned}$$

The equation of continuity is

$$\frac{1}{r} \frac{\partial (rv_r)}{\partial r} + \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z} = 0.$$