Measuring Cosmological Parameters
($\Omega_i$, $\Omega$, $H_0$, etc...)
Various Methods:

The different methods can be divided to several “major” groups of methods:

• Measuring local characteristics sensitive to cosmological parameters.
• Measuring behavior vs. time/z (luminosity, number counts etc.) at high redshift
• Looking at the young (and linear) universe: The Cosmic Microwave Background.
Measuring Hubble

In flat universe: \( \Omega_M = 0.28 \pm 0.085 \) statistical [\( \pm 0.05 \) systematic]

Prob. of fit to \( \Lambda = 0 \) universe: 1%
Friedman Equation

Homogeneity (RW metric) + Gravity ( $G_{\Box \Box} \Box \Box g_{\Box \Box} = 8\Box GT_{\Box \Box}$ )

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\Box G}{3} \Box_m \Box \frac{k c^2}{a^2} + \frac{\Box c^2}{3}$$

$$\Box_m = \Box_{m0} \frac{a_0^3}{a^3}$$

$$1 = \Box m + \Box k + \Box \Box$$

$$\Box m \equiv \frac{\Box m}{3H^2 / 8\Box G}$$

$$\Box k \equiv \frac{k c^2}{a^2 H^2}$$

$$\Box \Box \equiv \frac{\Box c^2}{3H^2}$$

$$\Box_{tot} \equiv \Box m + \Box \Box = 1 \Box \Box k \quad \text{closed/open}$$

$$q \equiv \frac{\Box \Box \Box}{\dot{a}^2} = \frac{1}{2} \Box m \Box \Box \Box \quad \text{decelerate/accelerate}$$

For $\Box \Box = 0$

- $\Box m << 1 \quad a \quad t \quad Ht = 1$
- $\Box m = 1 \quad a \quad t^{2/3} \quad Ht = 2/3$
- $\Box m > 1 \quad a \quad 1 \Box \cos \Box \Box \quad Ht < 2/3$
Energy = \( \frac{1}{2} V^2 \frac{GM}{R} \)

Curvature = \( 1 + \frac{m}{R} \)

Acceleration = \( \frac{m}{R} + 2 \)

\( m < 1 \)  \( \Rightarrow \)  Eternal expansion

\( m = 1 \)  \( \Rightarrow \)  Big Bang

\( m > 1 \)  \( \Rightarrow \)  Now & Here  \( \Rightarrow \)  Collapse
Dark Matter and Dark Energy

\[ W_m/2 - W_L = 0 \]

\[ W_L = 0 \]

mass - attraction

vacuum - repulsion

bound

unbound

open

flat

closed
Total Luminous Matter

By counting the total amount of starlight from galaxies, and the number of galaxies, one can obtain (using \( \frac{M}{M_{\text{sun}}}/\frac{L}{L_{\text{sun}}} \sim \text{few} \):}

\[ \Box \text{luminous} \quad \Box 0.01 \]

Is the universe “empty?” Where is the rest of the mass?
Dark Matter
Galaxy Rotation Curves

Flat Rotation Curve!

$V^2 = \frac{GM(R)}{R}$

$\square M(R) \quad R$

300,000 lyrs
On larger scales, more Mass is missing!
Cosmic Flows

POTENT Reconstruction

Mark III

X [h^{-1}Mpc]

SF1

X [h^{-1}Mpc]
Dark-matter density in supergalactic plane
Mass density in 3D

Great Attractor

Perseus Pisces

Great Void
Most matter: Dark Matter

Dynamics $\delta_m = 0.35 \pm 0.05$

Using BBNS, $\delta_{\text{baryons}} = 0.05$

What is the dark matter?
Cosmological Parameters

- LSS
- Unbound
- Bound
- Open
- Flat
- Closed
- Accelerate
- Decelerate
Distant supernovae
In flat universe: $\Omega_M = 0.28 \pm 0.085$ (statistical) $\pm 0.05$ (systematic)

Prob. of fit to $\Lambda = 0$ universe: 1%
Energy = $\frac{1}{2} V^2 \left[ \frac{GM}{R} \right]$ $\square R^2$

Curvature = $\square 1 + \square m + \square \frac{G^2}{V^2}$

Acceleration = $\square m + 2 \square \frac{G^2}{V^2}$
Acceleration

Vacuum energy is responsible for an effective “repulsion force”:

\[ V_L = 0.65 \pm 0.05 \geq V_m \]
SN Cosmology Project

Supernova Cosmology Project
Perlmutter et al. (1998)

\[ \Omega = \frac{\text{Energy Density}}{\text{Critical Density}} \]

- No Big Bang
- 99%
- 95%
- 90%
- 68%
- 42 Supernovae
- SNAPsat
- Target Statistical Uncertainty
- Flat Universe
- \( \Lambda = 0 \)
- Expands forever
- Recollapses eventually
- Closed Universe
- Flat Universe
- Open Universe

Graph showing the relationship between \( \Omega_\Lambda \) and \( \Omega_M \).
Cosmological Parameters

![Diagram showing the relationship between dark matter density (\(\Omega_m\)) and dark energy density (\(\Omega\)) in the context of cosmological parameters. The diagram illustrates regions corresponding to different cosmological scenarios: open, flat, and closed universes, as well as regions indicating accelerated and decelerated expansion.]

