Diffuse emission in the presence of an inhomogeneous spin-orbit interaction for the purpose of spin filtration

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A lateral interface connecting two regions with different strengths of the Bychkov-Rashba spin-orbit interaction can be used as a spin polarizer of electrons in two-dimensional semiconductor heterostructures [Khodas *et al.*, Phys. Rev. Lett. **92**, 086602 (2004)]. In this paper we consider the case when one of the two regions is ballistic, while the other one is diffusive. We generalize the technique developed for the solution of the problem of the diffuse emission to the case of the spin-dependent scattering at the interface and determine the distribution of electrons emitted from the diffusive region. It is shown that the diffuse emission is an effective way to get electrons propagating at small angles to the interface that are most appropriate for the spin filtration and a subsequent spin manipulation. Finally, a scheme is proposed of a spin filter device (see Fig. 9) that creates two almost fully spin-polarized beams of electrons.

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I. INTRODUCTION

Recently we proposed[1] to use a lateral interface between two regions of the two-dimensional (2D) electron gas with different strengths of the Bychkov-Rashba[2] spin-orbit (SO) interaction as a spin-polarizing element for the purposes of spintronics.[3,4] The lateral interface introduces the space-varying SO interaction which leads to spin-dependent refraction of spin carriers passing through the interface.[5] Consequently, an electron beam with a nonzero angle of incidence after passing through the interface splits into beams with different spin polarizations propagating in different directions (see Fig. 1 for the scattering at the lateral interface between the regions with and without SO interaction). Using an interface with an inhomogeneous SO interaction as a principal element of spin-based devices we outlined the schemes for a spin filter, spin guide, and a spin current switch (spin transistor). This program promises to build spintronics devices avoiding magnetic materials that are not conventional for the semiconductor industry. It can be realized in the gated heterostructures with a sufficiently strong Bychkov-Rashba SO interaction[6–8] by manipulating the gates.[1]

The effect of the separation of the trajectories of electrons with different spin polarizations (chiralities) influenced by the space-varying SO interaction has the same grounds as the double refraction (birefringence) of light in uniaxial crystals exploited in optical devices for the polarization of light. The separation of the trajectories of carriers of different spin has been observed recently[9] in the case of a homogeneous SO interaction as a result of action of a perpendicular magnetic field. The separation of the trajectories after reflection at a lateral potential barrier in the presence of the SO interaction has been observed in Ref. 10.

Two facts that can be useful for the purposes of the spintronics have been found in Ref. 1 in the analysis of the spin-dependent scattering of electrons incident on a lateral interface with a SO interaction varying in the direction normal to the interface. First, there exists an interval of outgoing angles $\theta^c < \theta < \pi/2$ where only electrons with a definite spin chirality can penetrate (see Fig. 1). If it is possible to collect electrons from this interval, one will have an ideal spin filter. Second, electrons of this chirality exhibit a total internal reflection for an angle of incidence in an interval $\varphi^c < \varphi < \pi/2$, where φ^c is a critical angle of the total internal reflection. It is clear from these observations that the electrons propagating at small angles to the lateral interface are most sensitive to the variation of the magnitude of the spin-orbit interaction, and therefore such electrons are most appropriate for spin control and manipulation. Hence one has to find a way to create (and collect) flows of electrons of high intensity that are almost tangential to the interface. In this paper we suggest using the diffuse emission[11,12] as a possible solution of this task.

We show that making one of the two regions connected by the lateral interface to be diffusive is an effective way to achieve a flat angular distribution of particles emitted from the diffusive region into the clean one. The effect of flattening of the angular distribution of the emitted electrons is the robust property of the diffuse emission which holds despite the presence of a spin-dependent reflection at the interface.

