

# Quantum-Fluctuation Effects in the Transport Properties of Ultrathin Films of Disordered Superconductors above the Paramagnetic Limit

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We study the transport in ultrathin disordered film near the quantum critical point induced by the Zeeman field. We calculate corrections to the normal state conductivity due to quantum pairing fluctuations. The fluctuation-induced transport is mediated by virtual rather than real quasiparticle excitations. We find that at zero temperature, where the corrections come from purely quantum fluctuations, the Aslamazov–Larkin paraconductivity term, the Maki–Thompson interference contribution, and the density of states effects are all of the same order. The total correction leads to the negative magnetoresistance. This result is in qualitative agreement with the recent transport observations in the parallel magnetic field of the homogeneously disordered amorphous films and superconducting two-dimensional electron gas realized at the oxide interfaces.

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**Introduction.**—According to the microscopic BCS theory [1], a magnetic field  $H$  extinguishes superconductivity. In the absence of spin-orbit interaction there are two basic mechanisms. The first one is the diamagnetic effect associated with the action of the field on the orbital motion of electrons forming a Cooper pair. The second, paramagnetic mechanism, is due to Zeeman splitting of the states with the same spatial wave function but with opposite spin. In the former case, the estimate for the upper critical field follows from the condition  $H_{c2}\xi^2 \simeq \Phi_0$ , where  $\Phi_0 = hc/2e$  is the flux quantum,  $\xi = \sqrt{\hbar D/\Delta}$  is the coherence length for the disordered superconductor with  $\Delta$  being energy gap, and  $D$  diffusion coefficient. In contrast, Zeeman splitting destroys superconductivity at a different critical field  $H_z$  that follows from the condition  $E_z \simeq \Delta$ , where  $E_z = g_L \mu_B H$  is the Zeeman energy,  $\mu_B = e\hbar/2mc$  is the Bohr magneton, and  $g_L$  is the renormalized giro factor. The ratio between the two fields is  $H_z/H_{c2} \sim k_F \ell \gg 1$ , where  $k_F$  is the Fermi momentum and  $\ell$  is the elastic scattering length. Thus, in bulk systems, the suppression of superconductivity is typically governed by the first diamagnetic mechanism. The situation changes in the case of restricted dimensionality. For example, in the case of a thin-film superconductor in a parallel field, the above ratio becomes  $H_z/H_{c2} \sim (k_F \ell)(d/\xi)$ , which can be small provided that the film is thin enough,  $d \ll \xi/k_F \ell$ , such that spin effects dominate.

The scenario of paramagnetically limited superconductivity has long history that goes back to pioneering works by Clogston and Chandrasekhar [2]. The first order phase transition from superconductor to paramagnet was predicted at the critical field approaching  $E_z = \sqrt{2}\Delta$  at low temperatures. In practice, the measured film resistance follows a hysteresis loop [3–7] instead of a sharp first order

transition, and the experimental phase diagram is qualitative as in Fig. 1. At low temperatures, the system remains superconducting as the field increases up to the superheating field; above the critical field, the film is trapped in a metastable state. At fields exceeding the superheating threshold the film becomes normal. When the field is reduced, the films stay normal down to the supercooling field  $E_z^{\text{sc}}(T)$ , which corresponds to the zero binding energy of a Cooper pair. At  $T = 0$ , the normal state is metastable in the interval  $\Delta < E_z < \sqrt{2}\Delta$  [8]. In this Letter, we study

