Low-Dimensional Electronic Systems – Problem Set 2

1. Consider the spin sector of a Tomonaga-Luttinger model in the presence of a backward scattering term

$$H = \int dx \left\{ \frac{v}{2} \left[K \left(\partial_x \theta \right)^2 + \frac{1}{K} \left(\partial_x \phi \right)^2 \right] + \frac{g_{1\perp}}{2(\pi\alpha)^2} \cos\left(\sqrt{8\pi}\phi \right) \right\}$$

a. Show that the fields $\phi_1 = \sqrt{\frac{\pi}{2}} (\theta + 2\phi)$ and $\phi_2 = \sqrt{\frac{\pi}{2}} (\theta - 2\phi)$ obey the commutation relations of bosonic fields describing left and right-moving *spinless* fermions. Use this result to define such fermions:

$\psi_L = \frac{1}{\sqrt{2\pi\alpha}} F_L e^{-i\phi_1}$ and $\psi_R = \frac{1}{\sqrt{2\pi\alpha}} F_R e^{-i\phi_2}$, where $F_{L,R}$ are Klein factors.

b. Show that in terms of these fermions

$$H = \int dx \left\{ v \left(\frac{1}{4K} + K \right) \left(\psi_L^\dagger i\partial_x \psi_L - \psi_R^\dagger i\partial_x \psi_R \right) + \frac{g_{1\perp}}{2\pi\alpha} \left(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L \right) + 2\pi v \left(\frac{1}{4K} - K \right) \psi_L^\dagger \psi_R^\dagger \psi_R \psi_L \right\}$$

c. Consequently, for $K = 1/2$ the problem is that of non-interacting fermions. Diagonalize H for this case by expressing the fields in terms of their Fourier components and using the transformation

$$c_{1k} = \alpha_k c_{Rk} + \beta_k c_{Lk}$$
$$c_{2k} = -\beta_k c_{Rk} + \alpha_k c_{Lk}$$

What is the resulting spectrum?