## Lecture 8

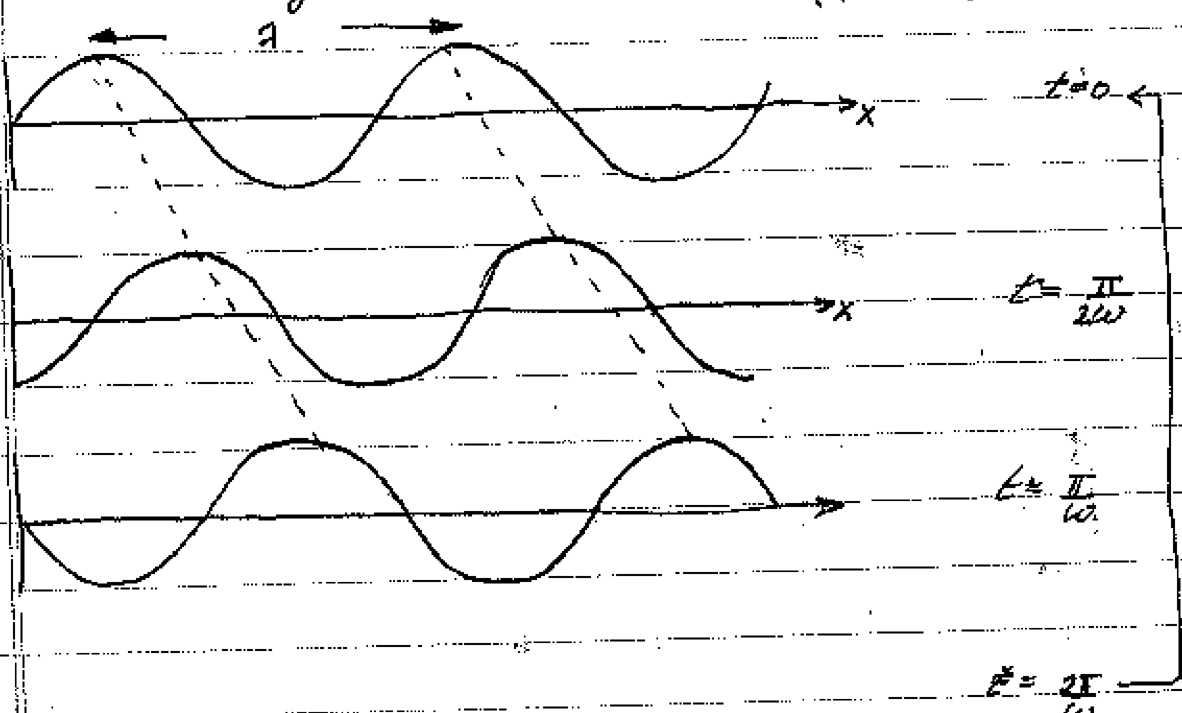
Last time we obtained solutions to the wave equation in a closed system. We found that they have the form of standing waves.

Today we are going to consider oscillations of open systems i.e. systems without an outer boundary.

Well, we already obtained the solutions that correspond to such oscillations. They are of the form

$$\psi(x,t) = e^{\pm i(kx \pm \omega t)} \quad \text{or equivalently} \quad \cos(kx \pm \omega t),\ \sin(kx \pm \omega t).$$

Let us look, say, at $A\sin(kx-\omega t) = A\sin\left(\frac{2\pi}{\lambda}x - \omega t\right)$




Something is moving to the right! Traveling Waves

You can convince yourself that similarly $\sin(kx+\omega t)$ describes a wave propagating to the left.

But *what* is propagating? : Different things:

1. The *phase*: Consider the phase of the wave $Ae^{i(kx-\omega t)}$, that is $\phi = kx-\omega t$ and ask the question where is the position along the wave at which $\phi$ has some constant value $\phi_0$?

   We easily find $kx-\omega t = \phi_0 \Rightarrow x = \dfrac{\phi_0+\omega t}{k}$

$\Rightarrow$ The positions of constant phase move to the right with velocity

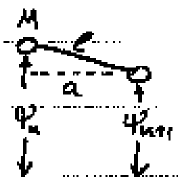
$$v_\phi = \frac{\omega}{k} \quad \text{: the \underline{Phase Velocity}}$$

This is the velocity at which individual crests in the wave propagate.

