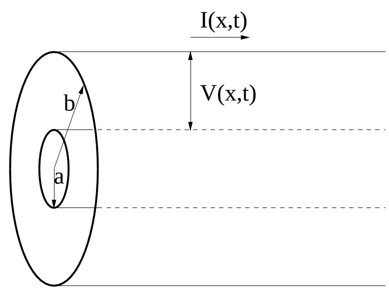
Homework Assignment #2 **Physics 17 Fall 99**

Due: Friday, October 22

1. Show that any function of the form $\psi(x,t) = \psi(x \pm vt)$ solves the wave equation
$$\frac{\partial^2\psi(x,t)}{\partial t^2} - v^2\frac{\partial^2\psi(x,t)}{\partial x^2} = 0\ . \tag{1}$$
Is it still true if we add to the l.h.s of (1) a term of the form $-\alpha v^2 \frac{\partial^4\psi(x,t)}{\partial x^4}$?

2. Consider a coaxial transmission line made out of two hollow conducting cylinders of radii a and b.



   (a) Calculate the capacitance per unit length $\mathcal{C}$ and the inductance per unit length $\mathcal{L}$ of the system (have a look at your EM course notes).

   (b) An oscillatory current is driven through the outer cylinder. Consider an infinitesimal segment of the transmission line. Derive a relation between the change in time of the current $I(x,t)$ through this segment and the change of the voltage difference $V(x,t)$ between the two cylinders across the segment.

   (c) Derive a relation between the change of $V(x,t)$ in time and the change of the current $I(x,t)$ across the segment.

   (d) Use these relations to obtain two equations involving $I(x,t)$ and $V(x,t)$ separately. What is the form of these equations?

   (e) Use the results of (2a) to calculate the velocity of the wave that propagates along the line.

3. The schrödinger equation
$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V_0\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}\quad , \tag{2}$$

describes the quantum mechanical propagation of a non-relativistic particle of mass m through a region of constant potential V_0. $\psi(x,t)$ is the "wave function" of the particle (you will learn its meaning in the basic quantum mechanics course) and $\hbar$ is Planck's constant divided by 2π.

(a) Find the general solution of Eq.(2).

(b) The rule is that the energy E of the particle is related to the frequency ω of the wave function by $E = \hbar\omega$. Find the wavelength λ of the wave function as a function of E.

4. Consider a line of pendulums placed at a distance a from each other. Each of the pendulums has string length l, mass M and is connected to its neighbors by springs of spring constant K. Denote by $\psi_n(t)$ the angle of the n-th pendulum relative to its equilibrium position.

   (a) Show that in the continuum limit the system is described by the following wave equation

$$\frac{\partial^2\psi(x,t)}{\partial t^2} - \frac{Ka^2}{M}\frac{\partial^2\psi(x,t)}{\partial x^2} + \omega_0^2\psi(x,t) = 0 \ , \tag{3}$$

   where $\omega_0^2 = g/l$. This equation is called the Klein-Gordon equation and it also describe the quantum mechanical propagation of relativistic particles.

   (b) Find the general solution of (3). How does it behave in the cases $\omega^2 > \omega_0^2$ and $\omega^2 < \omega_0^2$?

5. (a) Calculate the Fourier transform of a rectangular slit of width a

$$f_S(x) = \begin{cases} 1 & |x| < \frac{a}{2} \\ 0 & |x| > \frac{a}{2} \end{cases} \ . \tag{4}$$

   (b) Use the convolution theorem to calculate the Fourier transform of a multiple-slit function

$$f_{MS}(x) = \sum_{n=-\infty}^{\infty} f_S(x - nb) \ , \tag{5}$$

   where b is the spacing between the centers of the slits.

   (c) Prove Parseval's identity

$$\int_{-\infty}^{\infty} dt|f(t)|^2 = \int_{-\infty}^{\infty} d\omega|\tilde{f}(\omega)|^2 \ , \tag{6}$$

   where $\tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt e^{i\omega t} f(t)$ is the Fourier transform of $f(t)$.