Lecture 22 Reading Assignment French Ch 5, 6, 7.

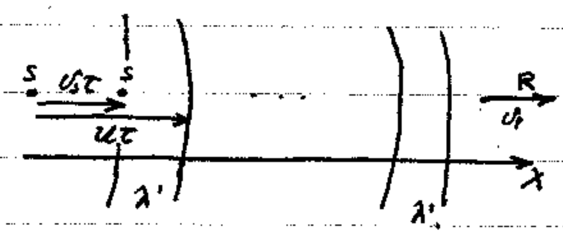
## The Doppler Effect

Consider first the non relativistic case:

A source of sound waves is moving along the $x$ axis with velocity $v_s$.
A receiver is moving in the same direction with velocity $v_r$.
We assume that the air is stationary.

If $\tau$ is the period of the wave emitted by $S$, and $u$ is the speed of sound the distance between consecutive wave fronts emitted by $S$ is:

$$\lambda' = (u - v_s)\tau$$




The speed of the wave relative to the receiver R is $u - v_r$.
$\Rightarrow$ The period as measured by R will be

$$\tau' = \frac{\lambda'}{u - v_r} = \frac{u - v_s}{u - v_r}\tau$$

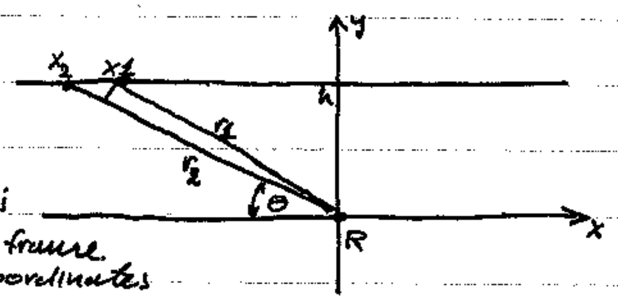
Or equivalently the relation between the emitted frequency $\omega$ and the received frequency $\omega'$ is $\left(\omega = \frac{2\pi}{\tau}\right)$

$$\omega' = \frac{u - v_r}{u - v_s}\omega \quad \Rightarrow \quad \omega' = \frac{1 - \frac{v_r}{u}}{1 - \frac{v_s}{u}}\omega$$

The result depends both on $v_r$ and $v_s$ since in this case we are measuring the velocities relative to the frame at which the medium carrying the sound - the air, is at rest. We will consider light shortly where there is no such frame and see what happens there.

Note that if the source and the receiver move towards each other say $u_s > 0$, $u_r < 0$ $\omega' > \omega$
while if they move away from each other $\omega' < \omega$.

Let us examine now the case where a source of light is moving at a constant velocity $v$ along the $x$ ~~axis but~~ direction but with $y = h$. The receiver is at rest at the origin.




The source emits light with period $\tau$ as measured in his rest frame.
I denoted by $x_1$ and $x_2$ the coordinates of the emision of conseoutive wave fronts as measured in R's rest frame.
Because of time dilation the time difference between the emission events as measured by R is

$$t_2 - t_1 = \gamma \tau$$

The light takes $r_1/c$ and $r_2/c$ respectively to reach R.

Thus the measured time separation between the arrival of the two wave fronts to R as measured by him is

$$\tau' = \left(t_2 + \frac{r_2}{c}\right) - \left(t_1 + \frac{r_1}{c}\right) = \gamma\tau - (r_1 - r_2)/c$$

If $x_2 - x_1 \ll r_1$ we have $r_1 - r_2 \simeq (x_2 - x_1)\cos\theta$

$$= v\gamma\tau\cos\theta$$

$$\Rightarrow \tau' = \gamma\tau\left(1 - \frac{v}{c}\cos\theta\right)$$

Or: $\omega' = \dfrac{\omega}{\gamma\left(1 - \frac{v}{c}\cos\theta\right)} = \omega\,\dfrac{\sqrt{1 - \left(\frac{v}{c}\right)^2}}{1 - \frac{v}{c}\cos\theta}$

Lets consider the case $h = 0 \Rightarrow \theta = 0$

$$\omega' = \omega\,\frac{\sqrt{\left(1+\frac{v}{c}\right)\left(1-\frac{v}{c}\right)}}{1 - \frac{v}{c}} = \omega\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \overset{v \ll c}{\simeq} \omega\sqrt{\left(1+\frac{v}{c}\right)^2} = \omega\left(1+\frac{v}{c}\right)$$

Compare this with the non relativistic result for the case $u_r = 0$

$$\omega' = \frac{1}{1 - \frac{v}{c}}\,\omega \overset{v \ll c}{\simeq} \omega\left(1 + \frac{v}{c}\right)$$

The important differences between the relativistic and non relativistic results:

1. The former contains both $v_r$ and $v_s$ separately. The latter depends only on the relative velocity. This is because light has no rest frame.

2. Relativity predicts a Doppler effect even when the source emits the light in a perpendicular direction to its direction of propagation. No such classical effect exist.
We then find

$$\omega' = \sqrt{1-\left(\frac{v}{c}\right)^2}\ \omega \qquad : \left(\theta = \frac{\pi}{2}\right)$$

$$\overset{v \ll c}{\simeq} \left[1 - \frac{1}{2}\left(\frac{v}{c}\right)^2\right]\omega$$

a second order effect in $\left(\frac{v}{c}\right)$.
Hard to observe but was confirmed experimentally.