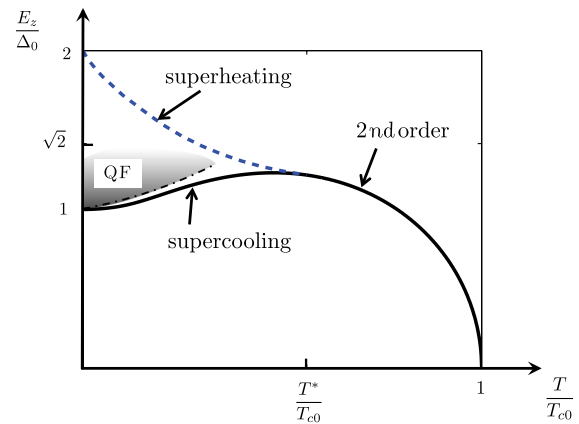


FIG. 1 (color online). Above the tricritical point  $T^*$  the second order paramagnet to superconductor transition occurs along the (black) solid line obtained from Eq. (1). At  $T < T^*$  this line becomes a supercooling part of the hysteresis, and the (blue) dashed line is its superheating part. The latter is obtained following Ref. [6]. The grey shaded area with the critical point  $(0, \Delta_0)$  as its lowest corner bounded by the black dashed line marks the region of quantum fluctuations (QF).

the transport properties near the supercooling field, which is determined by the equation [9]

$$\ln(T_c/T_{c0}) = \psi(1/2) - \text{Re}\psi(1/2 + iE_z^{\text{sc}}/4\pi T_c) \quad (1)$$

similar to that in the theory of paramagnetic impurities [10]. Here  $\psi$  is the digamma function and  $T_{c0} = T_c(H=0)$  is the critical temperature in the absence of a magnetic field. The zero-temperature solution of Eq. (1),  $E_z^{\text{sc}}(0) = \Delta$ , defines the quantum critical point (QCP), which is the premier interest of our study.

**Motivation.**—The renewed interest in the physics of paramagnetically limited superconductors is motivated by the rapid growth of its experimental realizations. Recent parallel magnetic field studies of two-dimensional superconducting systems were extended to much lower temperatures thus making it feasible to approach the limit of QCP. Tunneling spectroscopy of ultrathin Al and Be films revealed field-induced spin mixing and anomalous resonances in the density of states (DOS) [5,11,12]. The latter was successfully explained in theory [13,14], which emphasized the crucial role of superconducting pairing correlations in the paramagnetic state even far from the transition region. A surprising enhancement of superconductivity by a parallel magnetic field, deduced from the transport measurements, was observed in ultrathin, homogeneously disordered amorphous Pb films and the two-dimensional electron gas realized at the interface of oxide insulators  $\text{LaAlO}_3$  and  $\text{SrTiO}_3$  [15]. In addition, pronounced negative magnetoresistance (NMR), concomitant with the enhanced  $T_c$ , was reported. Although we do not dwell onto the issue of  $T_c$  enhancement in these systems (see Ref. [16] for the recent theoretical proposals), we show that transport anomalies, such as NMR, can be successfully addressed within BCS theory.

The issue of NMR in superconductors, either near the QCP or near the parallel field-tuned superconductor-insulator transition, was previously discussed in the literature experimentally [17,18] and attributed theoretically [19–21] to the proliferation of superconductive fluctuations [22]. These studies emphasized mainly the orbital effect of a magnetic field on the preformed Cooper pairs. In this Letter, we develop transport theory of paramagnetically limited ultrathin superconductors focusing on the quantum regime of zero temperature near the critical Zeeman field. The regime of classical fluctuations was partially discussed in the early papers [23–26].

**Theory.**—In the vicinity of the transition, transport properties of superconductors are governed by the fluctuation effects. These are famous paraconductivity phenomena introduced by Aslamazov and Larkin (AL) [27], Maki and Thompson (MT) [28], and also related DOS effects discussed first by Abrahams *et al.* [29]. We follow these classical papers and approach the problem based on the diagrammatic perturbation theory. Note that the technique based on the time-dependent Ginzburg–Landau formalism

applied for studying transport near QCP [30,31] accounts correctly only for the classical part of AL-type contribution to the conductivity, but it misses completely the quantum zero-temperature corrections. Microscopic approach takes care of all the contributions including the DOS part, resulting from the depletion of the normal state DOS by superconducting fluctuations, and also the MT interference term [19–21]. In fact, at  $T = 0$  where the corrections come from purely quantum fluctuations, these effects turn out to be of the dominant nature. In calculations we assume diffusive limit,

$$T \ll E_z, \quad \Delta \ll \tau^{-1} \ll \varepsilon_F. \quad (2)$$

Conditions (2) are satisfied in many experiments [5,11].

