Homework Assignment #1 **Physics 17 Fall 99**

Due: Friday, October 15

1. (a) Given 2 complex numbers z_1 and z_2 find the real and imaginary parts of z_1/z_2 in the Cartesian and polar representations.

   (b) Find expressions for $\sin(\theta)$ and $\cos(\theta)$ in terms of $e^{i\theta}$ and $e^{-i\theta}$.

2. Consider a system whose dynamics is described by the following fourth order differential equation

$$a\frac{d^4\psi}{dt^4} + b\frac{d^2\psi}{dt^2} + c\psi = 0 \qquad (1)$$

   where a, b and c are constants.

   (a) Use the complex variable technique to find the general (complex) solution for $\psi(t)$.

   (b) What happens to the frequencies of the system when the constant a is varied from 0 to ∞ ? Explicitly indicate how the system behaves when $a = 0$,$a = b^2/4c$ and $a \to \infty$.

3. The concept of an impedance Z is the complex generalization of the resistance to an arbitrary circuit element. It is defined through the relation $V = ZI$ where $V = V_0 e^{i\omega t}$ is the complex voltage across the element (V_0 and ω are real constants) and I is the complex current that is generated by V. Use this definition to find the impedance of

   (a) A resistor.

   (b) A capacitor.

   (c) An inductor.

   (d) A capacitor and an inductor in series. Plot the result as a function of ω.

4. In an oscillatory circuit a meaningful quantity is the power generated by a circuit element averaged over a cycle $\langle P\rangle = (1/T)\int_0^T dt' \mathrm{Re}[I(t')]\mathrm{Re}[V(t')]$, where $T = 2\pi/\omega$ is the period. Find an expression for $\langle P\rangle$ for an arbitrary circuit element in terms of its impedance Z and the complex current I.

5. Consider two pendulums, a and b, with the same string length l, but with different bob masses, M_a and M_b. They are coupled by a massless spring of spring constant K which is attached to the bobs.

(a) Show that the equations of motion (for small oscillations) are

$$M_a \frac{d^2\psi_a}{dt^2} = -M_a \frac{g}{l}\psi_a + K(\psi_b - \psi_a) \ , \tag{2}$$

$$M_b \frac{d^2\psi_b}{dt^2} = -M_b \frac{g}{l}\psi_b - K(\psi_b - \psi_a) \ . \tag{3}$$

(b) Solve these equations using the complex variable technique. What are the frequencies and the configurations of the two modes of the system?

(c) Show that by changing from the original coordinates ψ_a, ψ_b to new coordinates $\psi_1 = (M_a\psi_a + M_b\psi_b)/(M_a + M_b)$ and $\psi_2 = \psi_a - \psi_b$ the two coupled equations (2-3) decouple. Such coordinates are called normal coordinates. Solve these new equations and show that the solution that you get is equivalent to the one you obtained in (5b). What is the physical significance of ψ_1 and ψ_2 ?

(d) Find a superposition of the two modes that describes the motion of the system with the initial conditions at time $t = 0$ that both pendulums have zero velocity, that bob a has amplitude A, and that bob b has zero amplitude.

(e) Find expressions for $E_a(t)$,$E_b(t)$ and $E_s(t)$ - the energies of bob a, bob b and the spring for the solution you found in (5d). Plot them as a function of time. Verify that the total energy of the system is conserved.