Measuring Cosmological Parameters

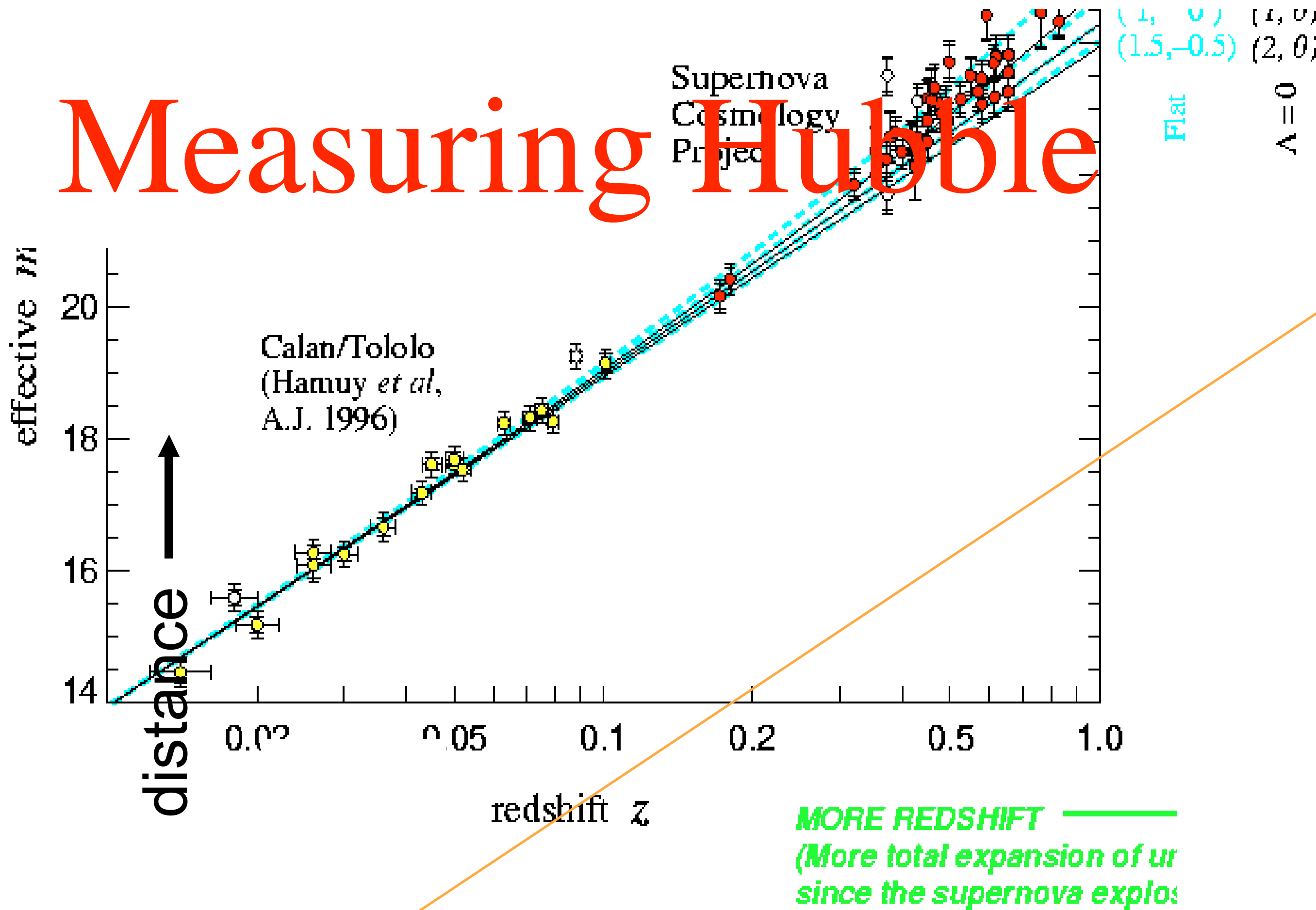
($\square_i$, $\square$, H_0, etc…)

Various Methods:

The different methods can be divided to several "major" groups of methods:

- Measuring local characteristics sensitive to cosmological parameters.
- Measuring behavior vs. time/z (luminosity, number counts etc.) at high redshift
- Looking at the young (and linear) universe: The Cosmic Microwave Background.

Measuring Hubble



velocity →

In flat universe: $\Omega_M = 0.28$ [± 0.085 statistical] [± 0.05 systematic]

Prob. of fit to $\Lambda = 0$ universe: 1%

Friedman Equation

Homogeneity (RW metric) + Gravity ($G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$)

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_m - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \qquad \rho_m = \rho_{m0}\frac{a_0^3}{a^3}$$

$$1 = \Omega_m + \Omega_k + \Omega_\Lambda$$

$$\Omega_m \equiv \frac{\rho_m}{3H^2/8\pi G} \quad \Omega_k \equiv -\frac{kc^2}{a^2H^2} \quad \Omega_\Lambda \equiv \frac{\Lambda c^2}{3H^2}$$

$$\Omega_{tot} \equiv \Omega_m + \Omega_\Lambda = 1 - \Omega_k$$ closed/open

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}\Omega_m - \Omega_\Lambda$$ decelerate/accelerate

for $\Omega_\Lambda = 0$

$\Omega_m << 1 \Rightarrow a \propto t \Rightarrow Ht = 1$

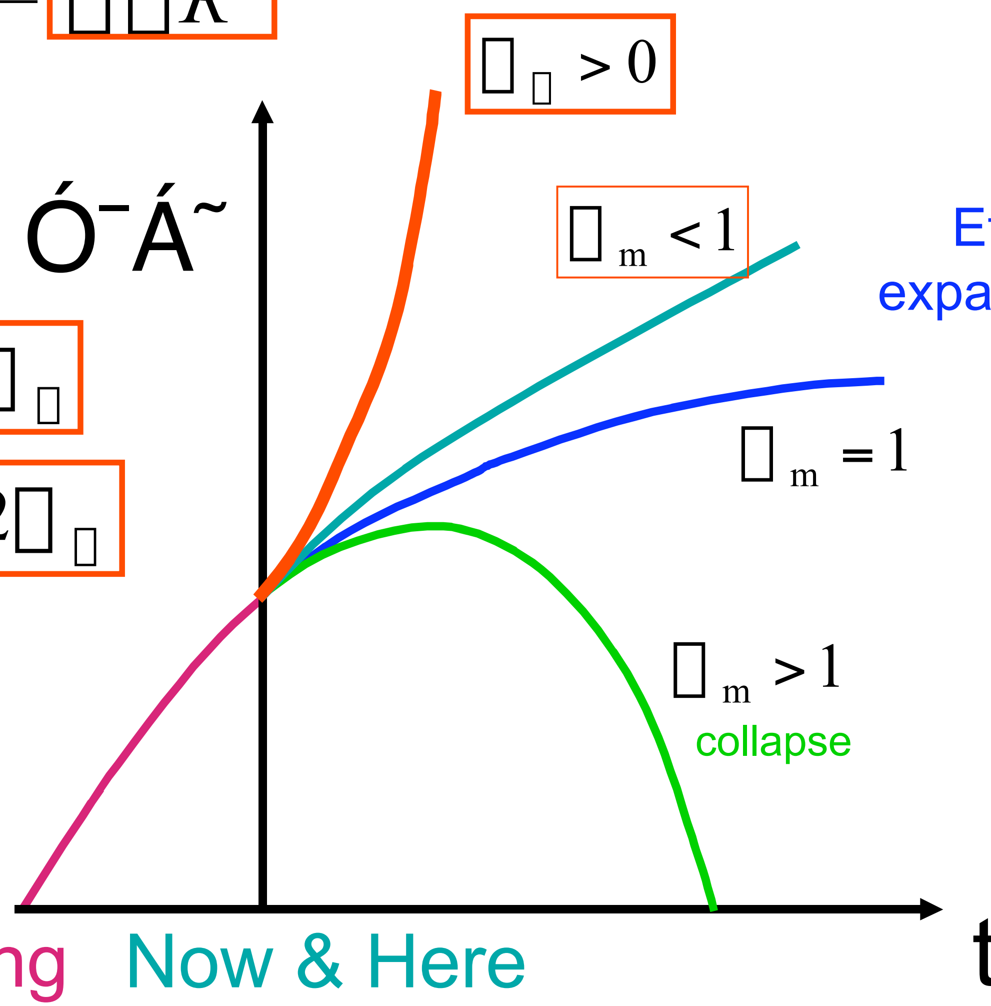
$\Omega_m = 1 \Rightarrow a \propto t^{2/3} \Rightarrow Ht = 2/3$

$\Omega_m > 1 \Rightarrow a \propto 1 - \cos\theta \Rightarrow Ht < 2/3$

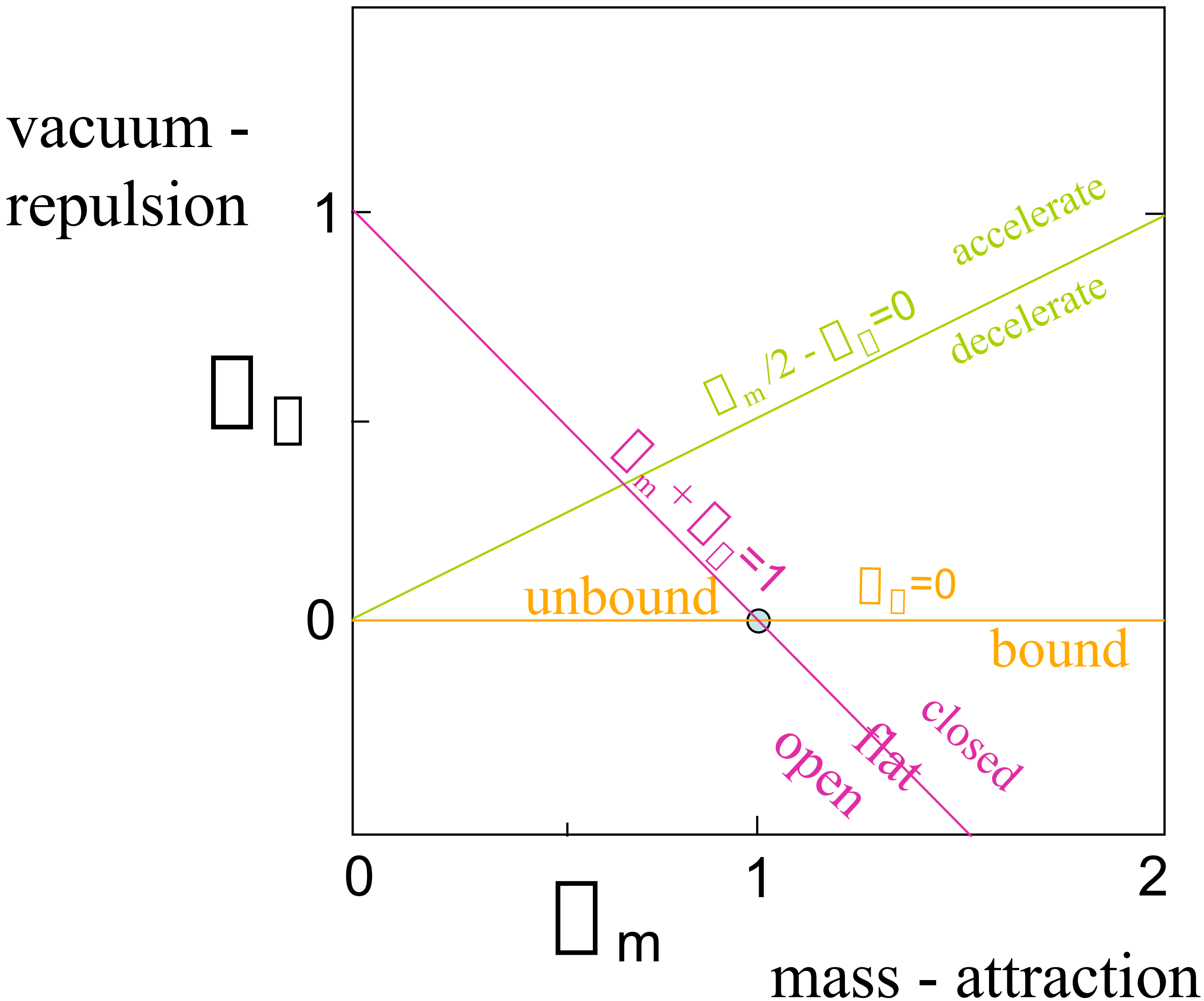
$$\text{Energy} = \frac{1}{2}V^2 \,\square\, \frac{GM}{R} \;\square\square R^2$$

$$\text{Curvature} = \square 1 + \square_m + \square_\square$$

$$\text{Acceleration} = \square\square_m + 2\square_\square$$



Dark Matter and Dark Energy



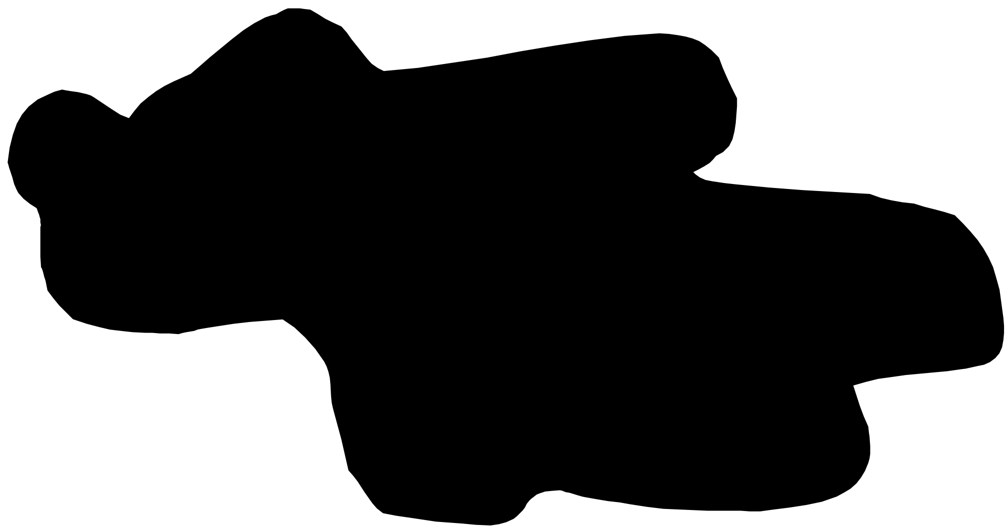
Total Luminous Matter

By counting the total amount of starlight from galaxies, and the number of galaxies, one can obtain (using (M/Msun)/(L/Lsun)~few):