Within Kubo linear response formalism conductivity is obtained from  $\sigma = -K^R(\omega)/i\omega$  by analytic continuation of the Matsubara current correlation kernel  $K(\omega_n) = -\int_0^{1/T} d\tau e^{i\omega_n\tau} \langle T_\tau J(\tau) J(0) \rangle$ . This kernel can be conveniently presented as a sum of three contributions  $K = K_{\text{AL}} + K_{\text{MT}} + K_{\text{DOS}}$ . The general expression for the AL term reads (hereafter  $\hbar = k_B = 1$ ):

$$K_{\text{AL}}(\omega_n) = -e^2 T \sum_{Q, \Omega_k} B_{Q, \Omega_k, \omega_n}^2 L_{Q, \Omega_k} L_{Q, \Omega_k + \omega_n}, \quad (3)$$

where  $\Omega_k = 2\pi kT$ . The triangular vertex function

$$B_{Q, \Omega_k, \omega_n} = T \sum_{\sigma, \varepsilon_m} \lambda_{Q, \varepsilon_{m+n}, \Omega_k - \varepsilon_m}^\sigma \lambda_{Q, \varepsilon_n, \Omega_k - \varepsilon_n}^\sigma J_{\text{AL}}^\sigma, \quad (4)$$

$$J_{\text{AL}}^\sigma = \sum_P v_P G_{P, \varepsilon_{n+m}}^\sigma G_{P, \varepsilon_n}^\sigma G_{-P+Q, -\varepsilon_n + \Omega_k}^{-\sigma}, \quad (5)$$

consists of two Cooperons

$$\lambda_{Q, \varepsilon_n, \varepsilon_m}^\sigma = \frac{\theta(-\varepsilon_n \varepsilon_m)}{\tau [DQ^2 + |\varepsilon_n - \varepsilon_m| - i\sigma E_z \text{sgn}(\varepsilon_n - \varepsilon_m)]} \quad (6)$$

and an integral over the block of three Green's functions with  $G_{P, \varepsilon_n}^\sigma = [i\varepsilon_n - \xi_P + \sigma E_z/2 + i\text{sgn}(\varepsilon_n)/2\tau]^{-1}$ . Here we used notations:  $\varepsilon_m = 2\pi T(m + 1/2)$ ,  $\xi_P = P^2/2m - \varepsilon_F$ ,  $v_P = \partial_P \xi_P$ ,  $\theta(\varepsilon)$ -step function, and  $\text{sgn}(\varepsilon)$ -sign function. Finally, propagator of fluctuating Cooper pairs in Eq. (3) is given by

$$L_{Q, \Omega_k}^{-1} = -\nu \left[ \ln \frac{T}{T_{c0}} - \psi\left(\frac{1}{2}\right) + \frac{1}{2} \sum_{\sigma=\pm} \Psi_{Q, \Omega_k}^\sigma \right], \quad (7)$$

where  $\Psi_{Q, \Omega_k}^\sigma = \psi\left(\frac{1}{2} + \frac{DQ^2 + |\Omega_k| + i\sigma E_z}{4\pi T}\right)$ . When calculating  $B$  vertex one should follow few basic steps [22]. (i) To the leading order in the momentum transferred  $Q$  one can approximate  $G_{-P+Q, -\varepsilon_n + \Omega_k}^{-\sigma} \approx G_{P, -\varepsilon_n + \Omega_k}^{-\sigma} + (v_P \cdot Q) \times (G_{P, -\varepsilon_n + \Omega_k}^\sigma)^2$ . (ii) Furthermore, one can neglect Zeeman energy as compared to the inverse scattering time in the Green's functions [provided the condition of Eq. (2)] and then complete  $P$  integration in a standard way

$\sum_P \rightarrow \nu \int d\xi_P \int \frac{dQ_P}{2\pi}$ . (iii) Next is the fermionic Matsubara  $\varepsilon_m$  sum in Eq. (4), which can be found in the closed form with the result

$$B_{Q,\Omega_k,\omega_n} = \frac{\nu Q_x D}{\omega_n} \sum_{\sigma} [\Psi_{Q,|\Omega_k|+\omega_n}^{\sigma} - \Psi_{Q,|\Omega_k|}^{\sigma} + \Psi_{Q,|\Omega_k+n|+\omega_n}^{\sigma} - \Psi_{Q,|\Omega_k+n|}^{\sigma}]. \quad (8)$$

