Lecture 19 Reading Assignment : French Chap. 3,4,5
F-I ch. 15

We have spent the last few weeks studying the nature of wave phenomena in general and light in particular.
We have seen that waves propagate through systems of coupled oscillators with characteristic velocity. This velocity is determined by the properties of the carrying medium and is defined in the frame which is at rest relative to the medium.
As we learned light is a wave in the EM field. It is then natural to speculate that there exist a frame in which the EM field is at absolute rest and where the velocity of light is c. By measuring the velocity of light in a frame that is in motion relative to the above mentioned rest frame one expects to find a different value than c. This kind of reasoning led physicists in their quest to find the absolute rest frame of light - the Ether, throughout the second half of the nineteenth century.
However, as hard as they tryed to detect the motion of the Earth relative to the Ether they failed time and again.
They always measured the same velocity of light c.
Probably the most important ~~exper~~ of these experiment was carried out by Michelson and Morley in 1887 and you are going to analize it in the discussion session.

This problem was actually related to another problem that Maxwell's equations have had, aparently, with the principle of relativity.
This principle was introduced in the context of mechanics by Galileo and Newton. Einstein promoted it to a statement about all the laws of physics:

Principle of Relativity: All inertial frames are equivalent with respect to all the laws of physics.

In order to understand the meaning of this statement we need to clearly define first few concepts.

We are interested in describing Events. An event is any physical occurance such as radioactive decay of a nucleus, bomb explosion, collision of galaxies etc.

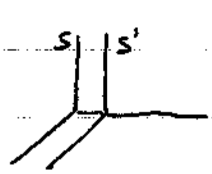
Physics is concerned with the quantative description of events. In order to do so we assign to each event an "address". There are many different ways one can do it and each of them defines a different Reference Frame. - its Coordinates

You may think about a reference frame as mesh of rods and clocks that fills space. Probably the symplest kind of a reference frame, and the one we are going to consider in the remining

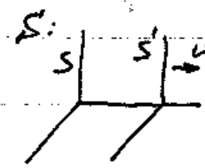
part of the course, is made out of a rectangular grid of rods of equal lengths (cartesian system) and synchronized clocks.
A possible way to synchronize the clocks is to send light from a clock at the origin of the frame at time 0 and to reflect it back to the origin from a clock at position A. Since experiments tell us that the velocity of light is always c then if the clock at the origin reads t_0 when it's hit by the reflected light the clock at A should be set in a way that gives a time $\frac{t_0}{2}$ for the event of the arrival of light from the origin.

It is crucial to understand that when we say that "a bomb went off at coordinates (x, y, z, t) of Spacetime", we are actually making a statement about the coincidence of events. We mean that the event of the explosion was simultaneous with the event of the display of the clocks at the end of the rods marking the point (x, y, z) turning t.
It is important for you to realize that when two events occur at two different points in space the time assigned to them is read from two different clocks!

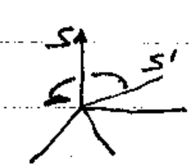
The reference frame S we described above is only one possible frame. Other frames can have their origin displaced relative to the origin of S :



, they can move relative to S:



S' rotate around S :



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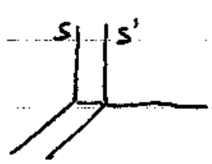
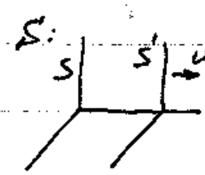
It is crucial to understand that when we say that "a bomb went off at coordinates (x, y, z, t) of <u>Spacetime</u>", we are actually making a statement about the <u>coincidence</u> of events. We mean that the event of the explosion was simultaneous with the event of the display of the clocks at the end of the rods marking the point (x, y, z) turning t.
It is important for you to realize that when two events occur at two different points in space the time assigned to them is read from two <u>different</u> clocks!

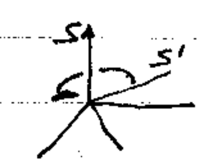
The reference frame S we described above is only one possible frame. Other frames can have their origin displaced relative to the origin of S :



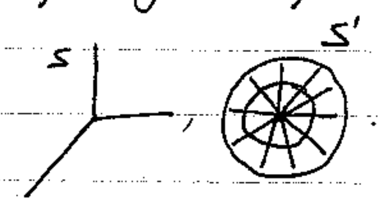
, they can move relative to S:



s' rotate around S :



or they can use a different system of rods (and possibly clocks).
For example a spherical coordinate system



The coordinates of an event in reference frames S and S' are related by a <u>Transformation Law</u>. For example if the coordinates of the event in S (the simple frame that we are using) are (x,y,z,t) and S' is a spherical coordinates frame at rest with respect to S, with the same origin as S, then the coordinates in S' are $(r,\varphi,\theta,t') = \left(\sqrt{x^2+y^2+z^2},\ tg^{-1}\left(\frac{y}{x}\right),\ tg^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right),\ t\right)$.

Physical laws are statements about the relations between events and they are expressed in mathematical form in terms of the coordinates of the events.

For example: Newton's first law states:

Every object free of the action of forces continues in a state of rest or of uniform motion in a straight line

In the cartesian reference frame S it means that the coordinates of the events defined by such an object are given by $(x,y,z,t) = (x(t), y(t), z(t), t)$ with $x = x_0 + u_x t$ $y = y_0 + u_y t$ $z = z_0 + u_z t$.

Reference frames in which Newton's first law holds true are called <u>Inertial Frames</u>.

You can easily convince yourself that if S is an inertial frame and S' is a frame moving at a constant velocity relative to S then S' is an inertial frame too. However if S' is rotating with respect to S it is not inertial.