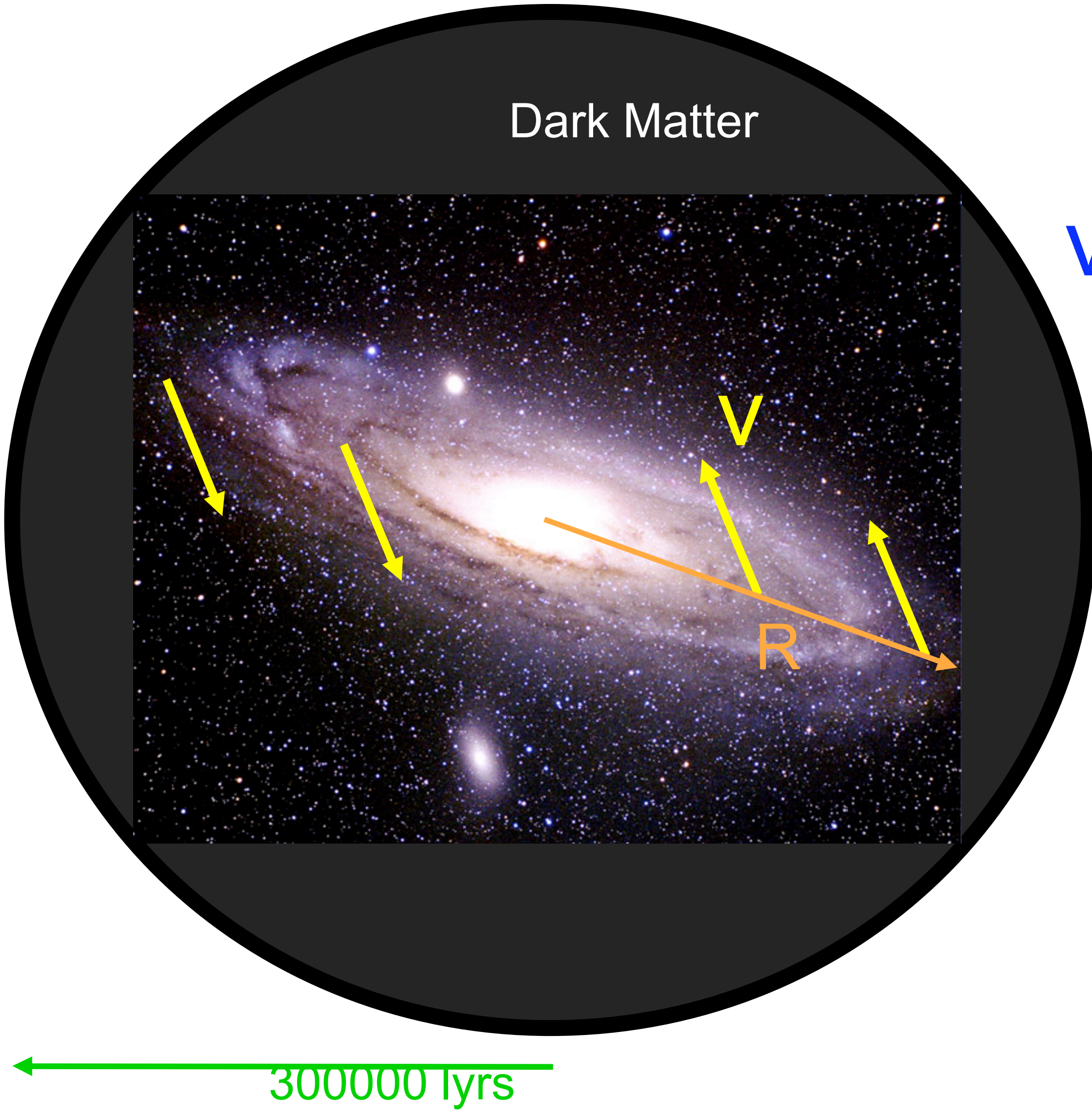
$$\Omega_{luminous} \approx 0.01$$

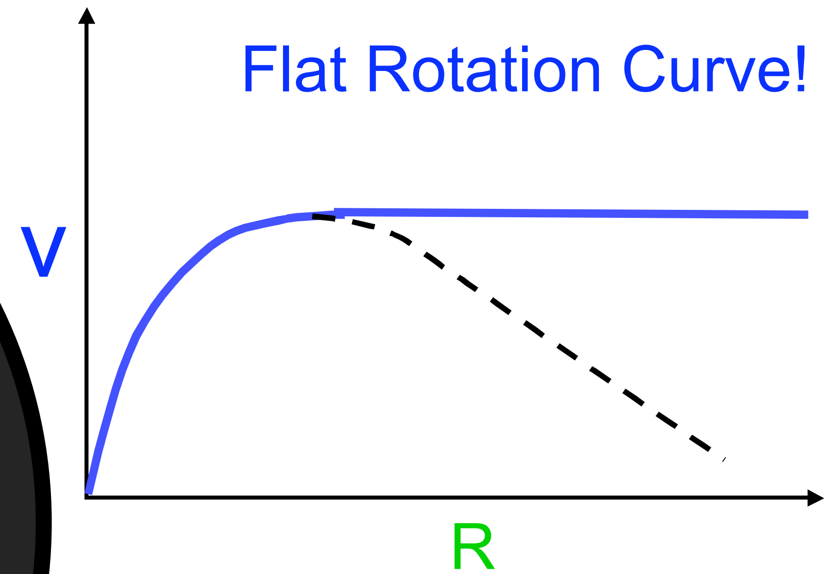
Is the universe "empty?" Where is the rest of the mass?

Dark Matter



Galaxy Rotation Curves

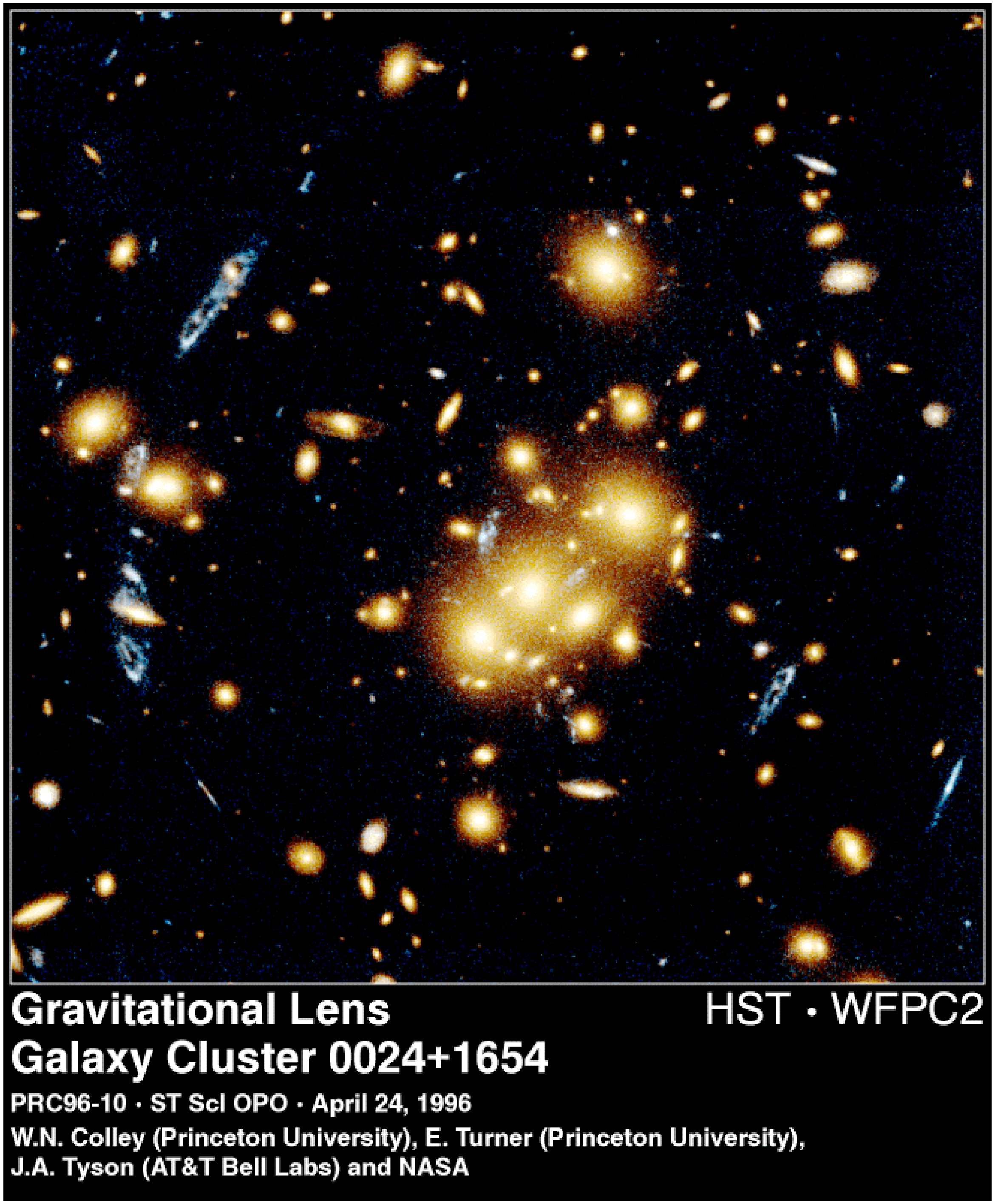




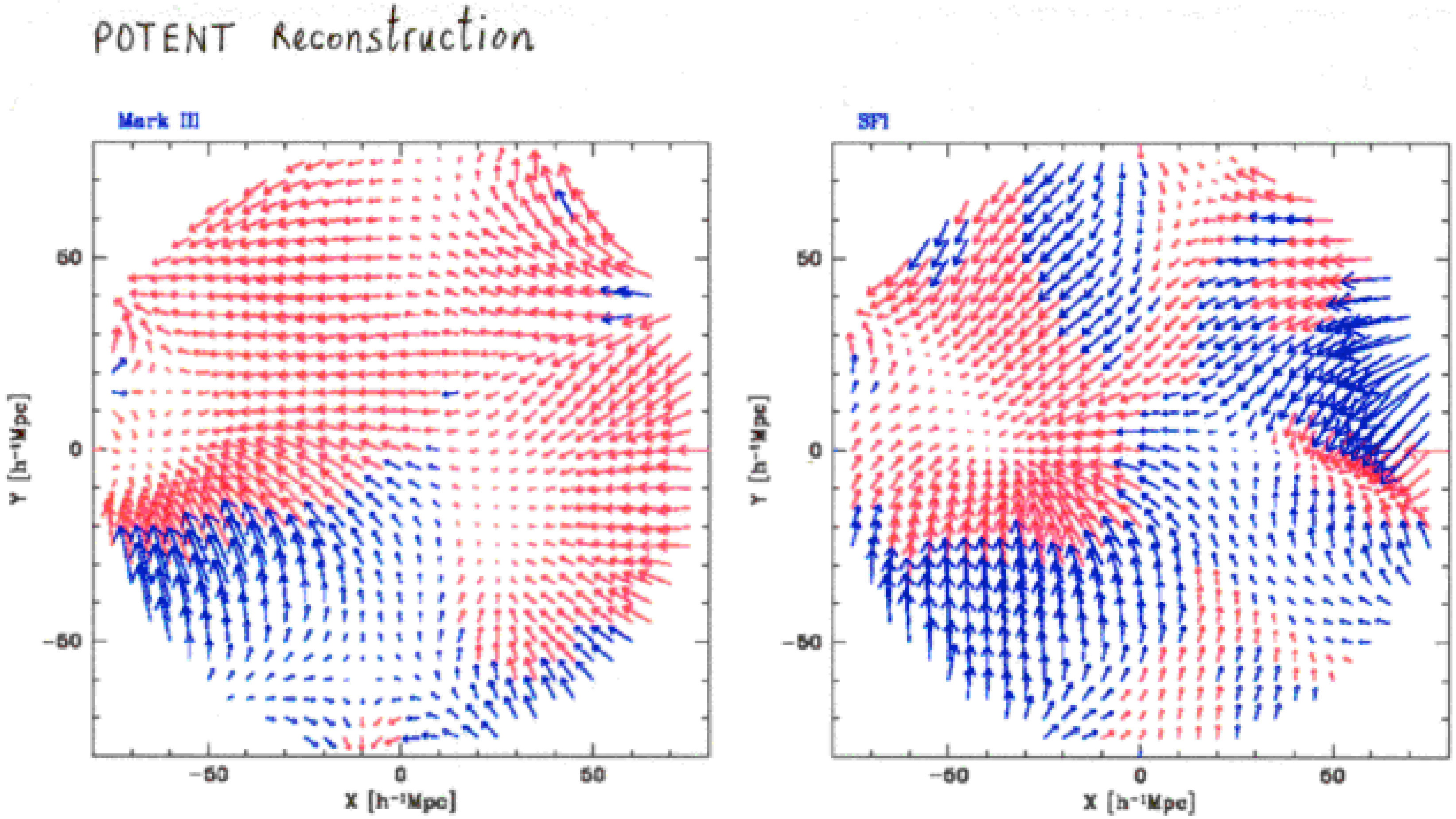
$$V^2 = \frac{GM(R)}{R}$$

$$\square \quad M(R) \quad R$$

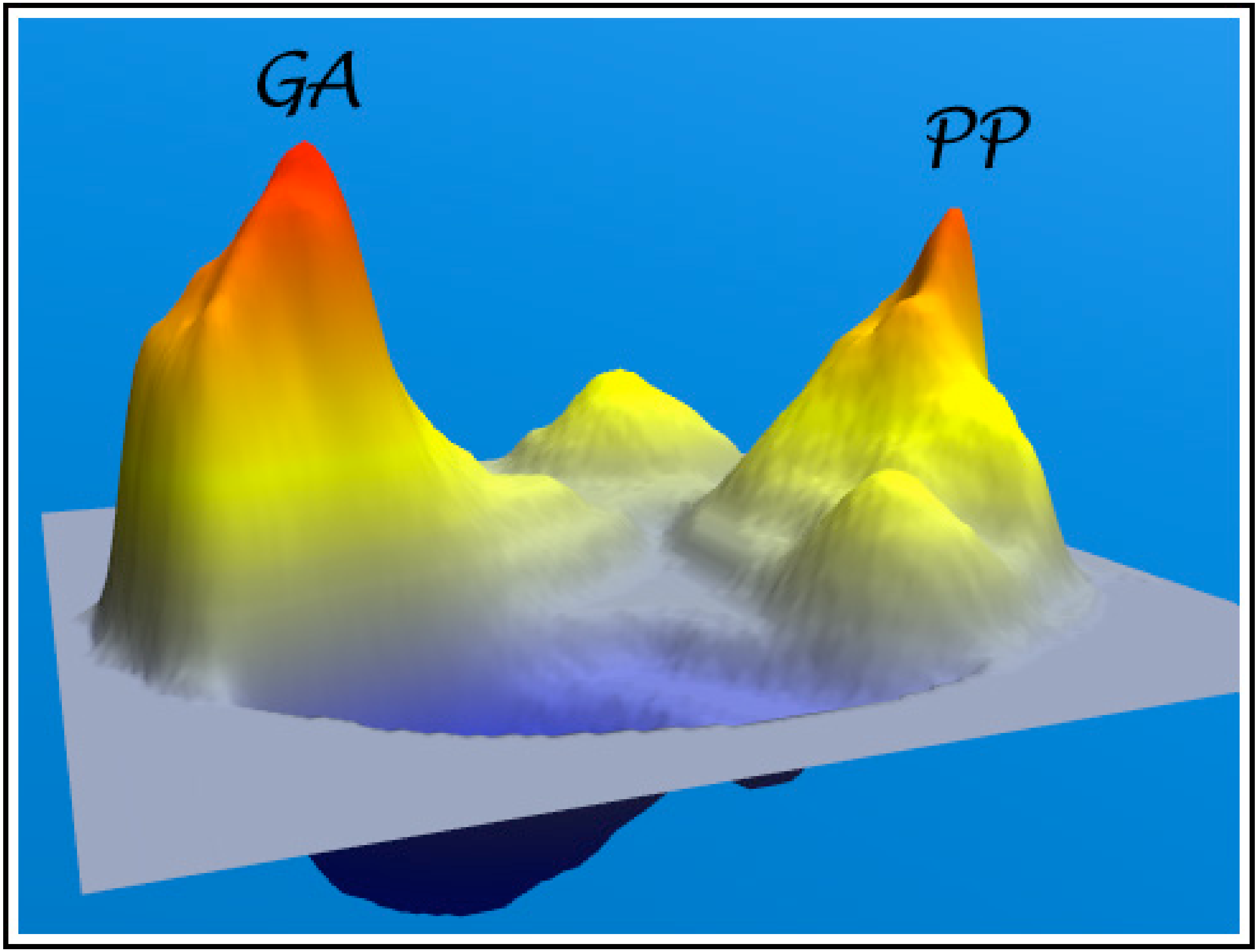
On larger scales, more Mass is missing!