(iv) The remaining step of calculation is bosonic  $\Omega_k$  sum followed by an analytical continuation  $i\omega_n \rightarrow \omega$ . The latter is accomplished via the contour integration over the circle with two-branch cuts at  $\text{Im}\Omega = 0, -\omega_n$  where the product of propagators in Eq. (3) has breaks of analyticity. After  $\omega$  expansion of  $K_{\text{AL}}^R(\omega)$  to the linear order one finds for the AL conductivity correction  $\sigma^{\text{AL}} = \sigma_{cl}^{\text{AL}} + \sigma_{q1}^{\text{AL}} + \sigma_{q2}^{\text{AL}}$ , where

$$\sigma_{cl}^{\text{AL}} = \frac{e^2}{4\pi T} \sum_Q \int_{-\infty}^{+\infty} \frac{d\Omega}{\sinh^2 \frac{\Omega}{2T}} (B_{Q,\Omega}^{RA})^2 (\text{Im} L_{Q,\Omega}^R)^2, \quad (9)$$

$$\sigma_{q1}^{\text{AL}} = \frac{e^2}{4\pi} \sum_Q \int_0^{\infty} d\Omega \coth \frac{\Omega}{2T} \text{Re} \{ [(B_{Q,\Omega}^{RA})^2 - (B_{Q,\Omega}^{RR})^2] \partial_{\Omega} (L_{Q,\Omega}^R)^2 \}, \quad (10)$$

$$\sigma_{q2}^{\text{AL}} = -\frac{e^2}{4\pi} \sum_Q \int_{-\infty}^{+\infty} d\Omega \coth \frac{\Omega}{2T} \{ \partial_{\omega} (B_{Q,\Omega-\omega}^{RR})^2 (L_{Q,\Omega}^R)^2 - \partial_{\omega} (B_{Q,\Omega-\omega}^{AA})^2 (L_{Q,\Omega}^A)^2 + \partial_{\omega} [(B_{Q,\Omega-\omega}^{RA})^2 - (B_{Q,\Omega,\omega}^{RA})^2] |L_{Q,\Omega}^R|^2 \}. \quad (11)$$

The superscript  $R(A)$  in the vertex function and propagators stands for the retarded (advanced) components while the subscript  $cl(q)$  refers to classical (quantum). This convention comes from the observation that as  $T \rightarrow 0$  classical contribution vanishes while quantum remains finite.

We turn now to the derivation of the MT contribution whose response kernel is given by

$$K_{\text{MT}}(\omega_n) = e^2 T \sum_{\Omega_k, Q} L_{Q,\Omega_k} \Sigma_{Q,\Omega_k,\omega_n}^{\text{MT}}, \quad (12)$$

where

$$\Sigma_{Q,\Omega_k,\omega_n}^{\text{MT}} = T \sum_{\sigma, \varepsilon_m} \lambda_{Q,\varepsilon_m+n,\Omega_k-n-\varepsilon_m}^{\sigma} \lambda_{Q,\varepsilon_m,\Omega_k-\varepsilon_m}^{\sigma \text{sgn}[\varepsilon_m \varepsilon_{m+n}]} J_{\text{MT}}, \quad (13)$$

$$J_{\text{MT}} = \sum_P \nu_P \nu_{Q-P} G_{P,\varepsilon_m+n}^{\sigma} G_{Q-P,\Omega_k-n-\varepsilon_m}^{-\sigma} G_{P,\varepsilon_m}^{\sigma} G_{Q-P,\varepsilon_m+\Omega_k}^{-\sigma}. \quad (14)$$

