Advanced Quantum Mechanics A – Problem Set 2

1. Show that in one dimension the Lippmann-Schwinger equation takes the form

$$\psi(x) = \frac{e^{ikx}}{\sqrt{2\pi}} - \frac{im}{\hbar^2 k}\int dx' e^{ik|x-x'|}V(x')\psi(x')$$

What is the asymptotic behavior of the solution for $|x| >> a$, where a is the extent of the scattering region? How many independent values does $f(\vec{k},\vec{k}')$ have?

2. Consider the one-dimensional scattering potential $V(x) = V_0\delta(x)$.

a. Solve the Lippmann-Schwinger equation exactly for this case.

b. Verify that $\psi(x)$, obtained in (a.), solves the Schrodinger equation.

c. Use this solution to form a Gaussian wave packet and calculate its exact time evolution. Plot (numerically) the probability density of the wave packet at different times.

d. Show that the scattering amplitude has a pole in the complex k plane if and only if $V_0 < 0$, and that the solution corresponding to this k describes a bound state of the potential. Write down its wave-function.

e. Show that in three dimensions the potential $V(\vec{r}) = V_0\delta(\vec{r})$ does not lead to any scattering.

3. a. Calculate within the first order Born approximation the scattering amplitude $f(\theta)$ from the spherical potential $V(r) = \begin{cases} V_0 & r < R \\ 0 & r > R \end{cases}$. What is its behavior in the limit $kR \ll 1$?

b. Calculate the imaginary part of the forward scattering amplitude $f(\theta = 0)$ using second order Born approximation in the limit $kR \ll 1$. Show that the optical theorem holds to second order in V_0, in the limit $kR \ll 1$.