

Low-Dimensional Electronic Systems – Problem Set 1

1. Derive a bosonized expression for the density of left moving electrons by using the bosonized form of the electronic operators. Specifically, show that

$$\lim_{x' \rightarrow x} : \psi_L^\dagger(x') \psi_L(x) : = \frac{1}{2\pi} \partial_x \phi_L$$

This procedure is called point splitting. What is the result for the density of right moving electrons?

2. Calculate the compressibility $\frac{\partial \rho}{\partial \mu}$ of a Luttinger liquid (here ρ is the long wavelength $q \sim 0$ component of the density) from the response to a perturbation of the form $\int dx \mu(x, t) \rho(x)$.

3. Calculate the correlation function of the pairing operator $\langle T_\tau O_p(x, \tau) O_p^\dagger(0, 0) \rangle$ in a Luttinger liquid, where $O_p(x, \tau) = \psi_R^\dagger(x, \tau) \psi_L^\dagger(x, \tau)$.