Momentum integration in the block of Green's functions  $J_{\text{MT}}$  is done under the same approximations as in the case of AL term described above. According to the standard convention [22], we split now MT term into the so-called regular and anomalous contributions:

$$\Sigma_{Q,\Omega_k,\omega_n}^{\text{MT(reg)}} = -\frac{\nu D}{\omega_n} \sum_{\sigma} [\Psi_{Q,|\Omega_k|+2\omega_n}^{\sigma} - \Psi_{Q,|\Omega_k|}^{\sigma}], \quad (15a)$$

$$\Sigma_{Q,\Omega_k,\omega_n}^{\text{MT(an)}} = -\frac{\nu D}{2(DQ^2 + \omega_n)} \times \sum_{\sigma} [\Psi_{Q,-|\Omega_k|+2\omega_n}^{\sigma} - \Psi_{Q,|\Omega_k|}^{\sigma}]. \quad (15b)$$

After the analytical continuation these translate into the conductivity correction  $\sigma^{\text{MT}} = \sigma_{\text{reg}}^{\text{MT}} + \sigma_{\text{an}}^{\text{MT}}$ , where

$$\sigma_{\text{reg}}^{\text{MT}} = -\frac{e^2 \nu D}{8\pi^3 T^2} \sum_{\sigma Q} \int_0^{\infty} d\Omega \coth \frac{\Omega}{2T} \text{Im} [L_{Q,\Omega}^R (\Psi_{Q,-i\Omega}^{\sigma})'], \quad (16a)$$

$$\sigma_{\text{an}}^{\text{MT}} = \frac{e^2 \nu D}{8\pi T} \sum_{\sigma Q} \int_{-\infty}^{+\infty} \frac{d\Omega}{\sinh^2 \frac{\Omega}{2T}} \frac{L_{Q,\Omega}^R [\Psi_{Q,i\Omega}^{\sigma} - \Psi_{Q,-i\Omega}^{\sigma}]}{DQ^2 + \Gamma_{\phi}}. \quad (16b)$$

In order to regularize logarithmically divergent momentum integral in the case of anomalous contribution we have introduced pair-breaking cutoff parameter  $\Gamma_{\phi}$ .

We finally discuss the DOS contribution to the conductivity. The latter is given by the expression similar to Eq. (12) with

$$K_{\text{DOS}}(\omega_n) = e^2 T \sum_{\Omega_k, Q} L_{Q,\Omega_k} \Sigma_{Q,\Omega_k,\omega_n}^{\text{DOS}}, \quad (17)$$

where

$$\Sigma_{Q,\Omega_k,\omega_n}^{\text{DOS}} = 2T \sum_{\sigma, \varepsilon_m} (\lambda_{Q,\varepsilon_m,\Omega_k-\varepsilon_m}^{\sigma})^2 J_{\text{DOS}}, \quad (18)$$

$$J_{\text{DOS}} = \sum_P \nu_P^2 (G_{P,\varepsilon_m}^{\sigma})^2 G_{P,\varepsilon_m+\omega_n}^{\sigma} \left[ G_{Q-P,\Omega_k-\varepsilon_m}^{-\sigma} + \frac{1}{2\pi\nu\tau} \sum_{P'} (G_{P',\varepsilon_m}^{\sigma})^2 G_{Q-P',\Omega_k-\varepsilon_m}^{-\sigma} \right]. \quad (19)$$

After completing the standard steps outlined above one arrives at the conductivity correction  $\sigma^{\text{DOS}} = \sigma_{cl}^{\text{DOS}} + \sigma_q^{\text{DOS}}$  in the form

$$\sigma_q^{\text{DOS}} = -\frac{e^2 \nu D}{16\pi^2 T^2} \sum_{\sigma Q} \int_{-\infty}^{+\infty} \frac{d\Omega [(\Psi_{Q,i\Omega}^{\sigma})' - (\Psi_{Q,-i\Omega}^{\sigma})']}{\sinh^2 \frac{\Omega}{2T}} L_{Q,\Omega}^R, \quad (20a)$$