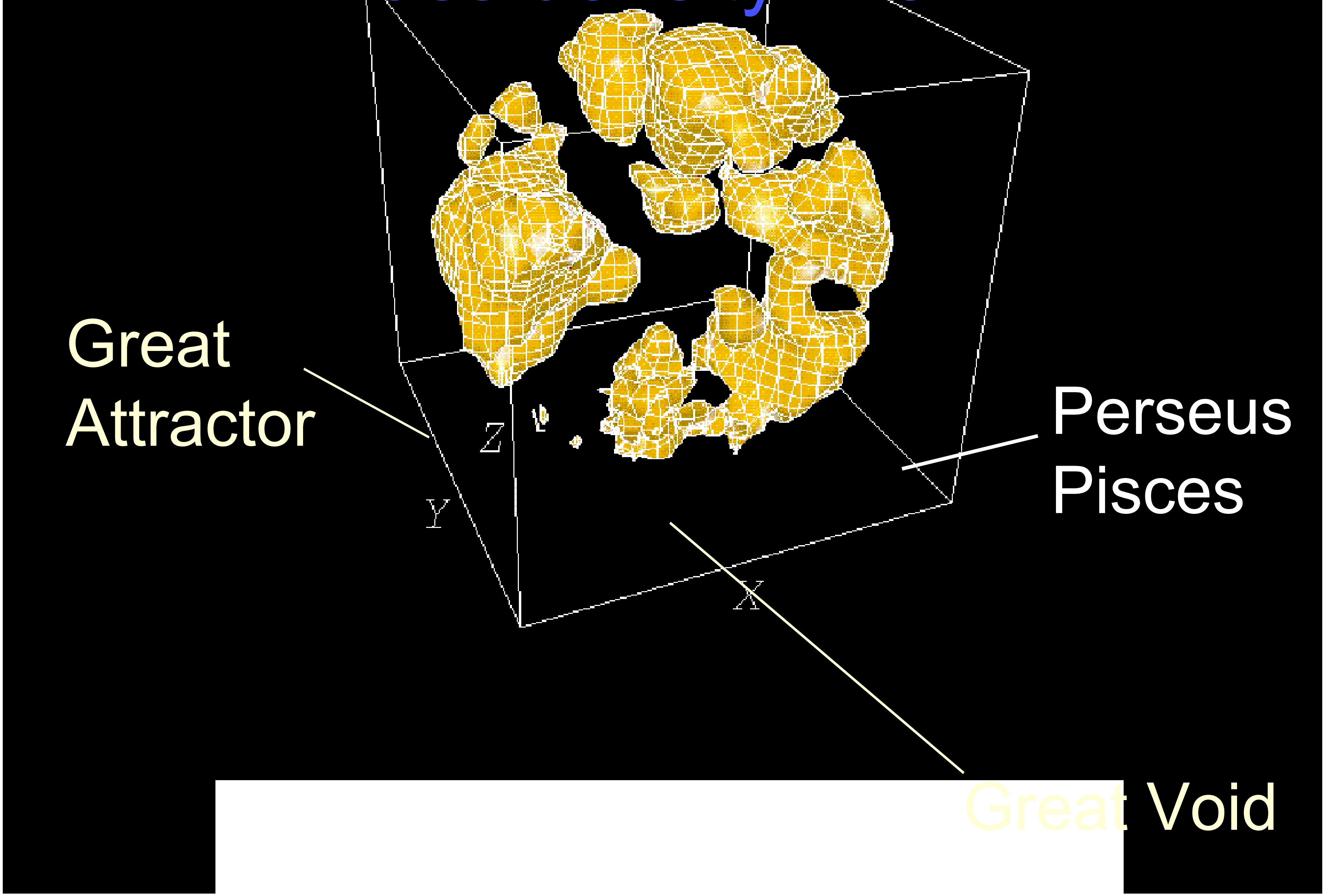
Cosmic Flows



Dark-matter density in supergalactic plane



Mass density in 3D



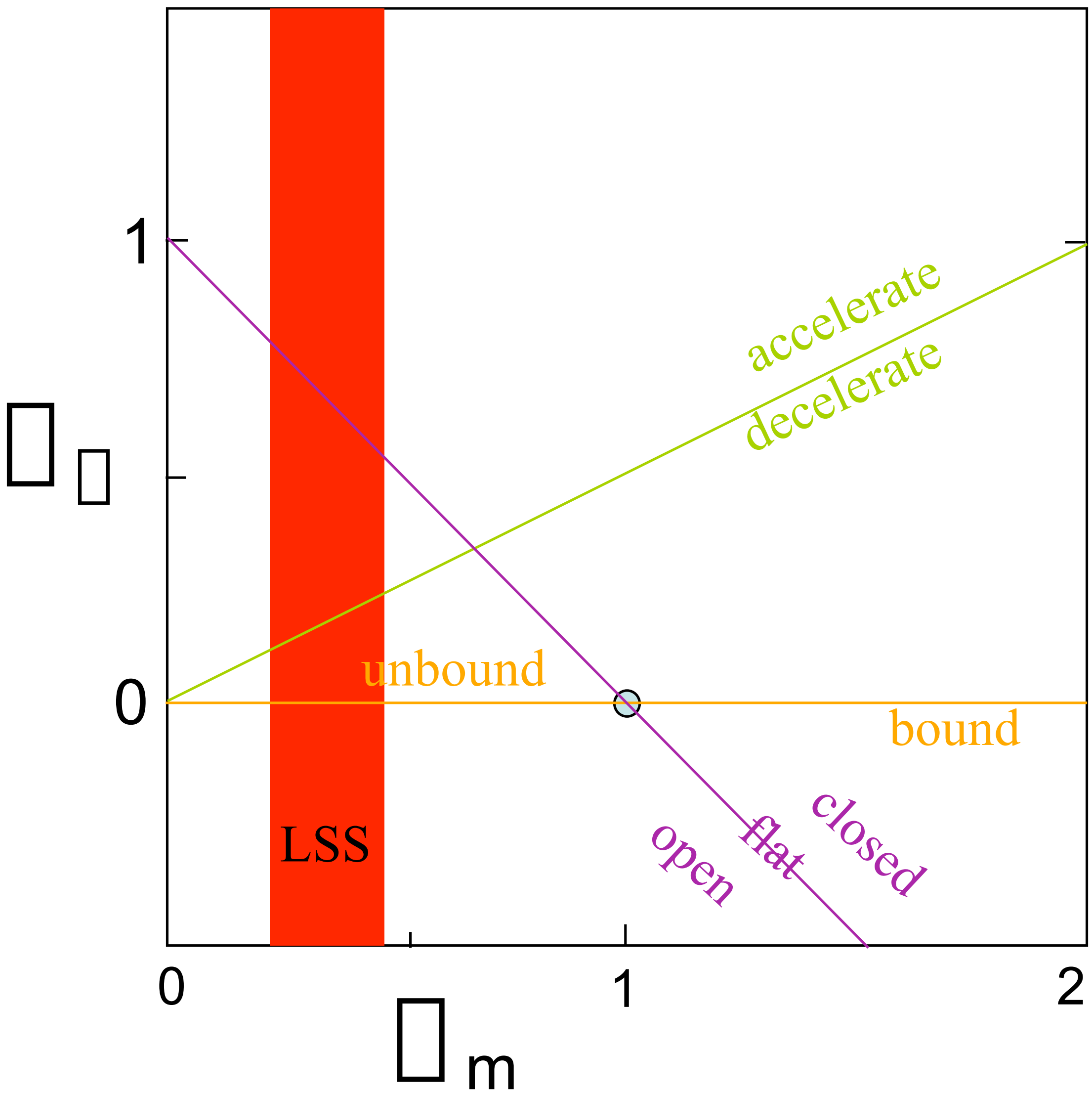
Most matter: Dark Matter

Dynamics $\Omega_m = 0.35 \pm 0.05$

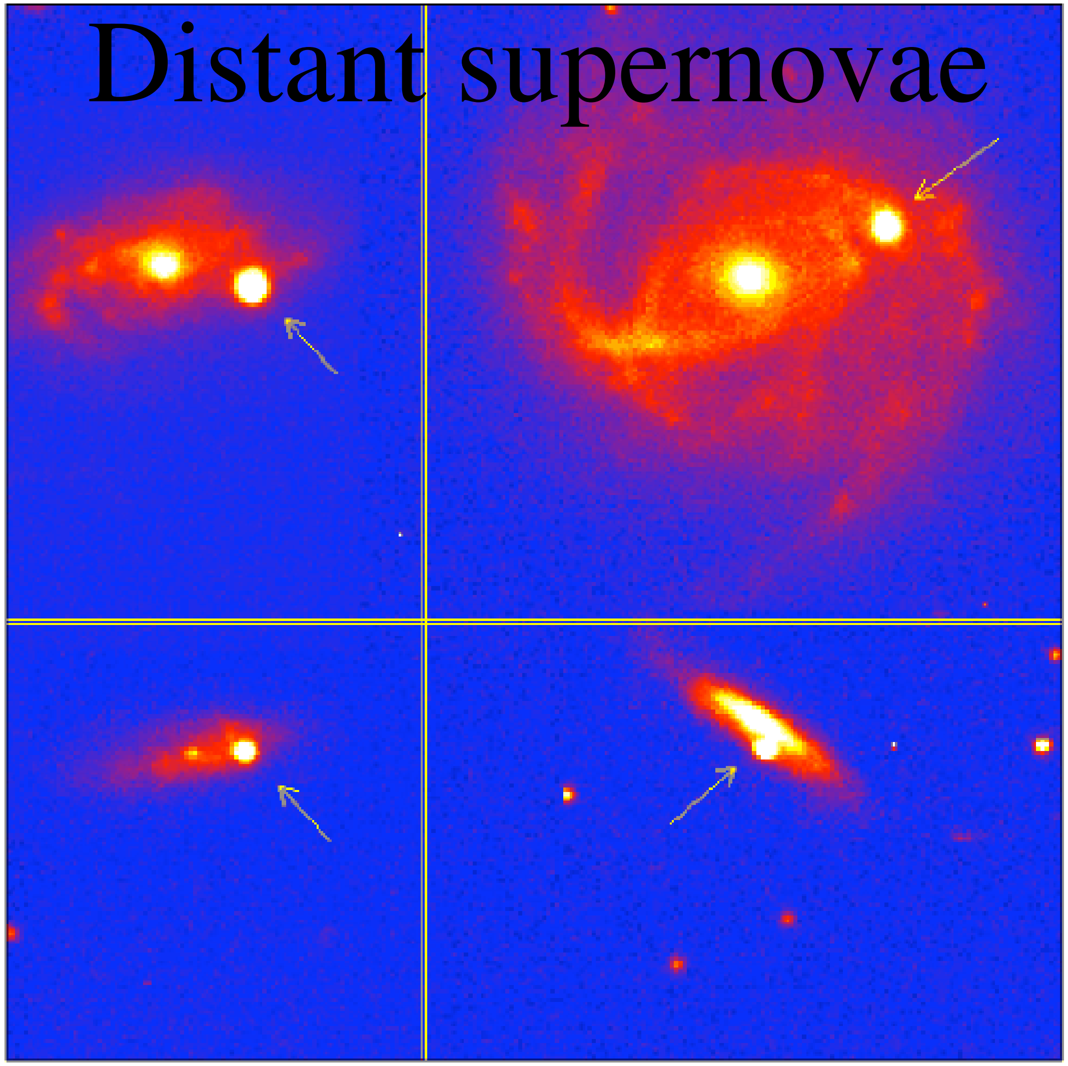
Using BBNS, cluster gas $\Omega_{baryons} \approx 0.05$

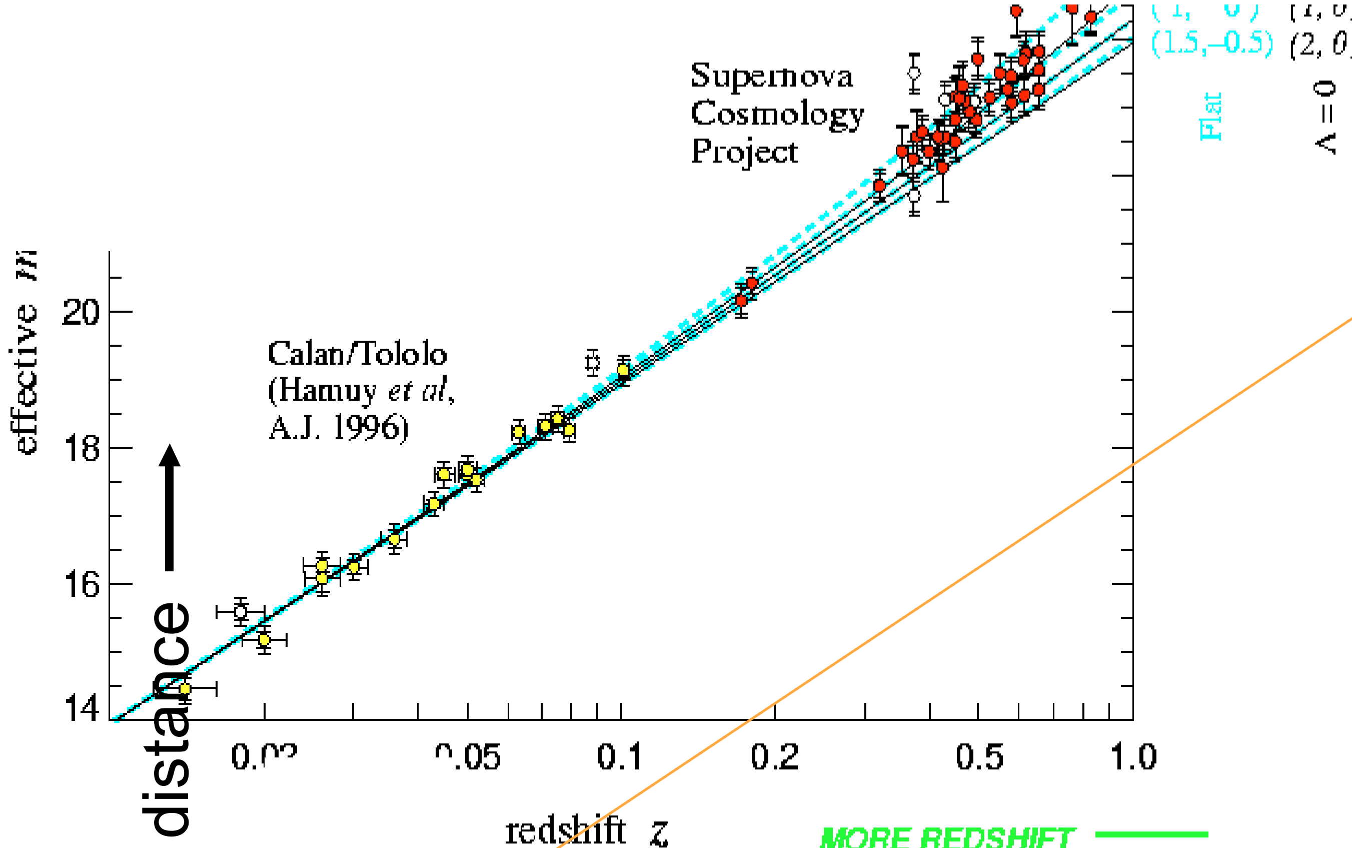
What is the dark matter?

