Density Perturbation in an open universe:

$\Omega_0 < 1$: for $z \gg 1$ $R \propto t^{2/3}$

The universe behaves like an Einstein-de Sitter model:

At $z \gg 1$ expect $\delta \propto D_{\Omega_0=1}(t) \propto R(t) \propto t^{1/2}$

Suppose that at some initial time, $z_i \gg 1$, the growing mode already dominates and $\delta(z_i) \sim \delta_i$.

It can be shown that

$$\frac{\delta(t \to \infty)}{\delta_i} = \frac{1}{\Omega_i^{-1} - 1}$$

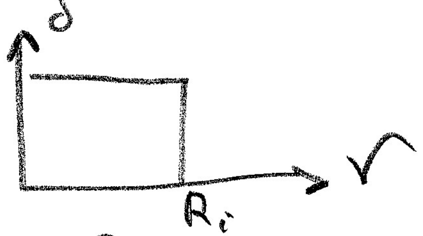
Namely there is a time after whic δ FREEZES and stops to grow.

$$z_{freeze} \sim \Omega_i^{-1} - 1$$

(Note $\Omega_0 = 1 \to z_{freeze} = -1 \Leftrightarrow t = \infty$)

Spherical Top-hat Model: Non-linear Evolution

Consider a spherical density perturbation:



Because of the spherical symmetry one can consider to perturbation to be a mini-Friedman universe

Equations of motion $\Rightarrow R(t) = A(1-\cos\theta)$ $\qquad A^3 = GMB^2$

$$t = B(\theta - \sin\theta)$$

The perturbation reaches max expansion at $\theta = \pi \Rightarrow R_{max} = 2A$

$$t = \pi B$$

$A = (R_{max}/2)$

$B = \left(\frac{GM}{(R_{max}/2)^3}\right)^{-1/2}$

$t_{max} \Rightarrow$

$$t_{max} = \pi \frac{1}{\sqrt{\frac{GM}{(R_{max}/2)^3}}}$$

$$\frac{R_{max}}{R_i} = \frac{1+\delta_i}{\delta_i - (\Omega_i^{-1}-1)} \sim \frac{1}{\delta_i - (\Omega_i^{-1}-1)}$$

$z_i \gg 1$

$\Omega(z_i) \sim 1$

For $\Omega_0 = 1$ $\quad \frac{R_{max}}{R_i} = \delta_i^{-1}$

Virialization: Collapse $t_{coll} = 2\, t_{max}$

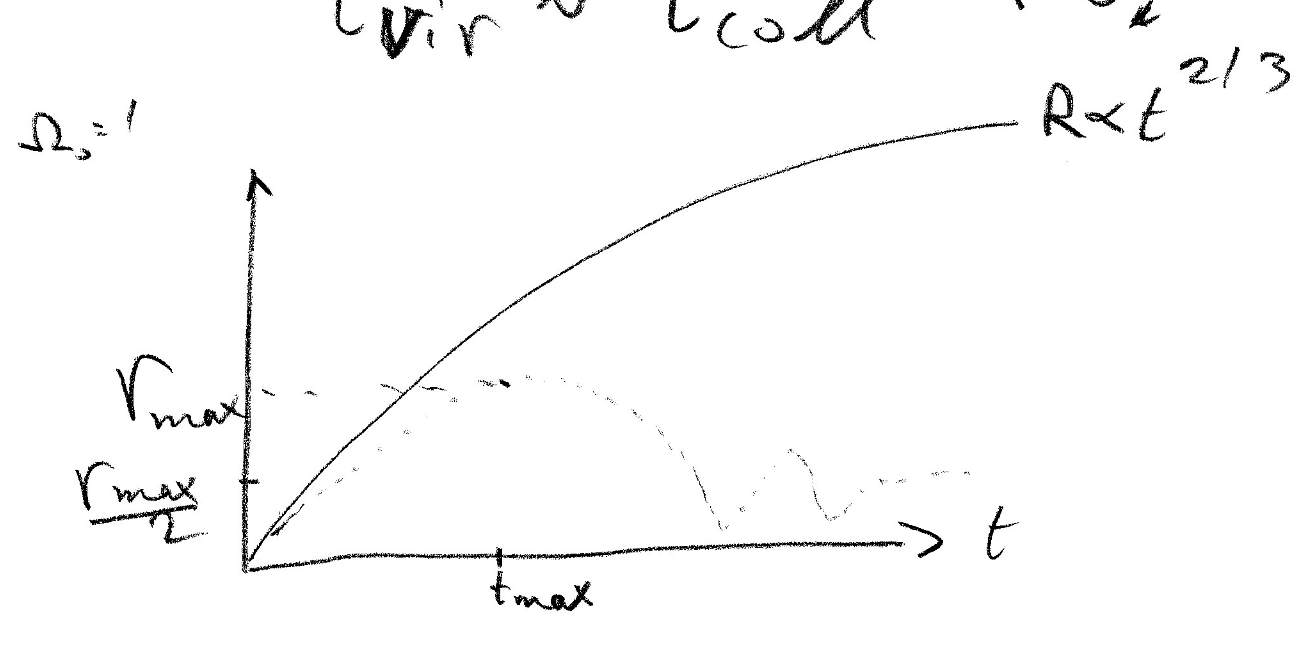
$R(t_{coll}) \equiv 0$

mathematically

At $t \sim t_{coll}$ virialization:

$$R_{vir} \sim \frac{R_{max}}{2}$$

$$t_{vir} \sim t_{coll} \propto \delta_i^{-3/2}$$



$\Omega_0 = 1.0$

$$\delta_{NL}(t_{max}) = \frac{9\pi^2}{16} \sim 5.5$$

$$\delta_L(t_{max}) = \frac{3}{20}\left(6\pi^2\right)^{2/3} \sim 1.06$$

$$1 + \delta_{vir} \sim 178\,\Omega_0^{-0.7}$$