~~*Energy*: What is the energy stored in the wave?~~
~~To answer this question~~

In the system that we are studying now: the elastic string $\omega = \sqrt{\frac{T_0}{\mu}}\, k$ and so $v_\phi = \sqrt{\frac{T_0}{\mu}}$. It is the same for all the waves regardless of their wave vector $k$ (or equivalently their wave length $\lambda$ or frequency $\omega$). We will see that this is also the case for EM waves in the vacuum. such waves are called <u>non dispersive waves</u> However when one considers more realistic models for the string or EM waves that propagate through dielectric media one usually finds phase velocity that depends on $k$. Such waves are called <u>Dispersive</u>.

2. <u>Energy</u> What is the energy stored in a wave? To answer this question let us go back to the descrete model and consider a bead and the spring connecting it to its neighbor:




The kinetic energy of the bead $= \frac{1}{2} M \left(\frac{d\psi_n}{dt}\right)^2$

The potential energy of the spring $= \frac{1}{2} K l^2 = \frac{1}{2} K \left[(\psi_{n+1} - \psi_n)^2 + a^2\right]$

Substructing the equilibrium energy (bead at rest and spring of length $a \Rightarrow \frac{1}{2} K a^2$) and summing over the beads and springs we get

$$E = \sum_n \frac{1}{2} M \left(\frac{d\psi_n}{dt}\right)^2 + \frac{1}{2} K \left(\psi_{n+1} - \psi_n\right)^2$$

$$= \sum_n a \left[\frac{1}{2} \underbrace{\left(\frac{M}{a}\right)}_{\mu} \left(\frac{d\psi_n}{dt}\right)^2 + \frac{1}{2} \underbrace{(Ka)}_{T_0} \left(\frac{\psi_{n+1} - \psi_n}{a}\right)^2\right]$$

$$\xrightarrow[\text{limit}]{\text{in the continuum}} \int dx \underbrace{\left[\frac{1}{2} \mu \left(\frac{\partial \psi}{\partial t}\right)^2 + \frac{1}{2} T_0 \left(\frac{\partial \psi}{\partial x}\right)^2\right]}_{\text{the energy } \underline{\text{density}}} \equiv \int dx \, \mathcal{E}(x)$$

The energy density for our wave ~~is~~ $A\sin(kx-\omega t)$ is:

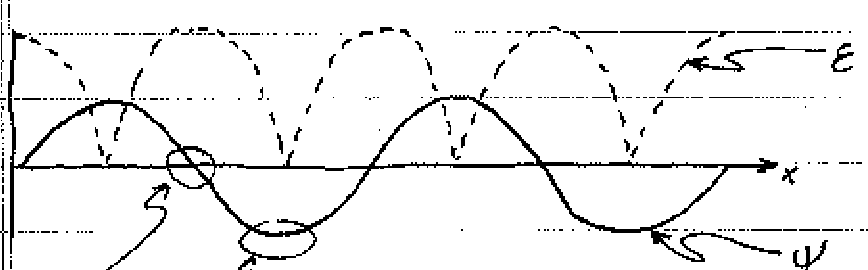
$$\mathcal{E}(x,t) = \frac{1}{2}\left(\mu\omega^2 + T_0 k^2\right) A^2 \cos^2(kx-\omega t)$$

$$= T_0 k^2 A^2 \cos^2(kx-\omega t) \qquad \Leftarrow \quad \omega = \sqrt{\frac{T_0}{\mu}}\, k$$

quadratic in the amplitude!

quadratic in the wave vector $\Leftrightarrow$ shorter wave length waves are more energetic!

Let's plot it at $t=0$




maximal bead velocity and spring stretching

here "bead" is at rest and spring not stretched

We thus see that the energy stored in the wave moves with it to the right.

From the discussion above it is also clear that the ~~mom~~

3. Momentum Density $\equiv \mu \frac{\partial \psi}{\partial t} = \cancel{\mu\omega\cos} - \mu\omega A\cos(kx-\omega t)$

travels with the wave.