Cosmological Parameters



Distant supernovae





MORE REDSHIFT
(More total expansion of u
since the supernova explo

velocity →

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Prob. of fit to $\Lambda = 0$ universe: 1%

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$$\text{Curvature} = \square 1 + \square_{m} + \square_{\square}$$

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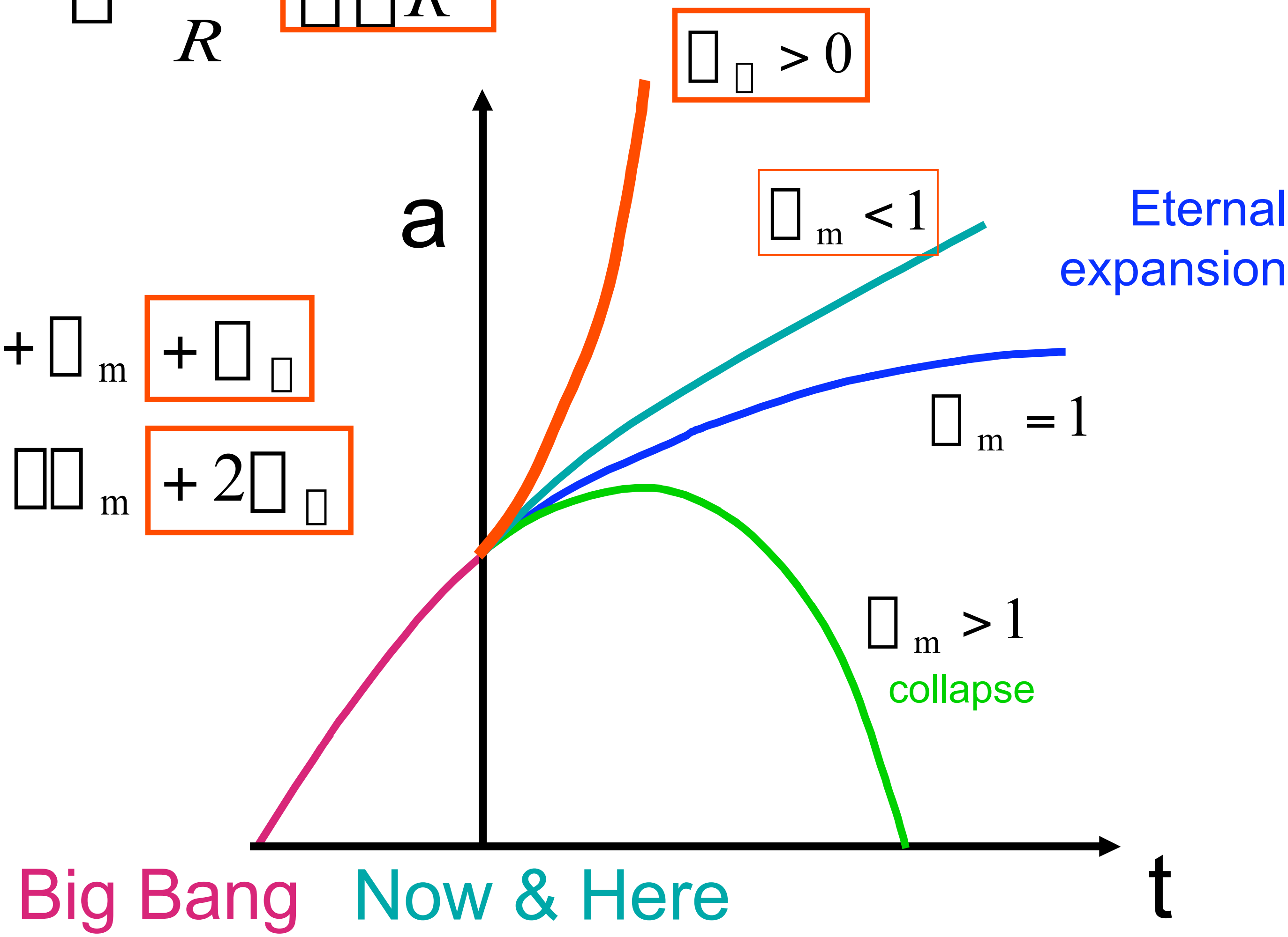
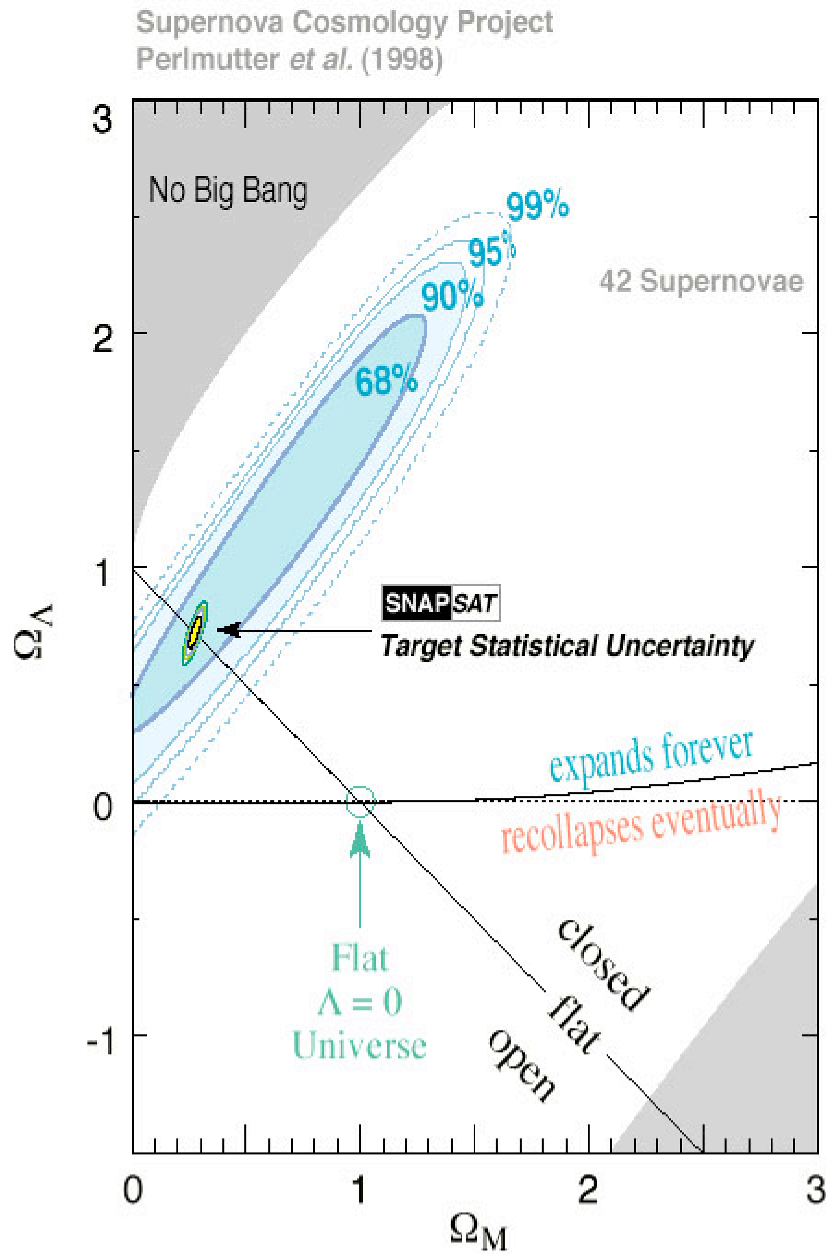


Acceleration

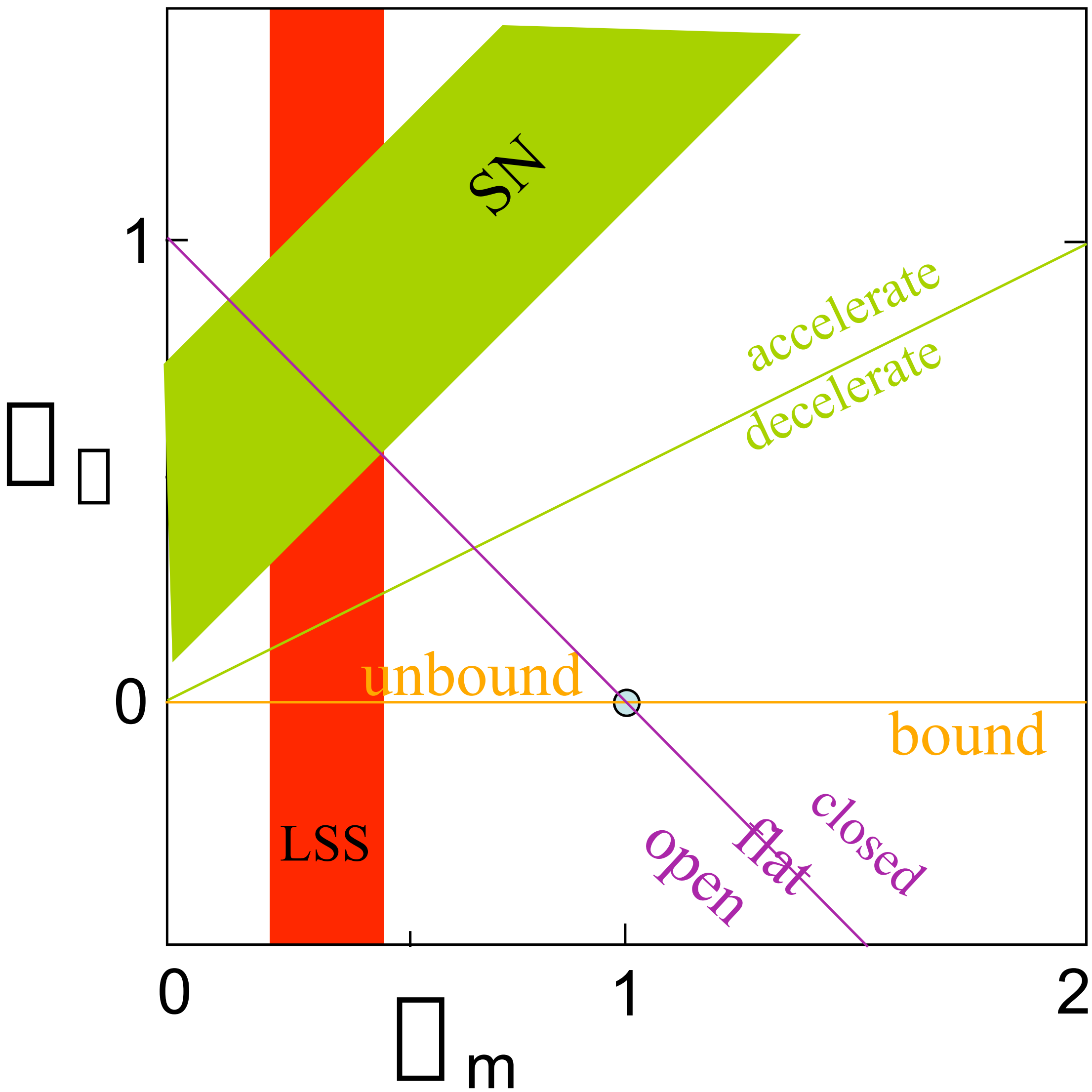
Vaccum energy is responsible for an effective “repulsion force”:

