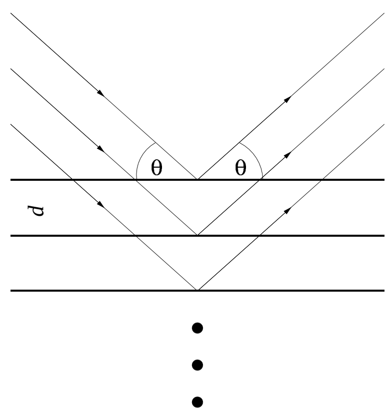
Homework Assignment #5 **Physics 17 Fall 99**

Due: Thursday, November 11

1. A plane wave of wavelength λ is specularly reflected (i.e. the angle of incidence θ equals the angle of reflection) from an infinite set of partially reflecting planes, separated by distance d. Assume that the planes can reflect waves coming from above but are transparent for waves arriving from below.

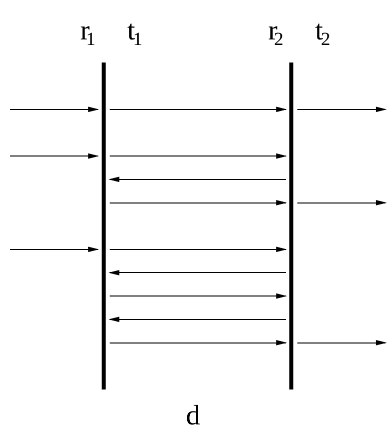


Show that the condition for the reflected waves to interfere constructively is

$$n\lambda = 2d\sin\theta \, , \tag{1}$$

where n is an integer. Such reflection is called Bragg reflection and (1) is known as the Bragg condition.

2. A Fabry-Perot interferometer is made out of two parallel partially reflecting plates separated by distance d.



The reflection and transmission coefficient of the left plate are r_1 and t_1 respectively, i.e., if the amplitude of the incident wave is 1 the amplitudes of the reflected and transmitted waves are r_1 and t_1 respectively. r_2 and t_2 are the corresponding coefficients of the right plate.

An incident plane wave of amplitude 1 and wavelength λ is passed through the interferometer. It can reach the other side either by going through both plates or after being reflected several times inside the interferometer, as I indicated in the figure.

(a) Write an expression for the amplitude of the wave that is transmitted to the right after n internal reflections (can n be odd?). Sum over all these amplitudes to find the total amplitude of the transmitted wave.

(b) Calculate the intensity of the transmitted wave. Plot it as a function of d/λ for several values of $r \equiv r_1 r_2$ (You might want to use Matlab or Mathematica for this purpose). What happens when r is increased from 0 to 1?

3. Find the diffraction pattern of light of wavelength λ from a system of N slits. The width of the slits is a and their centers are separated by a distance b. Hint: Use the following

   (a) The field amplitude far from the slits is the Fourier transform of their transmission function.

   (b) The diffraction pattern that we calculated for a single slit.

   (c) The convolution theorem.

4. A monochromatic plane wave of wavelength $\lambda = 0.5\mu$m (μm$= 10^{-6}$m$= 10^{-4}$cm) is passed through two slits. The distance between the centers of the slits is 2μm and their width is 0.2μm. One slit is covered by a $1/12\mu$m thick glass window with refractive index $n = 1.5$. Find the far field diffraction pattern.

5. The human brain is capable of resolving two points if their angular separation is about twice their diffraction width. The diameter of your pupil is typically 2mm. How far away is a car when you can barely resolve the two headlights? Take the distance between the headlights to be 2m and assume that they emit most of their light at wavelength of 5500Å.