- **Open Universe**: \(\Omega < 1\)
- **Flat Universe**: \(\Omega = 1\)
- **Closed Universe**: \(\Omega > 1\)
- **Accelerated Expansion**: \(\Omega_{\Lambda} > 0\)
- **Decelerated Expansion**: \(\Omega_{\Lambda} < 0\)

The diagram includes a shaded region labeled with 'SN' which likely represents a specific cosmological model or test case.
Cosmic Microwave Background

horizon big-bang

t=0

last-scattering surface

t_{dec} \sim 0.5 \text{Myr}

T \sim 4000 \text{K}

T = 2.7 \text{K}

here & now

t_{0} \sim 14 \text{Gyr}

translucent

neutral H

ionized p+e

opaque

\Omega_{\text{Thomson}} \sim 1/\text{m}^2
Spectrum of the Cosmic Microwave Background

COBE 1992

Plank black-body spectrum

\[ I(\varnothing) d\varnothing = \frac{2h}{c^2} \frac{\varnothing^3 d\varnothing}{e^{h\varnothing/kT} - 1} \]

I=energy flux per unit area, solid angle, and frequency interval
CMB Temperature

isotropy $T \approx 2.73K$

$\Delta T/T \approx 10^{-3}$

$\Delta T/T \approx 10^{-5}$

COBE DMR Microwave Sky at 53 GHz

dipole

COBE 1992 resolution ~10 degrees

MAP simulation 2003 resolution ~ few arcmin
CMB Temperature Fluctuations

COBE 1992

BOOMERANG
2002
Measuring Curvature

Known Size

Euclidean

Curved

Apparent Angle
Space’s Curvature

25°

closed
flat
open

BOOMERANG
CMB anisotropy

Angular power spectrum

\[ \frac{\Delta T(\theta, \phi)}{T} = \sum_{l=0}^{+\ell} \sum_{m=-\ell}^{\ell} a_{lm} Y_{lm}(\theta, \phi) \]

\[ C_l \equiv \langle |a_{lm}|^2 \rangle \]

\[ \langle \frac{T(\theta, \phi)}{T} \rangle^2 = \frac{l(l+1)}{2} C_l \]
Origin of Peaks

Horizon: $r_h \quad t \quad M$

$M_h \quad \Box r_h^3 \quad (t^{2/3}) \Box^3 t^3 \quad t$

Comoving sphere:

$a \quad t^{2/3} \quad M = \text{const.}$

fluctuations grow after entering the horizon

![Horizon: $r_h \quad t \quad M$](image)

-$\Box^3 r_h^3 (t^{2/3})$ growth
Latest Results WMAP

\[ \Omega_m + \Omega_b = 1.02 \pm 0.02 \]

\[ \Omega_b = 0.044 \pm 0.004 \]

\[ H_0 = 71 \pm 4 \text{ km/s/Mpc} \quad t_0 = 13.7 \pm 0.2 \text{ Gyr} \]
Cosmological Parameters

- SN
- CMB
- LSS

- accelerate
- decelerate
- unbound
- bound
- open
- closed
- flat
- decelerate
- accelerate
- unbound
- bound
- open
- closed
- flat
Age of an old star clusters
The Age of the Universe

**Best fit age of universe:** \( t_o = 14.5 \pm 1 \ (0.63/h) \) Gyr

**Best fit in flat universe:** \( t_o = 14.9 \pm 1 \ (0.63/h) \) Gyr
Big Bang Nucleosynthesis

\[ m_n > m_p \quad n + p + e^+ \]
only 12.5% \( n \) left after decaying to \( p \quad 75\% H + 25\% He \) (in mass)

At \( T \sim 10^9 \text{K} \) deuterium becomes stable and nucleosynthesis starts:
\[ p + n \quad d(pn) + \]
\[ d + p \quad ^3\text{He}(ppn) + n \quad ^4\text{He} \]

A minute later \( p \) becomes too cold to penetrate the Coulomb barrier by \( p \) in \( d \) and process stops. Rate \( n_p^2 \) abundances of \( d \) and \( ^3\text{He} \) decrease with \( b \)

\[ b = 0.04 \pm 0.01 \]

\[ b h^2 = 0.02 \quad \text{Density of Ordinary Matter (Relative to Photons)} \]