$$\square_{\square} = 0.65 \pm 0.05 \geq \square_{m}$$

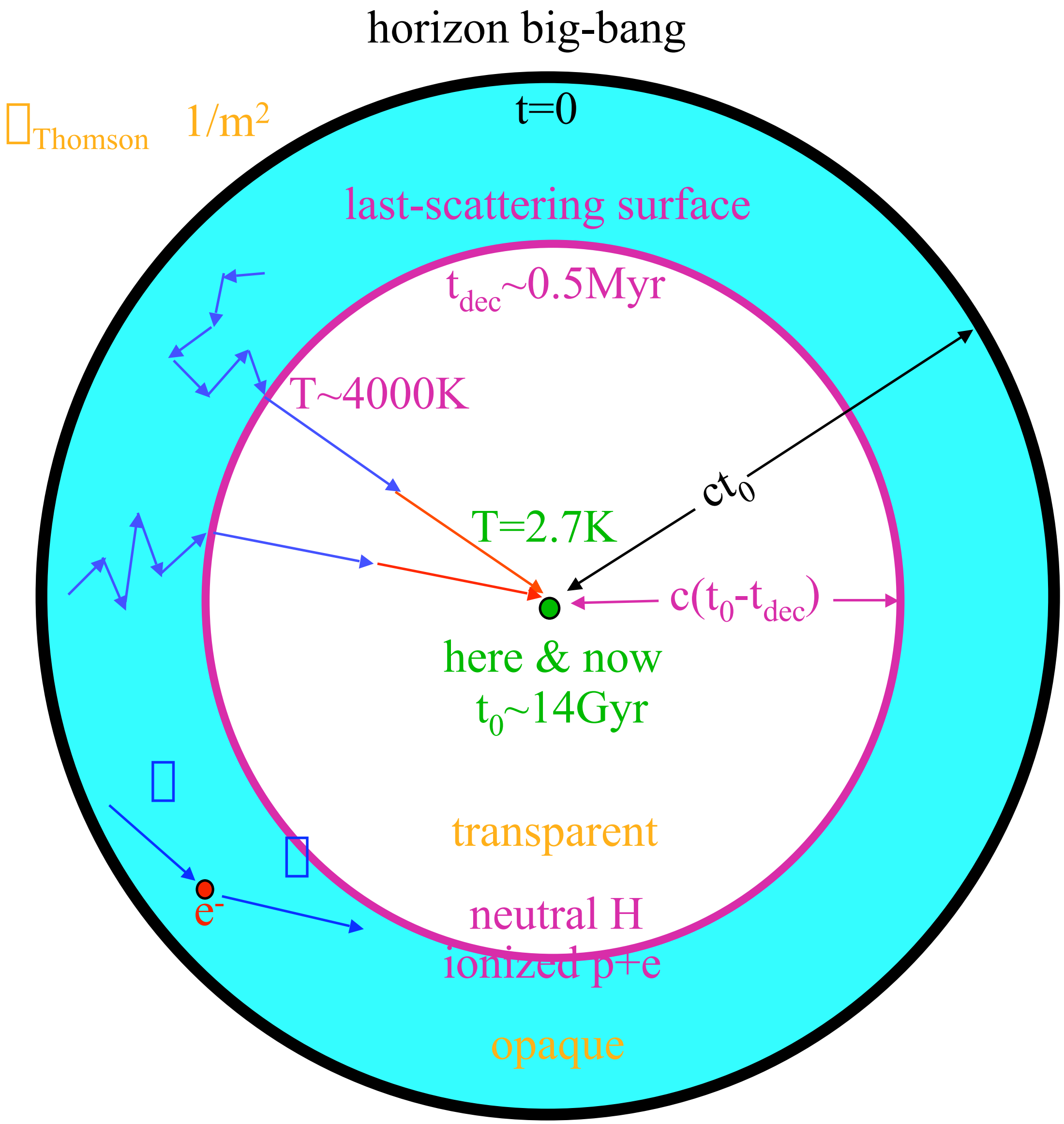
SN Cosmology Project

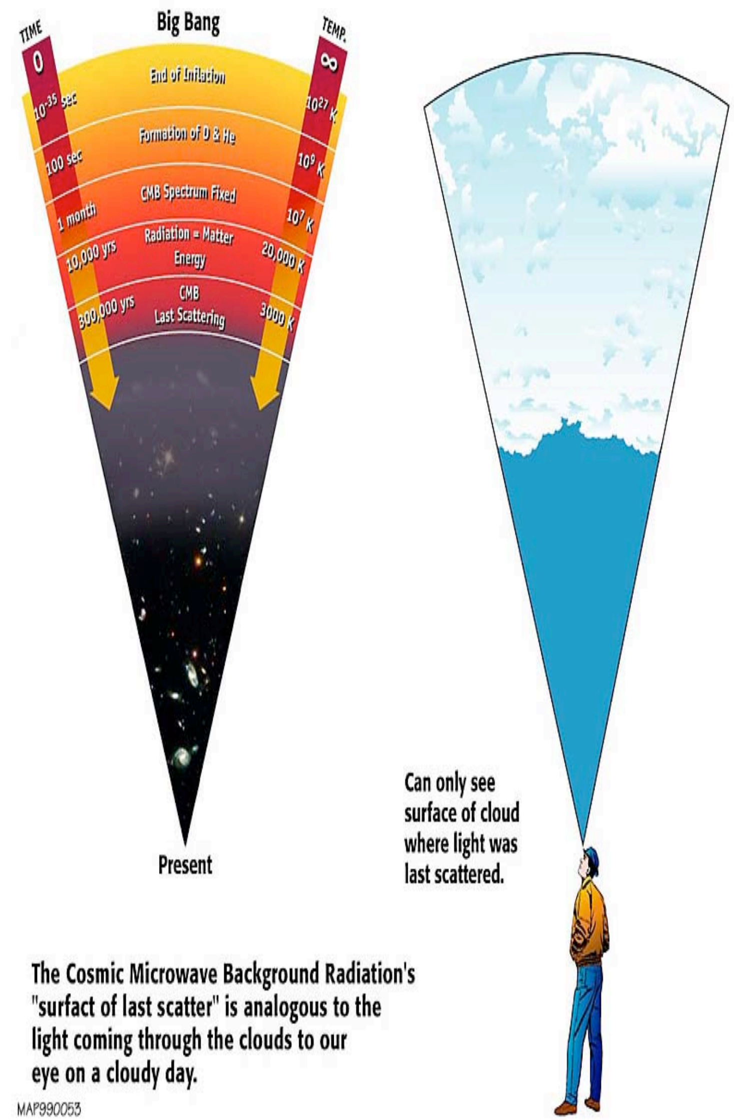


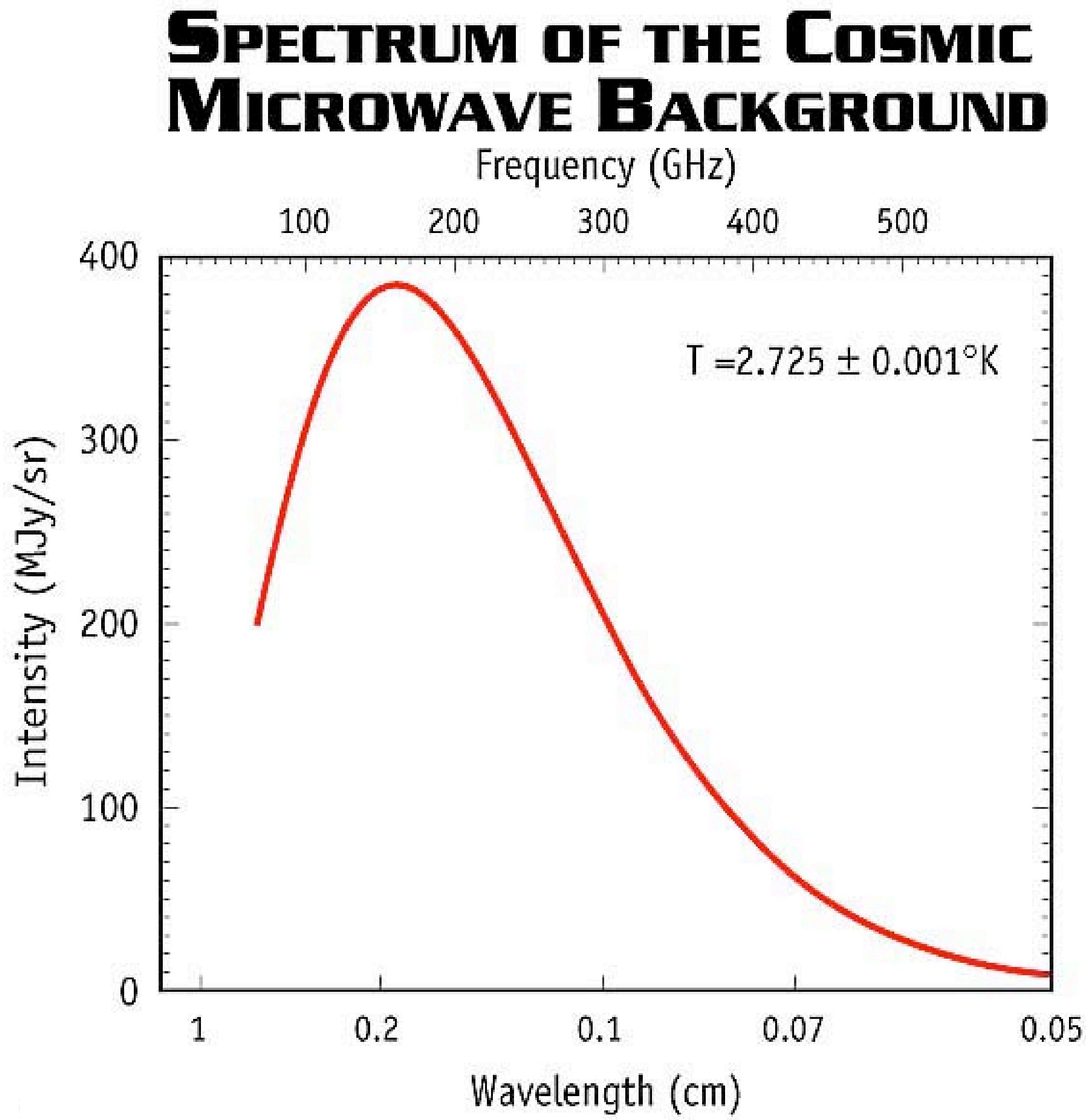
Cosmological Parameters



Cosmic Microwave Background







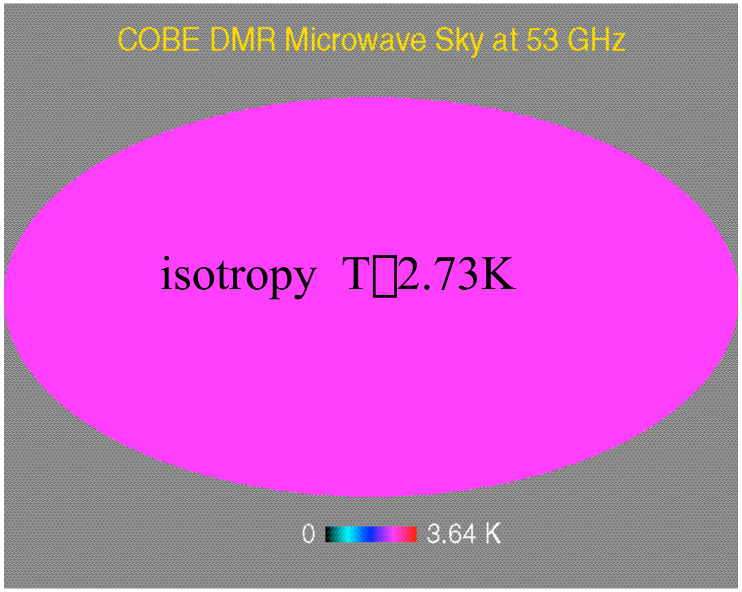
COBE 1992

Plank black-body spectrum

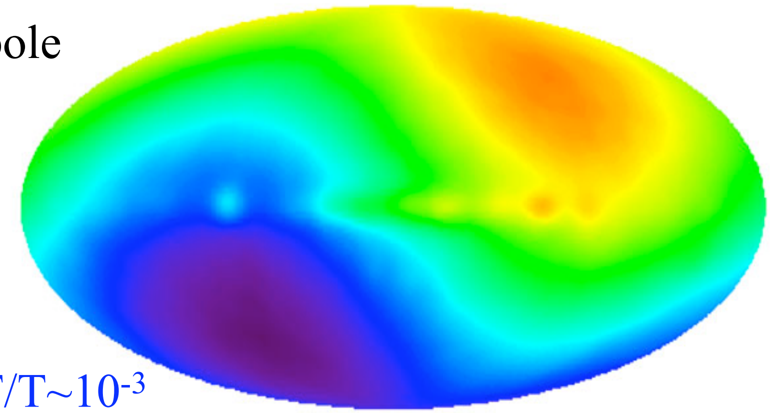
$$I(\nu)d\nu = \frac{2h}{c^2}\frac{\nu^3 d\nu}{e^{h\nu/kT} - 1}$$

