The Evolution of the Jeans Mass

z_{eq} = equality of matter & radiation densities: $\rho_m(z_{eq}) \equiv \rho_R(z_{eq})$

non-relativistic

$z \gg z_{eq}$: $\quad v_s = \left(\frac{\partial P}{\partial \rho}\right)_s^{1/2}$, $P = P_R + \underbrace{P_M}_{=0} \sim P_R \sim \frac{\rho_R c^2}{3}$

$$v_s \sim \frac{c}{\sqrt{3}}$$

Consider baryons: $z_{rec} < z < z_{eq}$

$$\left(\Omega_b h^2 > 4 \times 10^{-2}\right)$$

During that period $\quad z_{rec} < z \ll z_{eq}$

$\rho \sim \rho_M$, $P \sim P_R$

$$v_s = \frac{c}{\sqrt{3}} \frac{4}{3}\left(\frac{\rho_R}{\rho_M}\right)^{1/2} \sim \frac{c}{\sqrt{3}}\left(\frac{1+z}{1+z_{eq}}\right)^{1/2}$$

$$\sim 2 \times 10^8 \left(\frac{1+z}{1+z_{eq}}\right)^{1/2} \frac{m}{s}$$

$$\cancel{\lambda_J = v_s \sqrt{\frac{\pi}{G\rho}}}$$

$$\cancel{M_J = \frac{4\pi}{3} \rho_M \left(\frac{\lambda_J}{2}\right)^3}$$

For $z < z_{rec}$:

We have shown that $\underset{\nearrow}{v} \propto R^{-1}$

velocity perturbation
without density perturbation

$$\Rightarrow \quad T_M \propto \underset{\nearrow}{v^2} \propto R^{-2} \propto (1+z)^2$$

particles

$$T_R \propto R^{-1} \propto (1+z)$$

$$P \propto \rho^\gamma$$

$$P = n\, k_B T$$

$$v_s = \left(\frac{\partial P_M}{\partial \rho_M}\right)_S^{1/2}$$

$$= \left(\gamma \frac{k_B T}{m_p}\right)^{1/2}$$

Reminder: $\lambda_J = \left(\frac{\pi}{G\rho}\right)^{1/2} v_s$

$$M_J \equiv \frac{4\pi}{3} \rho_M \left(\frac{\lambda_J}{2}\right)^3$$

Reminder: $\rho(z \gg z_{eq}) \sim \rho_R = \rho_R(z_{eq}) \frac{(1+z)^4}{(1+z_{eq})^4}$

$$\rho(z \ll z_{eq}) \sim \rho_M = \rho_R(z_{eq}) \left(\frac{1+z}{1+z_{eq}}\right)^3$$

Show that: $M_J(z \gg z_{eq}) \sim M_J(z_{eq}) \left(\frac{1+z}{1+z_{eq}}\right)^{-3}$

$$M_J(z_{eq}) \sim 3.5 \times 10^{15} (\Omega h^2)^{-2} M_\odot$$

For $z_{rec} < z < z_{eq}$:

$$M_J(z) \sim \frac{\pi}{6} \rho_m \left[\frac{c}{\sqrt{3}} \left(\frac{1+z}{1+z_{eq}}\right)^{1/2} \left(\frac{\pi}{G\rho}\right)^{1/2} \right]^{1/3}$$

$$\sim M_J(z_{eq}) \sim \text{const}$$

For: $M_J(z < z_{rec}) \sim \frac{\pi}{6} \rho_m \left(\frac{\pi k_B T_M}{G m_p \rho_m} \right)^{3/2}$

$$\sim M_J(z_{rec}) \left(\frac{1+z}{1+z_{rec}}\right)^{3/2}$$

$T_{rec} \sim 1400°K$

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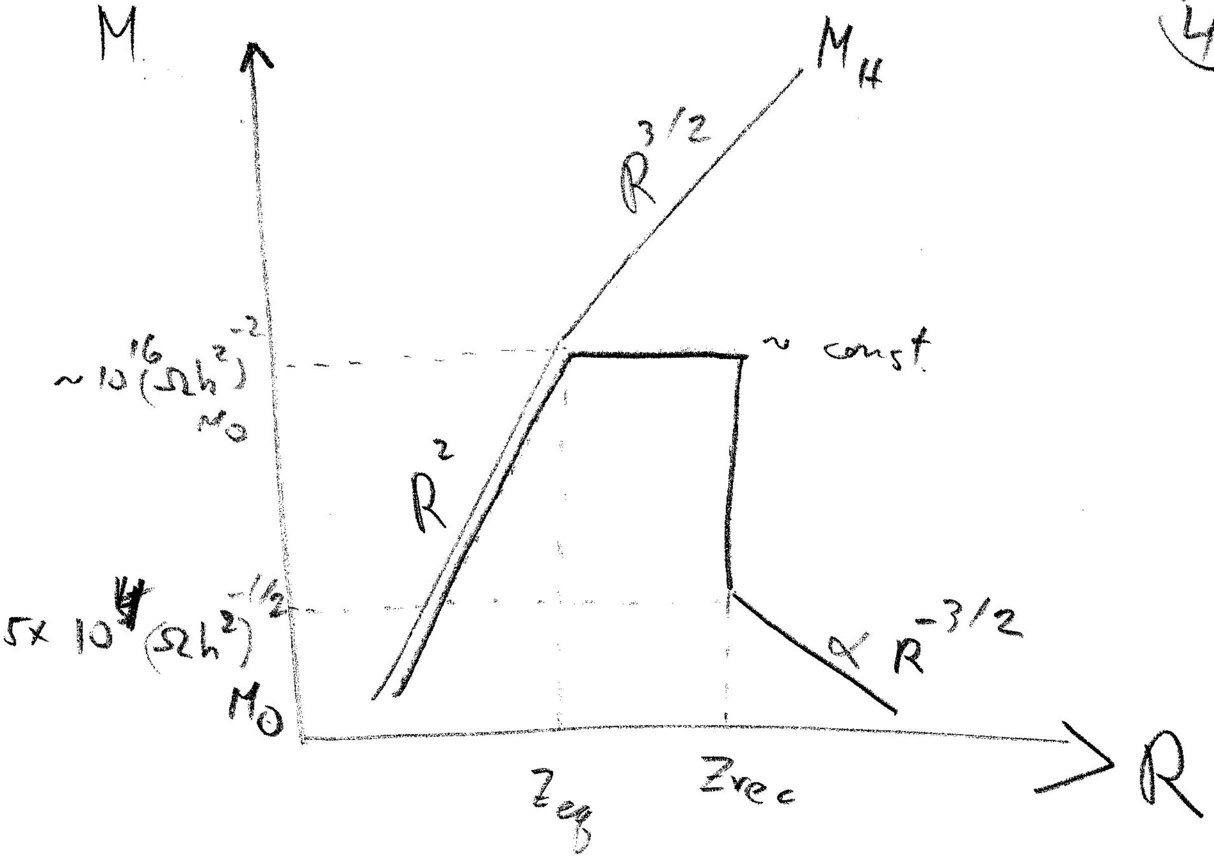
M
M_H
$R^{3/2}$
$\sim 10^{16}(\Omega h^2)^{-2}$
$M_\odot$
~ const
R^2
$5\times 10^{4}(\Omega h^2)^{-1/2}$
$\propto R^{-3/2}$
$M_\odot$
z_{eq}
z_{rec}
R