Jeans Instability

$$\tau_{pressure} \sim \frac{\lambda}{c_s}$$

$$c_s^2 = \left(\frac{\partial P}{\partial \rho}\right)_S$$

$$\tau_{grav} \sim \frac{1}{\sqrt{G\rho}}$$

$\lambda > \lambda_J = \dfrac{c_s}{\sqrt{G\rho}}$ gravitational instability

$\lambda < \lambda_J$ ~~damping~~ stability: sound waves

Continuity eq. $\dfrac{\partial \rho}{\partial t} + \underline{\nabla}(\rho \cdot \underline{v}) = 0$

Euler eq. $\underbrace{\frac{\partial}{\partial t}\underline{v} + (\underline{v}\cdot\underline{\nabla})\underline{v}}_{\text{acceleration}} = -\underbrace{\frac{1}{\rho}\underline{\nabla}P}_{\text{pressure gradient}} - \underbrace{\nabla\phi}_{\text{grav. acc.}}$

Small perturbation: $\rho = \rho_0 + \rho_1 + \dots$

$P_0, \rho_0 = \text{const.}$ $P = P_0 + P_1 + \dots$

In general one needs an equation of state:

$P = P(\rho, T)$ (say)

We assume adiabatic perturbations

$$\left(\frac{\partial P}{\partial \rho}\right)_S \equiv v_s^2$$

v_s ← sound speed

$P_1 = v_s^2 \rho_1$ (to 1st order)

$v_s^* = v_s(\rho_0, P_0)$

Linearization of E.O.M:

$$\text{①} \quad \frac{\partial \rho_1}{\partial t} + \rho_0 \underline{\nabla} \cdot \underline{v}_1 = 0$$

$$\text{②} \quad \frac{\partial \underline{v}_1}{\partial t} = -\frac{v_s^2}{\rho_0} \underline{\nabla} \rho_1 - \underline{\nabla} \phi_1$$

$$\nabla^2 \phi_1 = 4\pi G \rho_1$$

$$\left.\begin{array}{l} \frac{\partial}{\partial t} : \text{①} \\ \underline{\nabla} \cdot \text{②} \end{array}\right\} \Longrightarrow \quad \frac{\partial^2}{\partial t^2} \rho_1 - v_s^2 \nabla^2 \rho_1 = 4\pi G \rho_0 \rho_1$$

$$\rho_1(x, t) = \sum_{k,\omega} A(k,\omega) \exp[-i(\underline{k} \cdot \underline{r} - \omega t)]$$

Dispersion relation:

$$\omega^2 = v_s^2 k^2 - 4\pi G \rho_0$$

$\omega^2 > 0$ stability

$\omega^2 < 0$ instability

$$v_s^2 k_J^2 = 4\pi G \rho_0$$

$$\left\{\begin{array}{l} k_J \equiv \left(\frac{4\pi G \rho_0}{v_s^2}\right)^{1/2} \\ \lambda_J \equiv \left(\frac{\pi}{G \rho_0}\right)^{1/2} v_s \end{array}\right.$$

$$c_s^2 \equiv \left(\frac{\partial P}{\partial \rho}\right)_s$$

$$\delta(r,t) = \sum \delta_k(t)\, e^{-i k \cdot r}$$

$$\nabla \cdot (2),\ \frac{\partial}{\partial t}(1) \Longrightarrow \boxed{\ddot{\delta} + 2\frac{\dot{R}}{R}\dot{\delta} = \delta\left(4\pi G \rho_0 - \frac{c_s^2 k^2}{R^2}\right)}$$

Jeans scale appears also in the ~~expand~~ (co-moving) case.

Equation of motions for particles:

$$\underline{X} = R(t)\, \underline{r}$$

$$\dot{X} = \frac{\dot{R}}{R} X + R\dot{r} = \underbrace{H(t)\, \underline{X}}_{\text{Hubble expansion}} + R u$$

$$\ddot{X} = \underbrace{R\dot{u} + 2\dot{R} u}_{\underline{g}} + \underbrace{\frac{\ddot{R}}{R} X}_{g_0} = \underline{g} + \underline{g}_0$$