I=energy flux per unit area, solid angle, and frequency interval

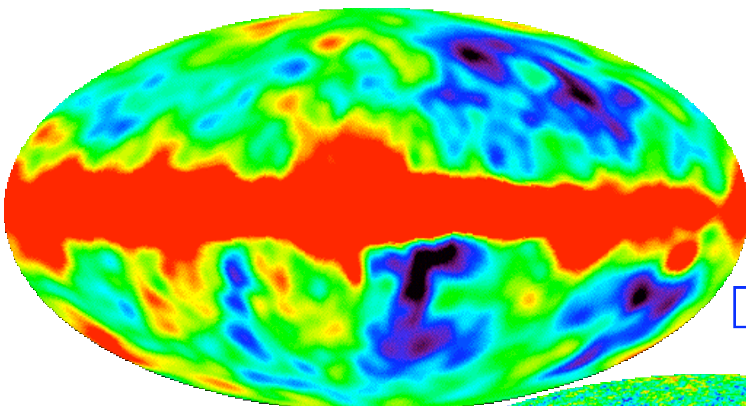
CMB Temperature



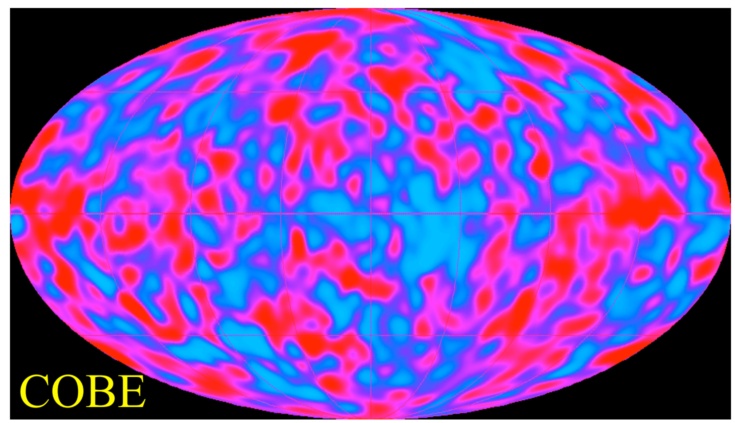
dipole



$\Delta T/T \sim 10^{-3}$

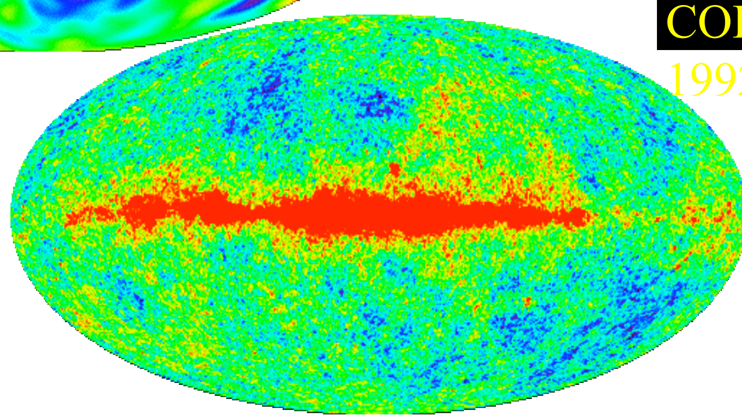


$\Delta T/T \sim 10^{-5}$



1992

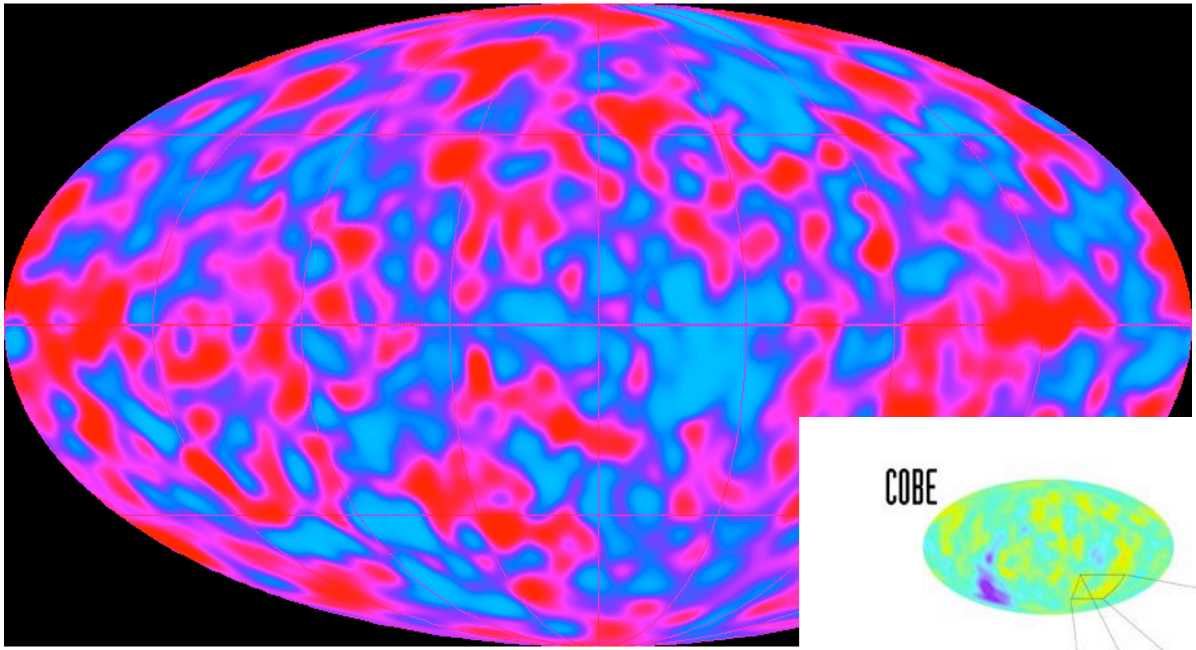
resolution ~10 degrees



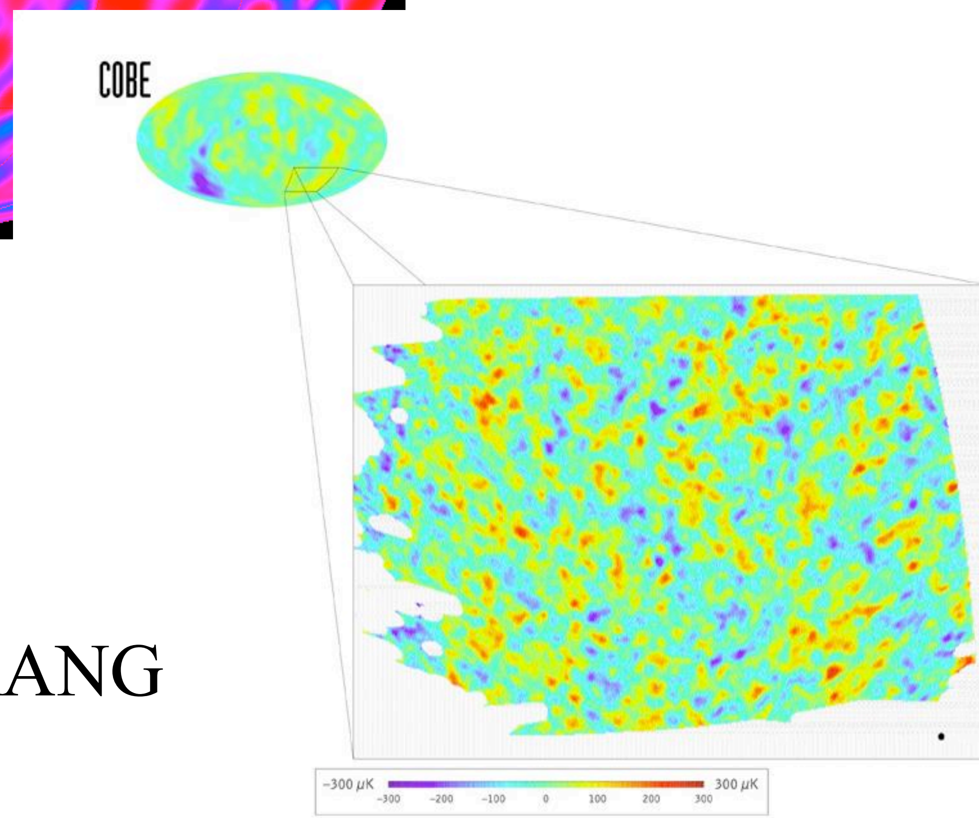
MAP simulation 2003

resolution ~ few arcmin

CMB Temperature Fluctuations

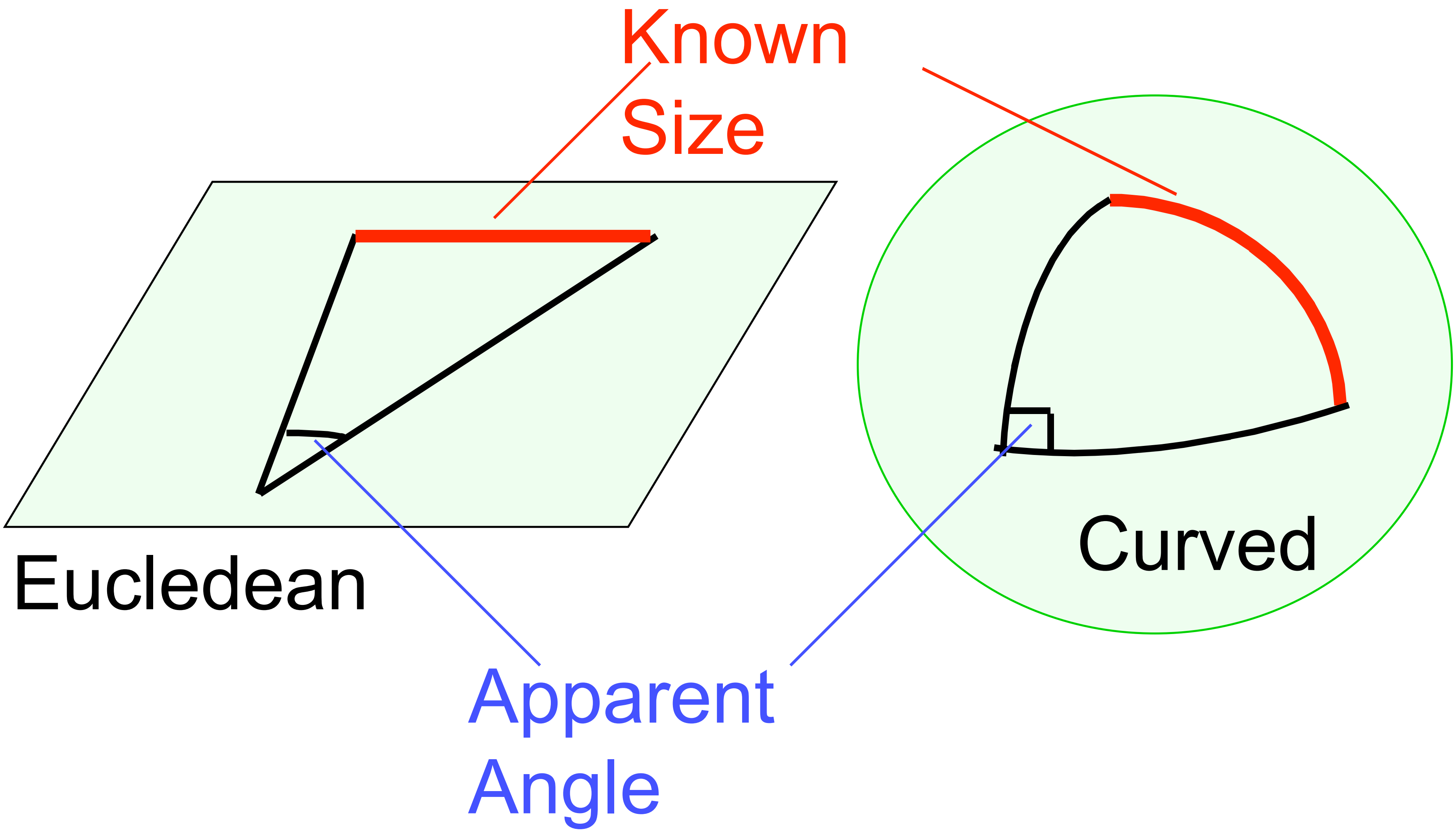


COBE 1992

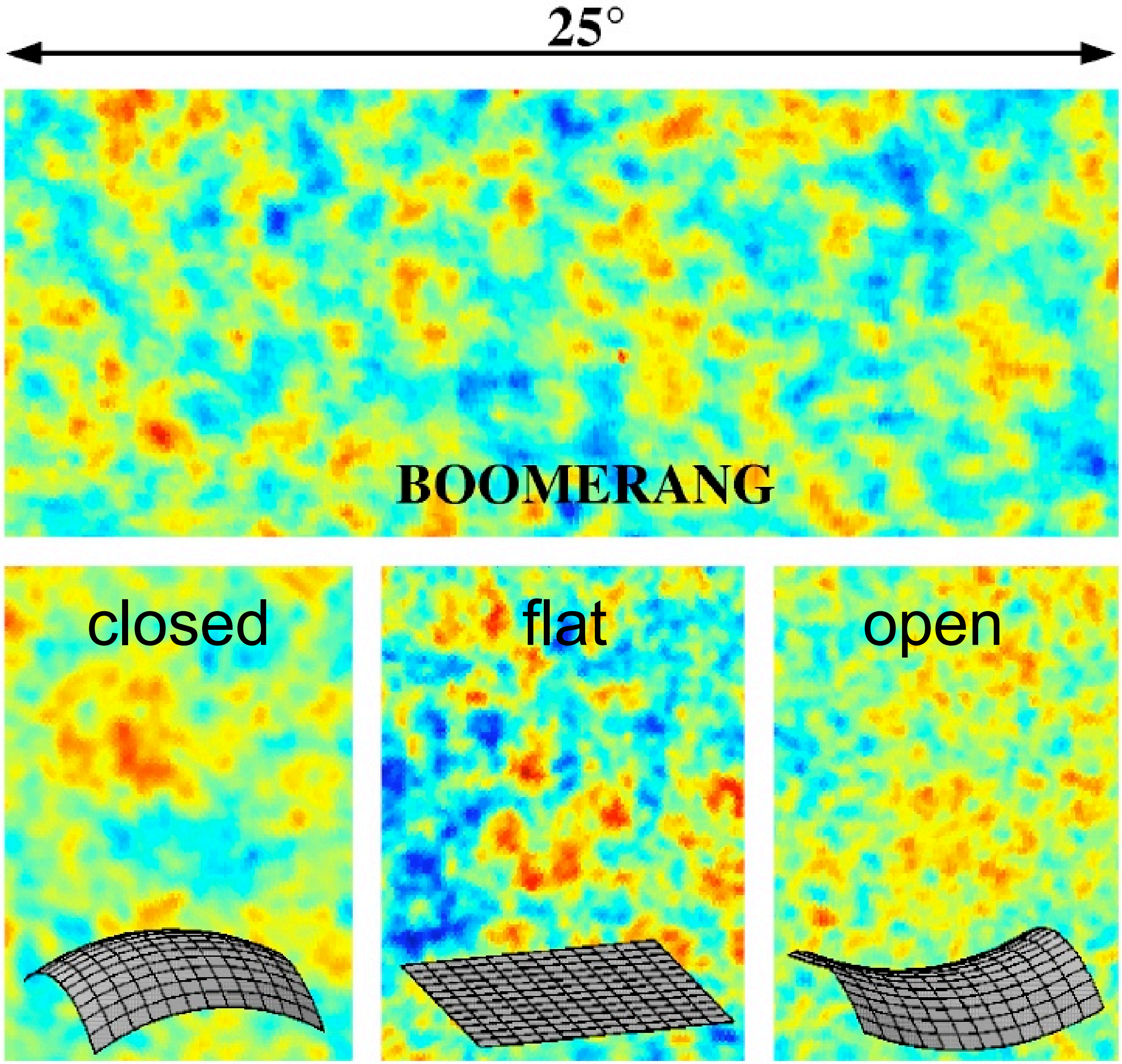


BOOMERANG
2002

Measuring Curvature



Space's Curvature



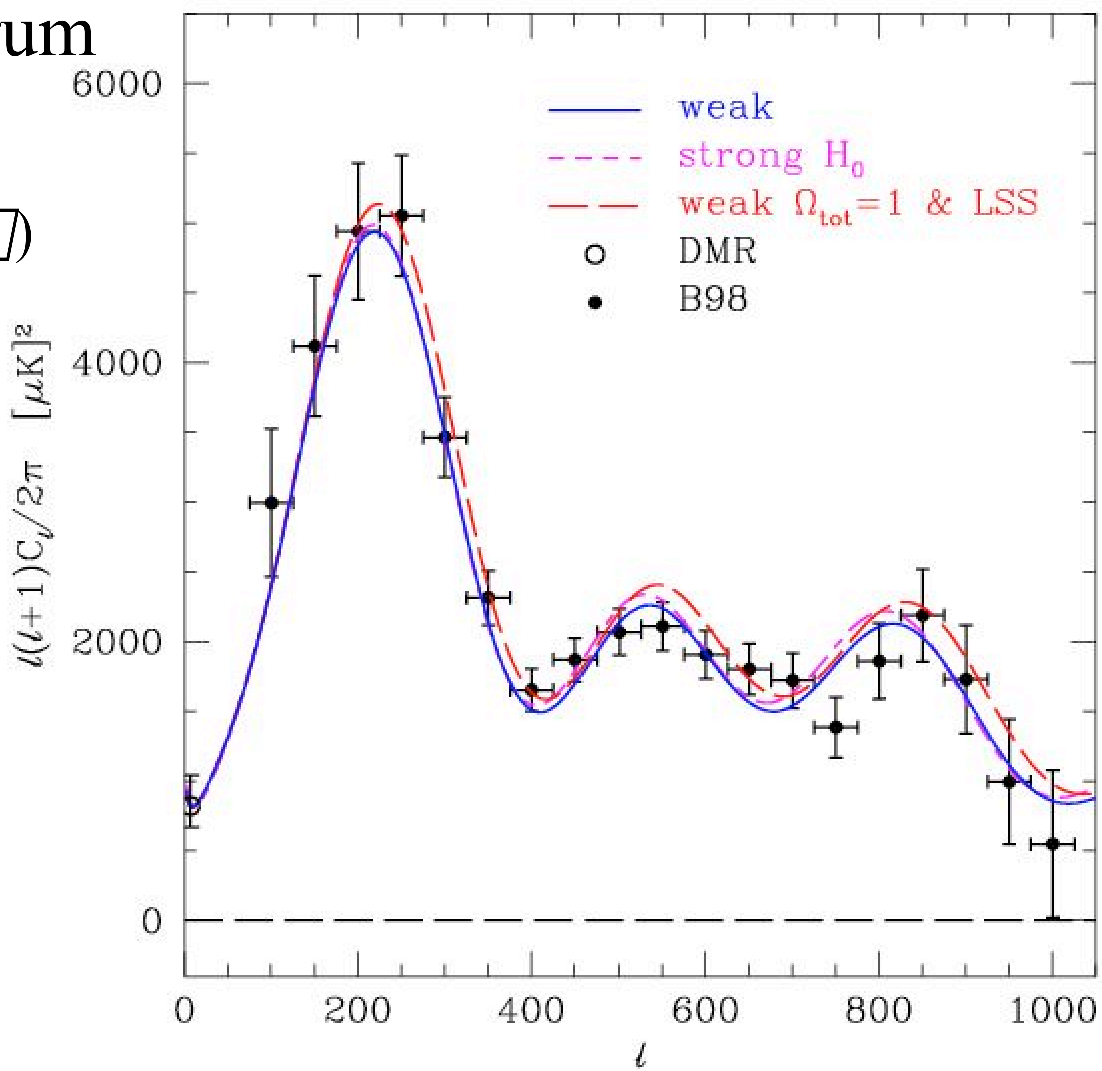
CMB anisotropy

Angular power spectrum

$$\frac{\Delta T(\theta,\phi)}{T} = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta,\phi)$$

$$C_l \equiv \left\langle |a_{lm}|^2 \right\rangle$$

$$\left\langle \left(\frac{\Delta T}{T}\right)^2 \right\rangle = \frac{l(l+1)}{2\pi} C_l$$