The principle of relativity then means that if an experiment is carried out in two inertial frames the coordinates that experimentalists in the two frames assign to the events of the experiment may be different. However, the relation between these coordinates - the physical laws that they deduce, will be the same.

For example: If an experimentalist in S measures the position of an object free of forces which is at rest at the origin he will assign the following coordinates to the object: $(0,0,0,t)$.

An experimentalist in a frame S' moving in the positive X direction with velocity v with respect to S will assign the coordinates $(-vt,0,0,t)$ to the same object. However they will both conclude that Newton's first law is obeyed.

Newton introduced the principle of relativity in the context of mechanics. Einstein generalized it to include all the laws of physics: If experimentalists in two different frames agree on Newton's first law they agree on everything else! (in the context of physics)

Prior to Einstein people beleived that:

1. Newtonian dynamics is correct.

2. Coordinates of an event (x, y, z, t) measured in an inertial frame S are related to the coordinates in a frame S' moving at a constant velocity v along the x direction relative to S, are related through the

<u>Galilean Transformation</u>:

$$x' = x - vt \qquad x = x' + vt'$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = t \qquad t = t'$$

Let us see that this is consistent with the principle of relativity. The basic statement of Newtonian mechanics is $\vec{F} = m\vec{a}$. Unless we have an explicit law of force $\vec{F} = m\vec{a}$ is only a definition of $\vec{F}$. So let us consider a force provided by the interaction between two bodies. Suppose it is a function only of their separation. Then we can put $F_{12} = f(x_1 - x_2)$. This is the force exerted on body 1 by body 2. Thus in frame S the equation of motion of body 1 is:

$$f(x_1 - x_2) = m_1 a_1 = m_1 \frac{d^2 x_1}{dt^2}$$

Since according to the Galilean transformation
$x_1' - x_2' = (x_1 - vt) - (x_2 - vt) = x_1 - x_2$ we have $F_{12}' = f(x_1' - x_2') = f(x_1 - x_2)$

Also $\frac{d^2 X_1}{dt^2} = \frac{d^2(X_1'+vt')}{dt'^2} = \frac{d^2 X_1'}{dt'^2}$ $\Rightarrow$ In Newtonian Mechanics the acceleration is <u>Invariant</u>

and since in Newtonian dynamics the inertial mass is a constant $m_1 = m_1'$ we get

$$F_{12}' = f(X_1'-X_2') = \frac{d^2 X_1'}{dt'^2} = m_1' a_1'$$

$\Rightarrow$ Newton's second law is invariant under Galilean transformations. Its form is the same in all inertial frame - in accordance with the principle of relativity.

Our conclusion holds for any force that depends on the <u>relative</u> positions of the bodies. It would fail if the force depends on their absolute position like $X_1^2 + X_2^2$. Nothing in our experience, however, revealed such a situation.

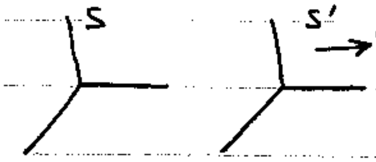
When people tried to check the invariance of the newly discovered Maxwell's equations under Galilean transformations they ran into trouble. To see why consider the wave equation for EM waves in vacuum. We saw that it is a consequence of Maxwell's equations in vacuum. Written for the fields in frame S' it reads:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

(we forget about the vector nature of E)

In S' we have according to Galileo transformation

$$\frac{\partial}{\partial x} = \frac{\partial x'}{\partial x}\frac{\partial}{\partial x'} = \frac{\partial}{\partial x'}$$



$$\Rightarrow \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial x'^2} \text{, similarly } \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial y'^2} \quad \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial z'^2}$$

But
$$\frac{\partial}{\partial t} = \frac{\partial x'}{\partial t}\frac{\partial}{\partial x'} + \frac{\partial t'}{\partial t}\frac{\partial}{\partial t'} = -v\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}$$

$$\Rightarrow \frac{\partial^2}{\partial t^2} = \left(-v\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}\right)^2 = v^2\frac{\partial^2}{\partial x'^2} - 2v\frac{\partial^2}{\partial x'\partial t'} + \frac{\partial^2}{\partial t'^2}$$

If we assume, as we do in Newtonian mechanics, that the charge and electric forces, and thus the electric fields are the same in S and S : $E' = E$ we get:

$$\left(1-\frac{v^2}{c^2}\right)\frac{\partial^2 E'}{\partial x'^2} + \frac{\partial^2 E'}{\partial y'^2} + \frac{\partial^2 E'}{\partial z'^2} + \frac{2v}{c^2}\frac{\partial^2 E'}{\partial x'\partial t'} - \frac{1}{c^2}\frac{\partial^2 E'}{\partial t'^2} = 0$$

$\Rightarrow$ The wave equation <u>is Not</u> invariant under Galilean transformation. An observer in S' will deduce that light behaves differently from the way it does in S – contrary to the principle of relativity. There are 3 possibilities:

1. The principle of relativity is wrong.
2. Maxwell's equations are wrong.
3. Galilean transformation is wrong.

It seemed almost obvious at the time that the newly discovered Maxwell's equations are wrong, so the thing to do was to change them in such a way that under Galilean transformations the principle of relativity would be satisfied. When this was tried, the new terms that had to be put into the equations led to predictions that did not exist when compared with experiments. It then gradually became apparent that Maxwell's equations were correct and that the trouble must be sought elsewhere.

Einstein firmly believed in the principle of relativity and the correctness of Maxwell's equations. As a logical consequence of that (since the velocity of light appearing in Maxwell's theory can not depend on the frame of reference, otherwise the theory will look differently in different frames), he obtained his second postulate:

The speed of light in vacuum always has the same value c.

But this meant that the law of Galilean transformations, and indeed Newtonian dynamics itself had to be modified.