Advanced Quantum Mechanics A – Problem Set 4

1. a. Find the Lagrangian for the electromagnetic field in the presence of sources (charge density and charge current density), whose Euler-Lagrange equations are Maxwell's equations.
b. Use this Lagrangian to derive an expression for the momentum density of the electromagnetic field.

2. a. Find the Lagrangian for the Schrodinger field coupled to the electromagnetic field, such that it produces the Schrodinger equation in the presence of such coupling.
b. Derive the expressions for the charge density and the current density in the presence of the coupling.

3. Derive an expression for the angular momentum operator of the electromagnetic field. Show that photons with circular polarization, created by $a_{\pm}^{\dagger}(\vec{k})$, carry an angular momentum component $\pm\hbar$ along the direction defined by $\vec{k}$.

4. Two point charges, each of mass M, are attached to the ends of a rod of length l. The rod is free to rotate in the $x-y$ plane around its mid-point. Calculate the rate of photon absorption by the system in the limit $\frac{\hbar}{M\,lc} \ll 1$ for the case of

a. Opposite charges.
b. Identical charges.