Peculiar Velocities

$$\frac{\partial}{\partial t}\delta = -\underline{\nabla}\cdot\underline{u}$$

$$\frac{\partial}{\partial t}u + 2\frac{\dot{R}}{R}u = -\frac{\underline{g}}{R}$$

$$-\underline{\nabla}\cdot\underline{g} = 4\pi G\rho_0 R\,\delta$$

$$\left[\begin{array}{l} \underline{\nabla} \to \nabla_r \\ \underline{U} = R\,u \\ \underline{g}(r,t) = G\rho_0 R\int d^3r'\,\delta(r',t)\,\frac{(\underline{r}-\underline{r}')}{|\underline{r}-\underline{r}'|^3} \end{array}\right.$$

$$\frac{\partial}{\partial t}\left(\frac{\underline{\nabla}\cdot\underline{g}}{4\pi G\rho_0 R}\right) = -\underline{\nabla}\cdot\underline{u}$$

$$\underline{u} = +\frac{\partial}{\partial t}\left(\frac{\underline{g}}{4\pi G\rho_0 R}\right) + \underbrace{\underline{F}(r,t)}_{\underline{\nabla}\cdot\underline{F}=0 \quad \text{decaying mode}}$$

Analysis applying to the growing mode:

$$\underline{g}(r,t) = G\rho_0 R\,\underset{\uparrow\ \text{growing mode}}{D(t)}\ \underbrace{\int d^3r'\,\delta(r')\,\frac{(\underline{r}-\underline{r}')}{|\underline{r}-\underline{r}'|^3}}_{\tilde{g}}$$

$$\underline{u} = +\frac{\tilde{\underline{g}}}{4\pi}\qquad \frac{d}{dt}D = -\frac{\tilde{\underline{g}}}{4\pi D(t)}\quad \cancel{D}$$

$$\underline{u} = +\frac{\underline{g}}{4\pi G\rho_0 R}\,\frac{1}{D}\,\frac{dD}{dt}$$

$$\Rightarrow\qquad \underline{u} = +\frac{H f(\Omega)\,\underline{g}}{4\pi G\rho_0 R},\qquad f(\Omega) = \frac{R}{D}\frac{dD}{dR} \sim \Omega^{0.6}$$

$$\underline{u}(r,t) = \frac{H}{4\pi} f(\Omega) \int d^3r' \, \delta(r') \frac{(\underline{r}-\underline{r}')}{|\underline{r}-\underline{r}'|^3}$$

Or:

$$\underline{u}(\underline{r},t) = \frac{2}{3} \frac{f(\Omega)\, \underline{g}}{H\,\Omega R}$$

$$\delta\underline{v}(r,t) = \frac{2}{3} \frac{f(\Omega)\, \underline{g}}{H\,\Omega}$$

Case of $\underline{\nabla}\cdot\underline{u} = 0$, $\underline{\nabla}\times\underline{u} \neq 0$

$$\frac{\partial}{\partial t} u + 2\frac{\dot{R}}{R} u = 0$$

$$u(r,t) = f(r)\, t^{\alpha}$$

$$\alpha \frac{u}{t} + 2\cdot\frac{2}{3}\frac{u}{t} = 0$$

$$\alpha = -\frac{4}{3}$$

$$u \propto t^{-4/3}$$

$$\Rightarrow v \propto t^{-2/3} \propto \frac{1}{R}$$