$\Rightarrow$ angle $\theta = 2\pi/l$

Origin of Peaks

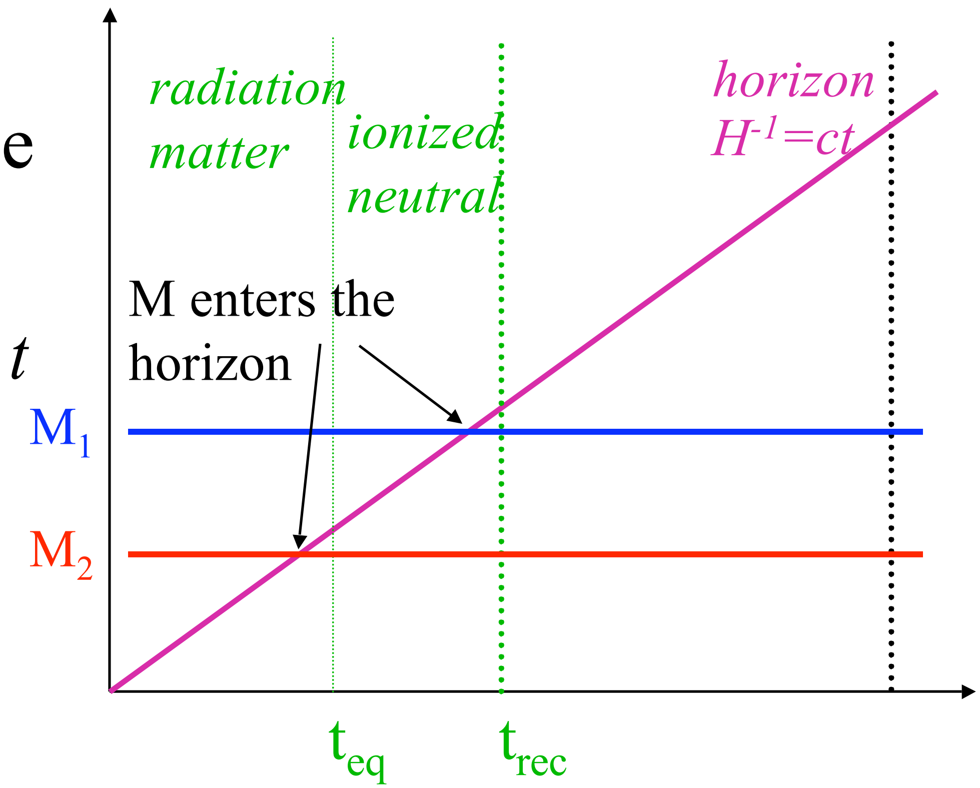
Horizon: scale

$r_h \quad t$ M

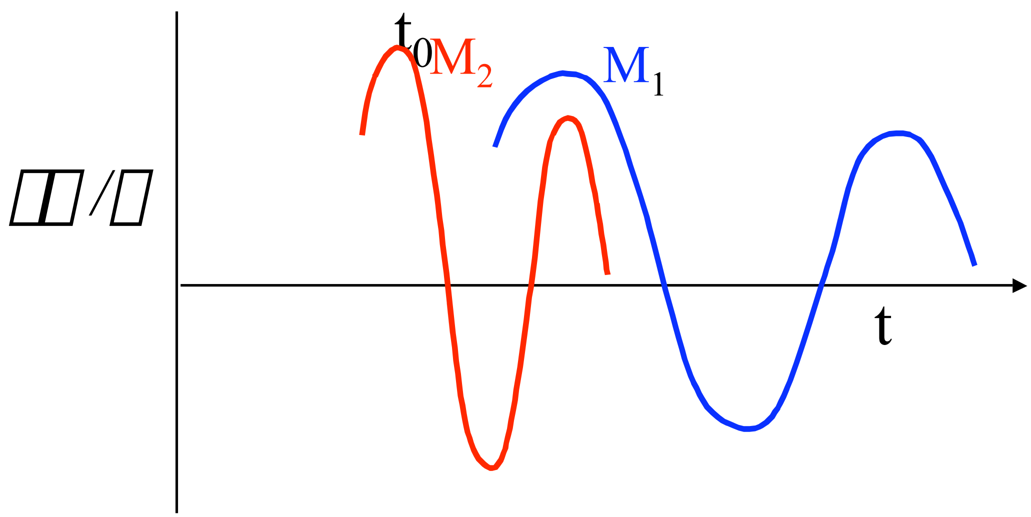
$$M_h \;\square\; r_h^{3} \quad (t^{2/3})^{\square 3}\, t^{3} \quad t$$

Comoving sphere:

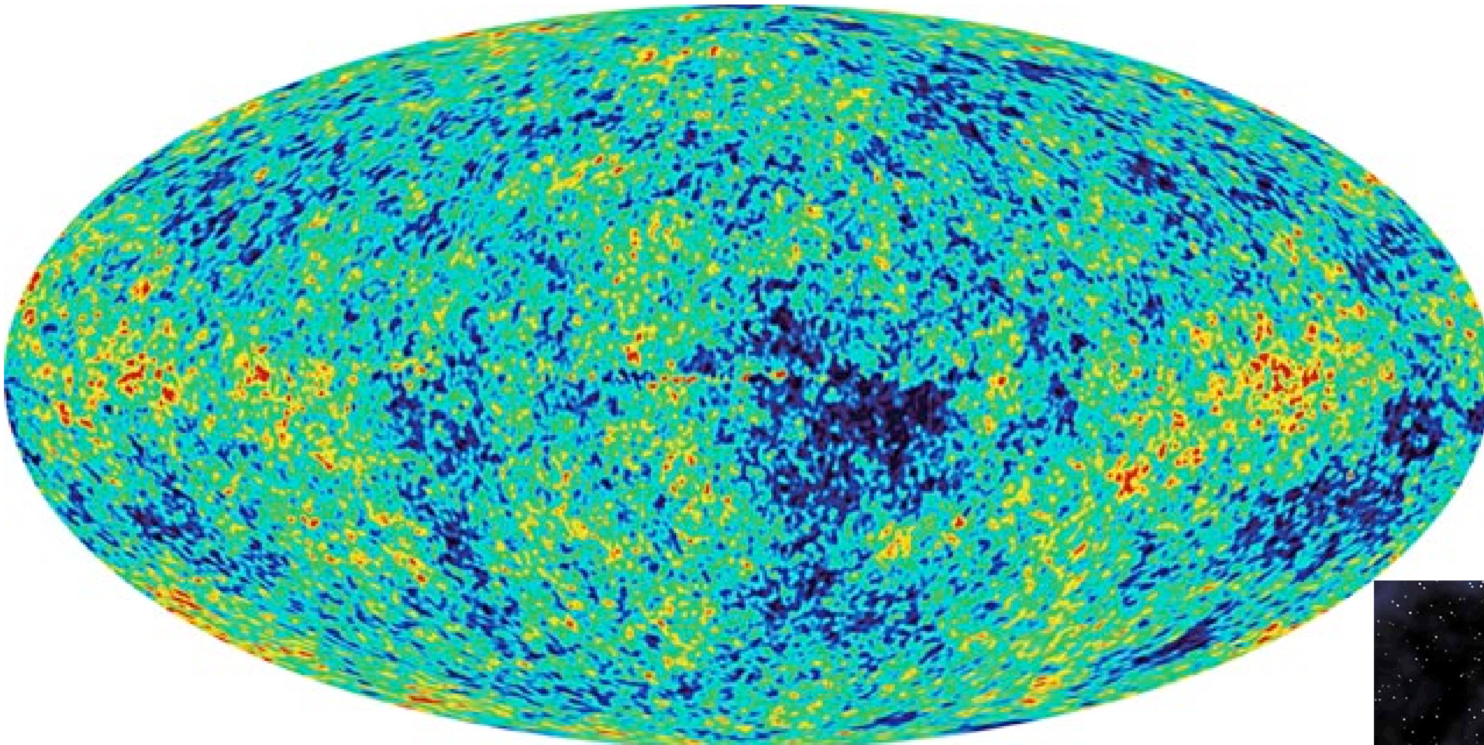
$$a \quad t^{2/3} \qquad M = const.$$

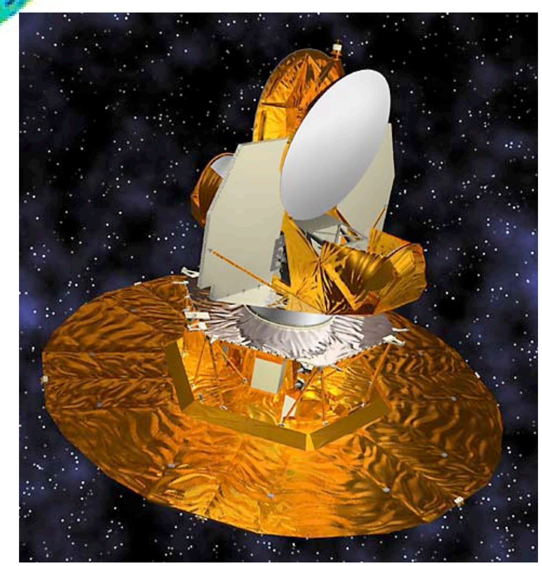


fluctuations grow after entering the horizon



Latest Results WMAP

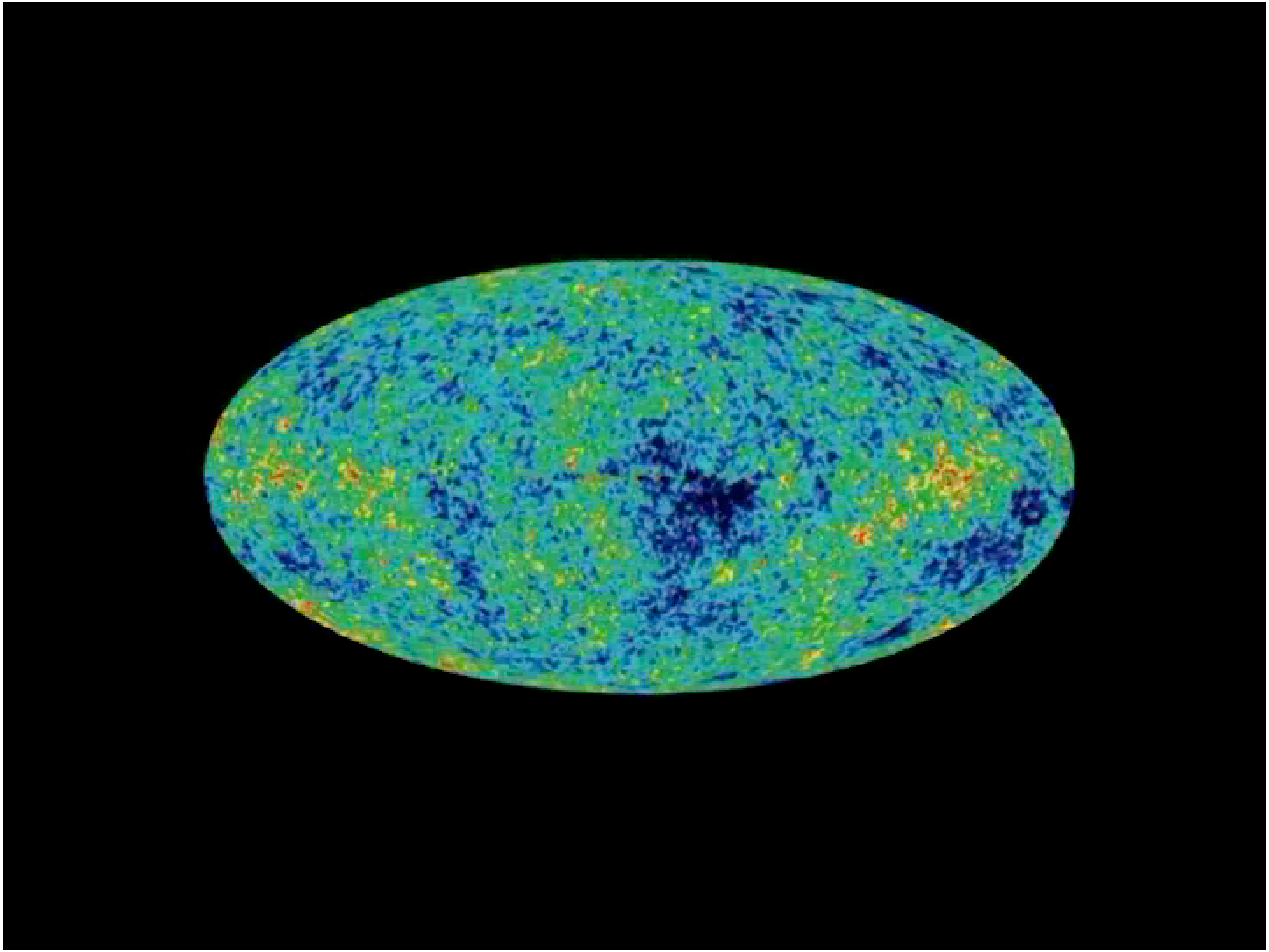


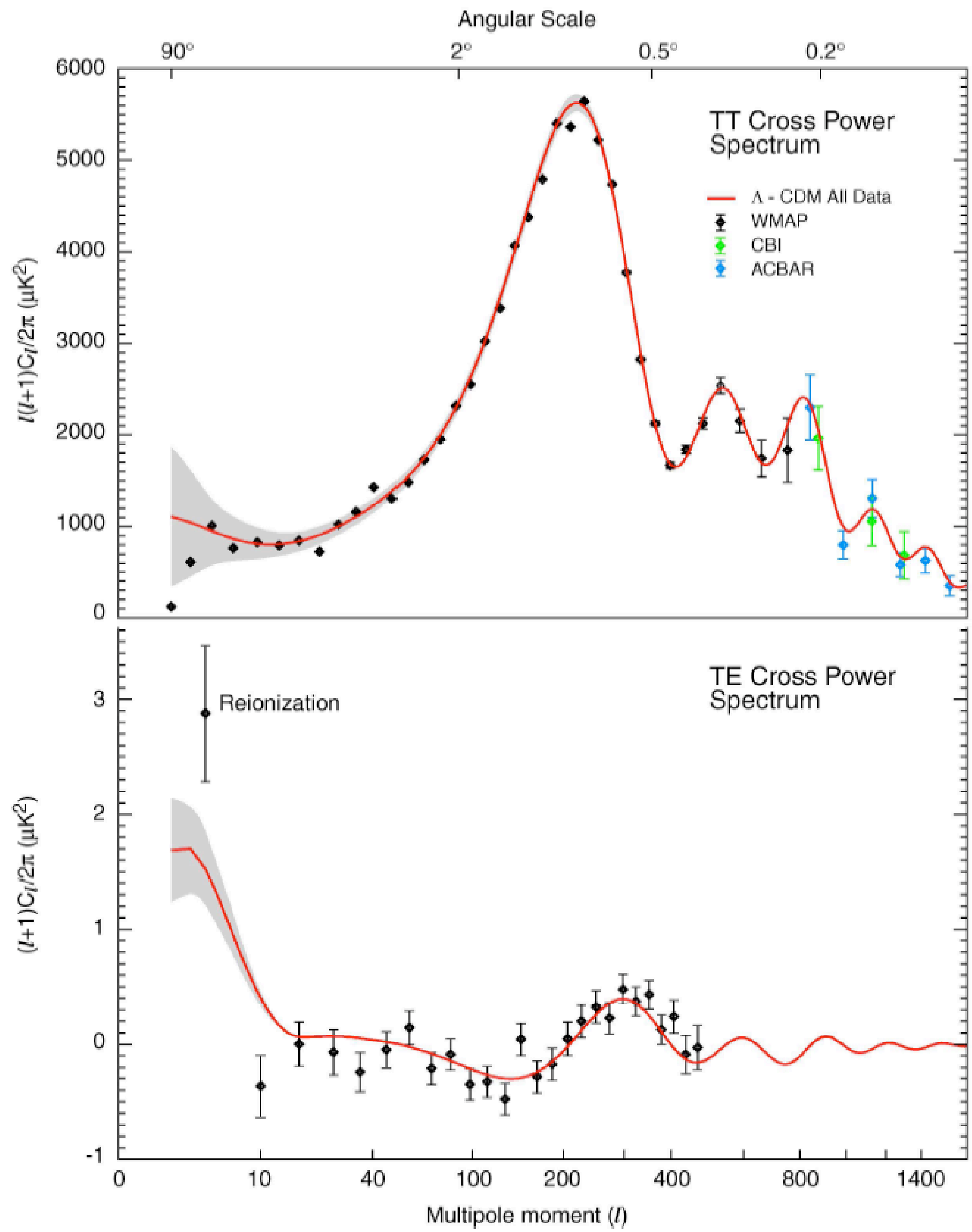


$$\Omega_m + \Omega_\Lambda = 1.02 \pm 0.02$$

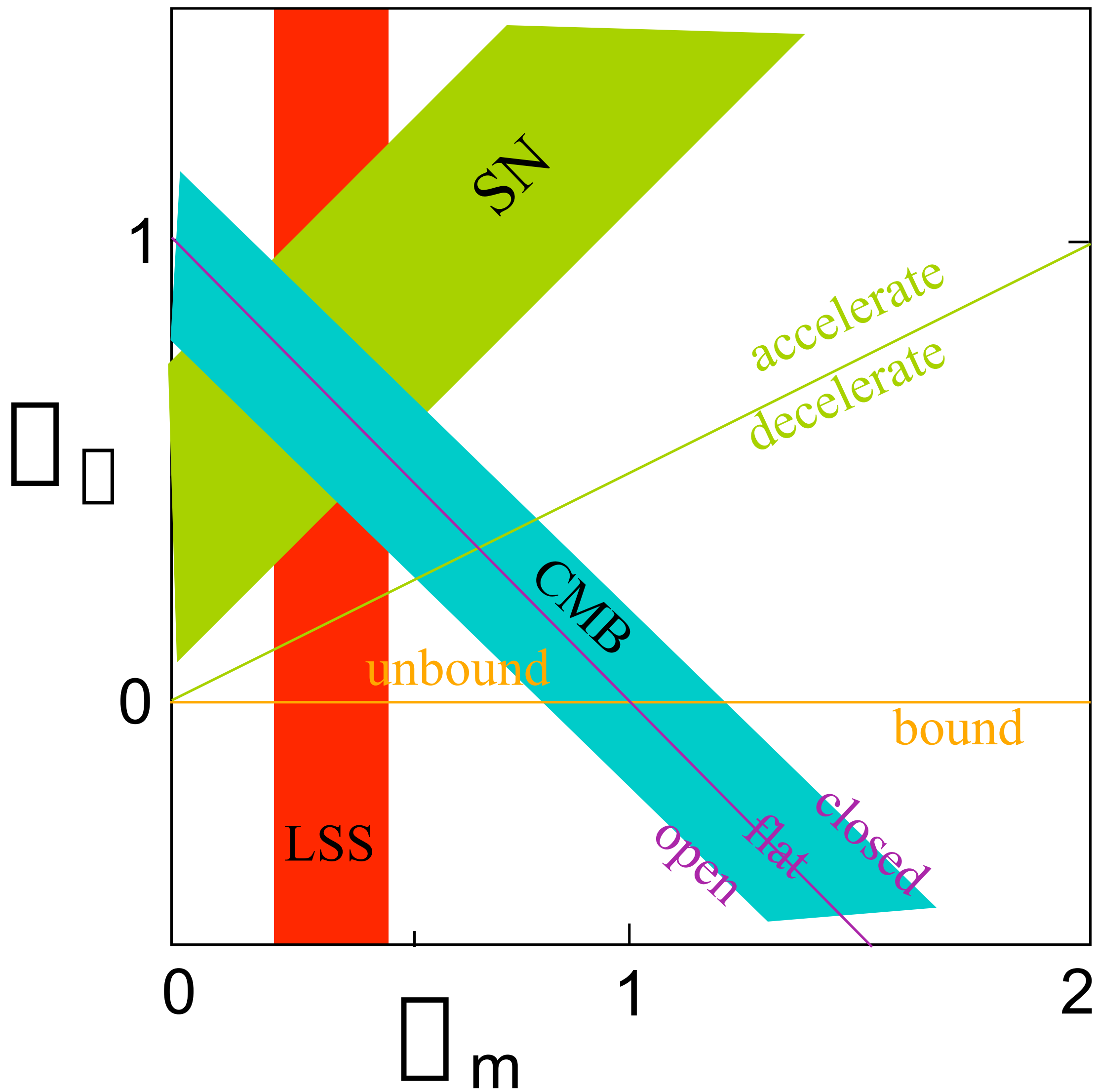
$$\Omega_b = 0.044 \pm 0.004$$

$$H_0 = 71 \pm 4 km/s/Mpc \qquad t_0 = 13.7 \pm 0.2 Gyr$$

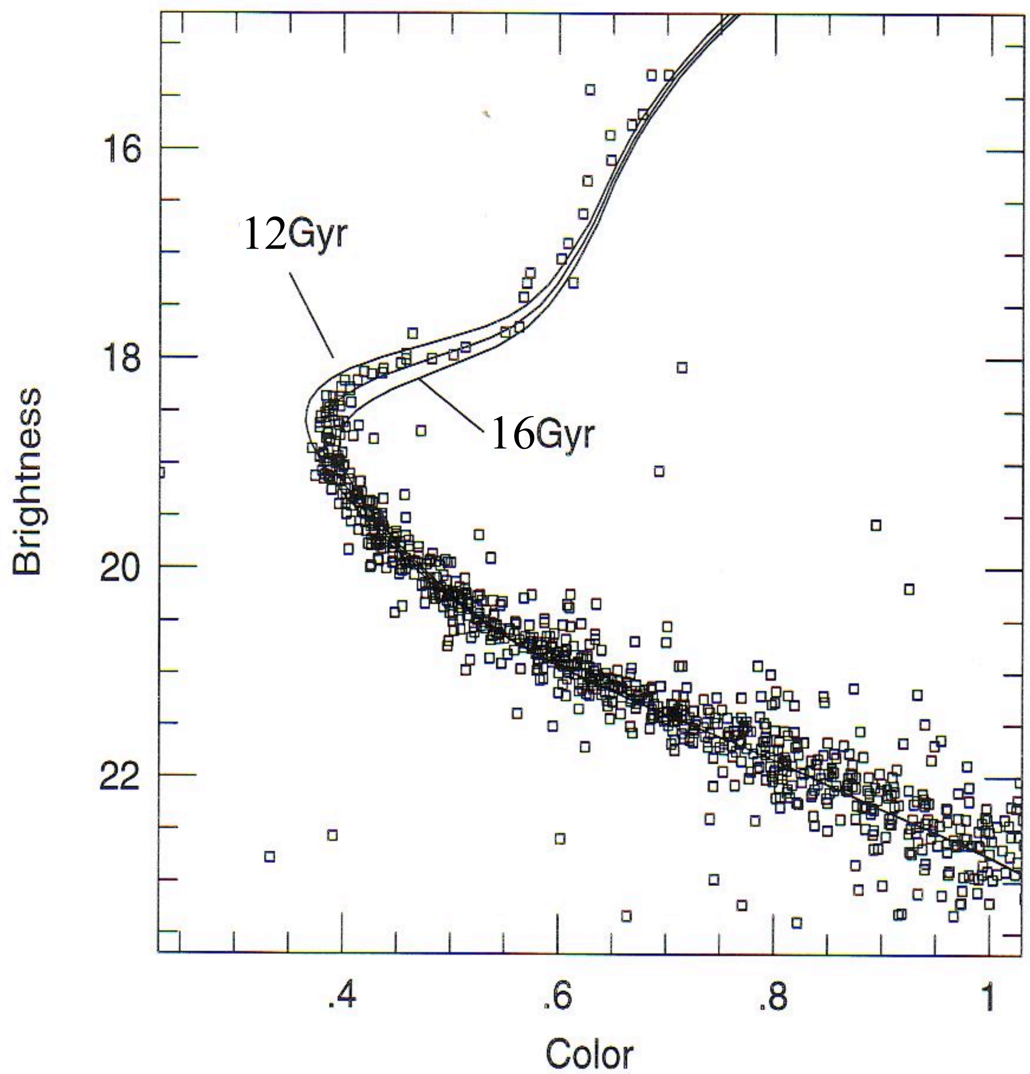



Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
l(l+1)C_l/2π (μK²)
3
2
1
0
-1
Reionization
TE Cross Power Spectrum
(l+1)C_l/2π (μK²)
0
10
40
100
200
400
800
1400
Multipole moment (l)

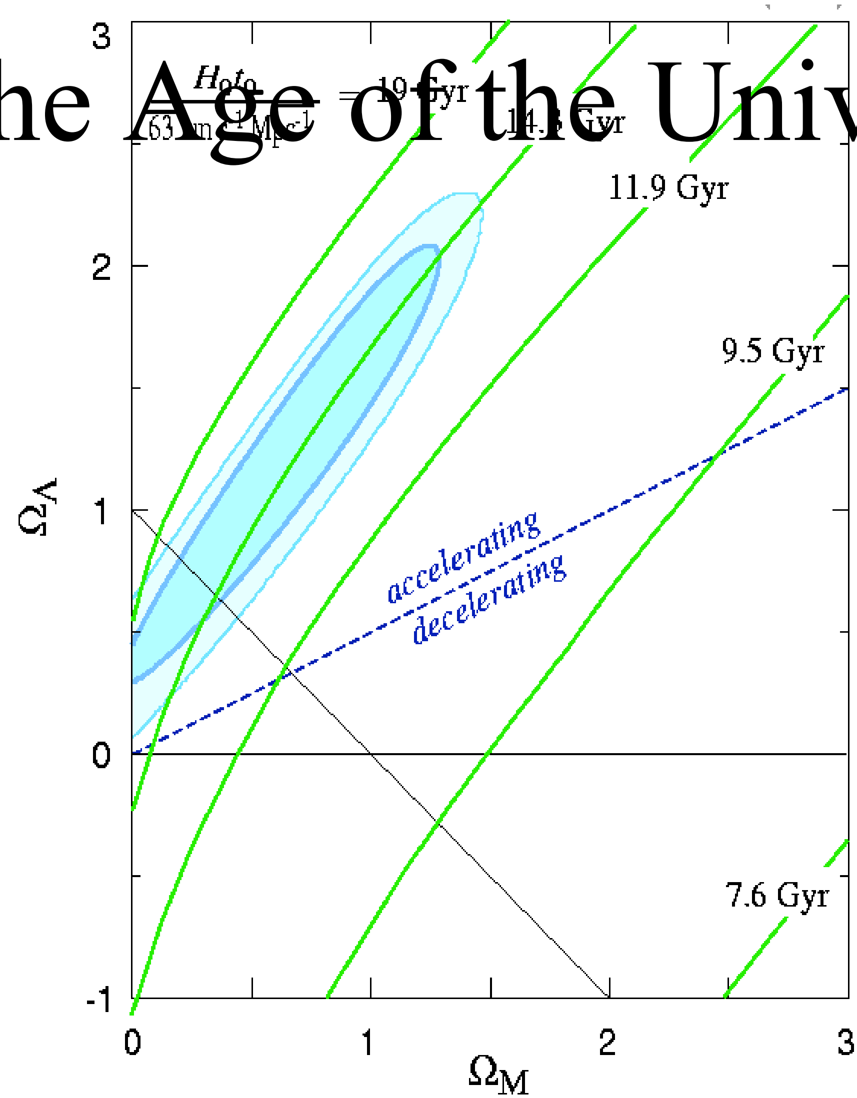
Cosmological Parameters



Age of an old star clusters



The Age of the Universe



Best fit age of universe: $t_o = 14.5 \pm 1 \;\; (0.63/h)$ Gyr

Best fit in flat universe: $t_o = 14.9 \pm 1 \;\; (0.63/h)$ Gyr

Big Bang Nucleosynthesis

$m_n > m_p \quad \Rightarrow \quad n + \nu \to p + e^-$
only 12.5% n left after decaying to
$p \to$ 75% H + 25% He (in mass)

At T~10^9K deuterium becomes stable and nucleosynthesis starts:

$$p + n \to d(pn) + \gamma$$

$$d \xrightarrow{p} {}^3He(ppn) \xrightarrow{n} {}^4He$$

A minute later p becomes too cold to penetrate the Coulomb barrier by p in d and process stops. Rate $\propto n_p^2 \Rightarrow$ abundances of d and ^3He decrease with Ω_b

$\Omega_b = 0.04 \pm 0.01$