$$\sigma_q^{\text{DOS}} = \sigma_{\text{reg}}^{\text{MT}}. \quad (20b)$$

The equality between the two contributions in Eq. (20b) has parallels with the original fluctuation transport considerations at  $T - T_c \ll T$ . In the original near- $T_c$  problem, the typical energy of diffusing pairs  $DQ^2 \sim T - T_c$  is smaller than the thermal energy of quasiparticle  $\sim T$ . In our case,  $E_z$  adds to the energy of pairs making it bigger than  $T$ . Correspondingly, unlike the near- $T_c$  case, the off shell energy of a pair,  $2\varepsilon \sim T$ , falls below the pair

excitation energy set by  $E_z$ . This causes a sign inversion of the energy denominator associated with the unbound intermediate state and the correction (20b) turns to be positive. In general, derived above conductivity corrections are applicable at any field  $H$  and temperature  $T$  above the transition. In the following, we discuss limiting case of interest.

**Results.**—It is convenient to regroup all contributions and present total conductivity correction as the sum of zero-temperature ( $\delta\sigma_q$ ) and finite-temperature ( $\delta\sigma_T$ ) terms, namely,

$$\delta\sigma(H, T) = \delta\sigma_q(H) + \delta\sigma_T(H, T). \quad (21)$$

The first term here is determined by the quantum AL [Eqs. (10) and (11)] and DOS [Eq. (20b)] contributions, and also regular part of the MT conductivity [Eq. (16a)]. The remaining terms define  $\delta\sigma_T$ . The magnitude of  $\delta\sigma_q$  decreases monotonically with increasing field; this leads to a NMR at zero temperature. At finite temperature, based on how the QCP is approached, there are several regimes that show different  $T$  and  $H$  dependencies, which should be experimentally accessible. Below we focus on QCP only and extract the leading singularity in  $\delta\sigma_q$  as the function of the Zeeman field. Thermal contribution  $\delta\sigma_T$  and various crossover regimes will be discussed elsewhere [32].

At zero temperature  $\Psi_{Q,\pm i\Omega}^\sigma \rightarrow \ln[(DQ^2 \pm i\Omega + i\sigma E_z)/4\pi T]$  and the pair propagator can be taken in the leading pole approximation

$$L_{Q,\Omega}^{R(A)} \approx -\frac{2\Delta_0^2/\nu}{E_c^2 - (\Omega \pm iDQ^2)^2}, \quad (22)$$

which is obtained from Eq. (7) under the conditions  $DQ^2 \ll \Delta_0$  and  $|E_c \pm \Omega| \ll \Delta_0$ . Here  $\Delta_0 = \pi T_{c0}/2\gamma_E$  where  $\ln\gamma_E \approx 0.57$  is the Euler constant, and  $E_c = \sqrt{E_z^2 - \Delta_0^2}$ . The branch cut of the propagator (due to the logarithmic structure) also contributes to  $\delta\sigma_q$  but gives the subleading singularity. Within the same accuracy we compute vertex functions:

$$(B_{Q,\Omega(-\omega),\omega}^{RR(AA)})^2 = \frac{8\nu^2 D}{E_z^4} DQ^2 (DQ^2 \mp i\Omega)(DQ^2 \mp i\Omega - 2i\omega), \quad (23)$$

$$(B_{Q,\Omega,\omega}^{RA(AR)})^2 = \frac{8\nu^2 D}{E_z^4} (DQ^2)^2 (DQ^2 - 2i\omega). \quad (24)$$

All together this leads to the conductivity correction near the Zeeman field—induced QCP

$$\delta\sigma_q(H) = \frac{2e^2}{\pi^2} \ln\left(\frac{E_z}{E_z - \Delta_0}\right), \quad (25)$$

which is obtained within the logarithmic accuracy. Equation (25) is the main result of this Letter.

**Discussions.**—The conceptual difference of our analysis from the problem of fluctuation-induced transport close to  $T_c$  is that unpaired particles have finite excitation energy  $E_z$ , see Eq. (6). As a result, the activation probability of such pairs is suppressed exponentially  $\propto \exp(-E_z/T)$ . We argue that while in the standard case the real gapless pairs are only important, in our case such pairs are always virtual.

