Advanced Quantum Mechanics A – Problem Set 1

1. A tritium atom, $^{3}\mathrm{H}$, initially in its ground state, undergoes a β decay where it emits a 17 KeV electron and turns into a $^{3}\mathrm{He}$ ion. Calculate the probability to find the $^{3}\mathrm{He}$ ion in its ground state. Justify the use of the sudden approximation.

2. Consider a one-dimensional system comprised of three particles. Two of the particles are heavy, with a mass M, and we denote their coordinates by x_1 and x_2. The third particle, of a much lighter mass m, resides in between them at a coordinate x_3. The three particles are connected by springs of natural length d and spring constant k. The Hamiltonian is given by

$$H = \frac{p_1{}^2}{2M} + \frac{p_2{}^2}{2M} + \frac{p_3{}^2}{2m} + \frac{1}{2}k(x_3 - x_1 - d)^2 + \frac{1}{2}k(x_2 - x_3 - d)^2$$

a. Calculate the spectrum within the Born-Oppenheimer approximation.

b. The Hamiltonian can be diagonalized exactly. Find the exact energy levels and compare them to the approximate results found in a.

3. A spin $\vec{S}$ (integer or half-integer) is placed in a time-dependent magnetic field $\vec{B}(t)$. The interaction between the two is described by the Hamiltonian $H = -\mu \vec{B} \cdot \vec{S}$. Calculate the Berry phase of the adiabatic states of the spin.