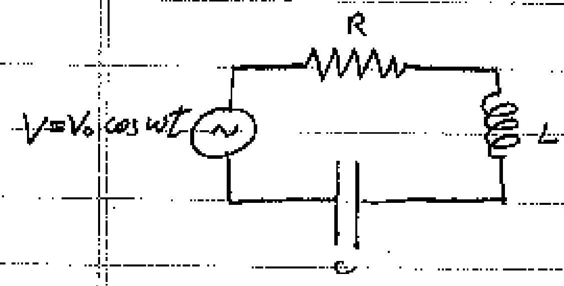
Lecture 5 + demonstration : forced harmonic oscillator

Now let's add a voltage generator to the circuit



The equation now reads $L \frac{d^2 q_T}{dt^2} + R \frac{d q_T}{dt} + \frac{q_T}{C} = V_T = V_0 \cos \omega t$

We will now treat V as well as q as complex quantity

Writing $q = q_R + i q_i$, $V = V_R + i V_i$ get

$$\left[L \frac{d^2 q_R}{dt^2} + R \frac{d q_R}{dt} + \frac{q_R}{C} \right] + i \left[L \frac{d^2 q_i}{dt^2} + R \frac{d q_i}{dt} + \frac{q_i}{C} \right] = V_R + i V_i$$

$\Rightarrow$ Thus if we take $V = V_0 e^{i\omega t}$ the real part of q satisfies the equation satisfied by the real physical quantities.

Put now $q = A e^{i\omega t}$ to get

$$\left[L (i\omega)^2 + R (i\omega) + \frac{1}{C} \right] A \cancel{e^{i\omega t}} = V_0 \cancel{e^{i\omega t}}$$

$$\Rightarrow A = \frac{V_0}{L(\omega_0^2 - \omega^2) + i R \omega} \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$

Can write $\frac{A}{V_0}$ as $\rho e^{i\theta}$ in the following way

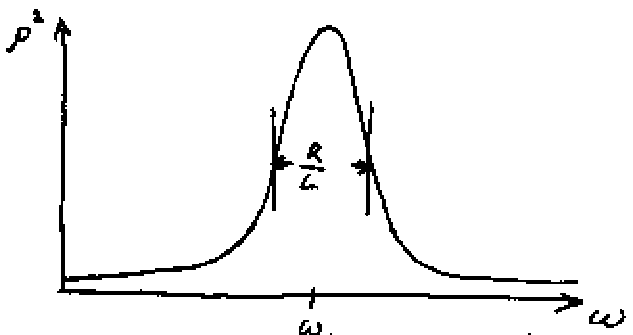
$$\frac{A}{V_0} = \frac{L(\omega_0^2-\omega^2) - iR\omega}{\left[L(\omega_0^2-\omega^2)+iR\omega\right]\left[L(\omega_0^2-\omega^2)-iR\omega\right]}$$

$$= \frac{L(\omega_0^2-\omega^2) - iR\omega}{L^2(\omega_0^2-\omega^2)^2 + (R\omega)^2}$$

$$\Rightarrow \quad \rho^2 = \frac{L^2(\omega_0^2-\omega^2) + (R\omega)^2}{\left[L^2(\omega_0^2-\omega^2)^2 + (R\omega)^2\right]^2} = \frac{1}{L^2(\omega_0^2-\omega^2)+(R\omega)^2} = \left[\left(\mathrm{Re}\,\frac{A}{V_0}\right)^2 + \left(\mathrm{Im}\,\frac{A}{V_0}\right)^2\right]$$

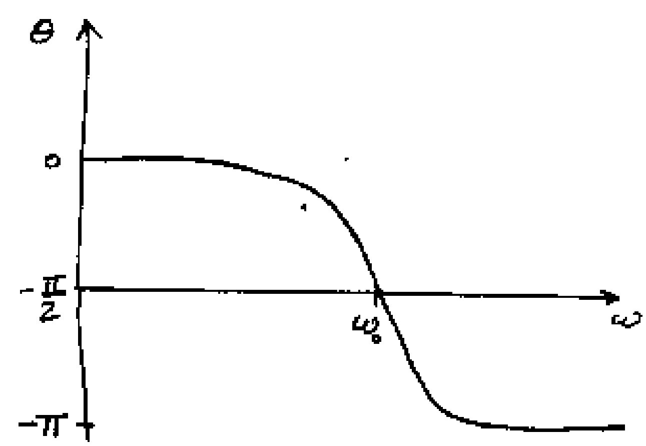
$$\tan\theta = -\frac{R\omega}{L(\omega_0^2-\omega^2)} = \left[\frac{\mathrm{Im}\ A/V_0}{\mathrm{Re}\ A/V_0}\right]$$

$$\Rightarrow \quad q = V_0\,\rho\, e^{i(\omega t+\theta)}$$



The Quality factor: $Q = \frac{\text{resonance frequency}}{\text{width of resonance}}$

$= \frac{\omega_0 L}{R}$



* The response of the system is peaked ~~for~~ when the ~~ext~~ frequency of the external perturbation matches the natural frequency ω_0 of the system: the frequency it have without external force or dissipation.

A Resonance

* Without any dissipation the responce diverges when $\omega = \omega_0$.
Dissipation limits this divergency. As we decrese R the peak gets sharper and sharper.
* Another typical property of resonances is the fact that the phase ~~shi~~ of the oscillations changes by π when ω is varied through the resonance frequency.

→ Read Sec 23.4 in F-I about various resonances occuring in nature

Discussion Session: 1. Oscillations in a ~~s~~ two body system, modes → W 1.4 2. Beats → W 1.5

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Resonances occur in many other circumstances (See Feynman)
1. Bridges in earthquakes
2. Oscillation of ions in a crystal (like salt - NaCl) induced by EM field
3. Resonance ~~in~~ ~~particle~~ elementary particle physics - seen when scattering particles off each other - vibrations of "internal" d.o.f.