Let us illustrate this point by taking AL correction as an example. Consider first standard case near- $T_c$ . In Eq. (3) the triangular vertex Eq. (8) can be estimated as  $B_{Q,\Omega,\omega} \propto DQ_x \partial \Pi_{Q,\Omega} / \partial \Omega$ . Here  $\Pi_{Q,\Omega} = L_{Q,\Omega}^{-1} + g^{-1}$  is a particle-particle polarization operator with momentum  $Q$  entering in a  $DQ^2 - i\Omega$  combination. At small momenta we can take  $\Pi_{0,\Omega}$  in the clean system. The imaginary part of the polarization operator  $\text{Im}\Pi \approx \int d\xi [n(-\xi_p + \Omega)n(\xi_p) - \tilde{n}(-\xi_p + \Omega)\tilde{n}(\xi_p)]\delta(\Omega - 2\xi) = \nu(\Omega/2) \tanh \frac{\Omega}{2T}$ , where the particle and hole occupation numbers are  $n(\varepsilon) = (1 + e^{\varepsilon/T})^{-1}$ ,  $\tilde{n}(\varepsilon) = 1 - n(\varepsilon)$ . The real part, due to virtual pairs  $\text{Re}\Pi \approx \log|(\Omega^2 - T^2)/\omega_d^2|$ , is a familiar Cooper logarithm. The imaginary part contribution is  $B_{Q,\Omega,\omega} \propto DQ_x/T$ . In contrast, the real part contribution vanishes at  $\Omega = 0$  due to the particle-hole symmetry,  $\nu(\Omega) = \nu$ . The expansion in  $\Omega \sim T - T_c \ll T$  yields a correction small in the parameter  $(T - T_c)/T_c \ll 1$ .

In the presence of a Zeeman field the situation is very different. The pair activation rate,  $\text{Im}\Pi \approx \nu(\Omega)[n(\omega/2 - E_z/2) - n(\omega/2 + E_z/2)]$ , gives exponentially suppressed contribution  $\propto DQ_x \exp(-E_z/T)/T$ . The real part, due to virtual pair excitation, can be obtained by the Kramers–Kronig relation,  $\text{Re}\Pi \approx \log|(\Omega^2 - E_z^2)/\omega_d^2|$ . Its contribution to  $B_{Q,\Omega,\omega}$  is suppressed only algebraically  $\propto DQ_x T/E_z^2$ . Unlike the standard case the virtual quasi-particles make a dominant contribution to the triangular vertex excitations. The algebraic suppression of vertexes is most pronounced in the case of the AL and is manifested in additional factors of  $DQ^2$ ,  $\Omega$  in Eqs. (23) and (24), which makes it logarithmic in  $E_z/E_c$ . Note that in the case of near- $H_{c2}$  problem [19] the AL contribution is also suppressed due to the current matrix elements connecting adjacent Landau levels.

The regular MT and DOS contributions are proportional to a second derivative of the real part of the polarization operator  $\text{Re}\Pi_{Q,\Omega}$ . Since the latter is finite at  $\Omega = 0$ , these contributions are as singular as AL terms.

We have checked explicitly that other contributions such as diffusion coefficient renormalization as well as contributions with only one or no Cooperon vertexes are either small or nonsingular. Since the temperature can be set to zero in integrations over fast fermion degrees of freedom, the additional factors of  $\tau$  result in small prefactors  $\tau E_z$ ,  $\tau DQ^2$ , or  $\tau \Omega$ .

**Outlook.**—The spin-orbit scattering and finite thickness effects modify the fluctuation transport, due to the finite spectral weight in the particle—particle channel at zero



frequency. Addition of a finite spin-orbit scattering introduces a finite lifetime  $\Gamma^{-1}$  to the Cooperon. At lowest temperatures the superconductivity survives if this scattering is not too strong,  $\Gamma \ll E_z$  with a somewhat lower critical field. While  $E_z$  approaches the supercooling transition from above the results obtained in this Letter are expected to crossover to a different regime at  $\Gamma \approx E_c$ . The finite film thickness affects the crossover in a similar way. Inclusion of these effects was shown to be necessary for quantitative analysis of measurements of the DOS [12]. These relevant perturbations as well as the regime of close proximity to the supercooling line will be studied elsewhere [32].

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