Homework Assignment #4 **Physics 17 Fall 99**

Due: Thursday, November 4

1. The electric field of a plane EM wave in vacuum is given, in CGS units, by

$$E_x = 0 \, , \quad E_y = 0.5 \cos\left[2\pi \times 10^{10}\left(\frac{x}{c} - t\right)\right] \, , \quad E_z = 0 \, . \tag{1}$$

   (a) Find the wavelength, frequency, polarization and direction of propagation of this wave.

   (b) Find the magnetic component of the wave from Maxwell's equation.

   (c) Calculate the Poynting vector and its time average over one cycle.

2. Consider a standing EM wave with electric component polarized in the y direction

$$E_y(x,t) = E_0 \cos(kx)\cos(\omega t) \, . \tag{2}$$

   (a) Find the magnetic component of the wave and show that it is related to the electric component in the following manner

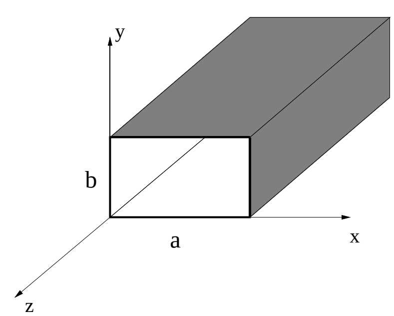
$$B_z = E_y\left(x - \frac{\lambda}{4}, t - \frac{T}{4}\right) \, , \tag{3}$$

   where λ and T are the wavelength and period respectively.

   (b) Find the energy density of the wave.

   (c) Calculate the Poynting vector and the time average of the energy flux through a cross-section which is perpendicular to the x axis.

3. In a metallic rectangular wave guide the lowest frequency electric mode (TE) is given by



$$\mathbf{E} = E_0 \sin\left(\frac{\pi}{a}x\right)\cos(kz - \omega t)\,\mathbf{j} \, , \tag{4}$$

$$\mathbf{B} = -E_0\frac{kc}{\omega}\sin\left(\frac{\pi}{a}x\right)\cos(kz - \omega t)\,\mathbf{i} + E_0\frac{\pi c}{\omega a}\cos\left(\frac{\pi}{a}x\right)\sin(kz - \omega t)\,\mathbf{k} \, ,$$

where **i**, **j** and **k** are the unit vectors in the x, y and z directions.

(a) Show that this is a solution to Maxwell's equations. What is the dispersion relation $\omega = \omega(k)$ one should use in order for this statement to be true? Is it the same as the one for EM waves in vacuum? Calculate the phase and group velocities of the wave.

(b) Check that the solution obeys the appropriate boundary conditions i.e. that the electric field is perpendicular to the conducting surfaces and that the magnetic field is parallel to them.

(c) Calculate the Poynting vector and the electromagnetic energy density.

(d) Calculate the average energy flux (averaged over the cross-section of the wave guide and one time cycle).

(e) Calculate the average energy density (averaged over a piece of length $\lambda = 2\pi/k$ of the waveguide and one time cycle).

(f) Using the results of (3d) and (3e) find the velocity at which the energy is propagating in the wave guide. Show that it is equal to the group velocity that you found in (3a).

4. A linearly polarized EM wave with electric component

$$\mathbf{E} = E_0 \cos(kz - \omega t)\, \mathbf{i}\,, \tag{5}$$

for $z < 0$, is passed through ideal linear polarizers placed at $z = 0$.

(a) What is the electric component of the wave for $z > 0$ if the first polarizer is oriented along the x direction and a second polarizer, placed just next to it, is oriented along the y direction?

(b) Repeat (4a) but now consider the case where a third polarizer, oriented in 45° to the x axis, is placed between the other two polarizers.

(c) Repeat (4b) but now with the middle polarizer rotating with angular frequency Ω.