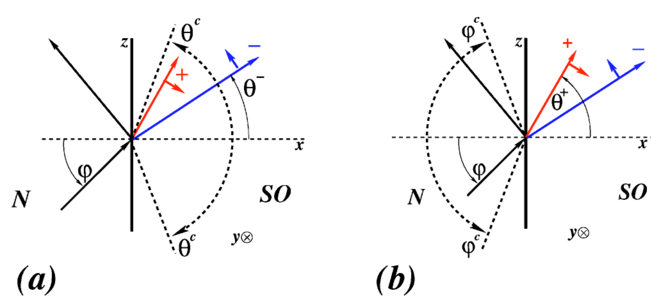

FIG. 1. (Color online) The refraction of electrons at the interface between the regions with (SO) and without (N) spin-orbit interaction. A beam incident at angle φ splits after the refraction into two beams with "+" and "−" chiralities that propagate at angles $\theta^{\pm}$ (denoted by red and blue colors, respectively). (a) θ^c determines the limited aperture for − chirality; in the angle interval $\theta^c < \theta < \pi/2$ only electrons of + chirality can penetrate. (b) φ^c is an angle for total internal reflection for electrons of the + chirality.

Due to the flatness of the angular distribution a substantial portion of electrons propagates at small angles to the interface and these electrons are suitable for the spin filtration and subsequent spin manipulation. In Fig. 9 a scheme of a device is presented which operates as a spin filter with a high level of spin-polarization of a filtered current.

The paper is organized as follows. In Sec. II we present the results[1] of the analysis of scattering of electrons at the lateral interface between the two regions with different magnitudes of the Bychkov-Rashba term. In Sec. III we analyze the transport of the 2D electrons in the presence of the Coulomb interaction near the interface between the diffusive and ballistic regions. In Sec. IV we reconsider the problem of the diffuse emission (Milne problem[11,12]) for an arbitrary dimension. In Sec. V we generalize the technique developed for the problem of the diffuse emission to the case of the spin-dependent scattering at the lateral interface. We use a semiclassical approach with electrons moving along the classically allowed trajectories when the spin of electrons is the only element treated quantum-mechanically. For this purpose, we analyze the transport near the interface in terms of the spin density matrix. In the Summary a scheme of a spin filter device that creates two beams of electrons of a very high level of spin polarization is presented.

II. SPIN-DEPENDENT SCATTERING AT THE LATERAL INTERFACE

Consider a two-dimensional electron gas confined in the xz plane by a potential well in the semiconductor heterostructure. Generally, the potential well has the shape of an asymmetric triangle, and, consequently, there is a direction of asymmetry, $\hat{\mathbf{l}}$, perpendicular to the electron gas plane. This leads to the appearance of the specific spin-orbit interaction term[2] in the Hamiltonian, $\alpha(\mathbf{p}\times\hat{\mathbf{l}})\sigma$. We will consider the case when the parameter α varies along the x direction. The direction of $\hat{\mathbf{l}}$ is chosen as $\hat{\mathbf{l}}=-\hat{\mathbf{y}}$. Generally, the Hamiltonian has the form

$$H_R = \frac{1}{2m}p_x^2 + \frac{1}{2m}p_z^2 + B(x) + \frac{1}{2}(\hat{\mathbf{l}}\times\sigma)[\alpha(x)\mathbf{p} + \mathbf{p}\alpha(x)]. \tag{1}$$

Here $B(x)$ describes the varying bottom of the conduction band which may be controlled by gates. The current operator corresponding to this Hamiltonian contains a spin term: $\mathbf{J} = \mathbf{p}/m + \alpha(x)(\hat{\mathbf{l}}\times\sigma)$. The presence of spin in the current operator implies that in the process of scattering at the lateral interface with varying α the continuity conditions for the wave function involves the spin degrees of freedom of the electrons. This makes the electron scattering at the interface to be spin-dependent.

To diagonalize the Hamiltonian with the Bychkov-Rashba term in the regions of constant α one has to choose the axis of the spin quantization along the direction $(\hat{\mathbf{l}}\times\mathbf{p})$. Then for the two chiralities (referred to below as "+" and "−" modes) the dispersion relations are given by

$$E^{\pm} = \frac{p^2}{2m} \pm \alpha p + B. \tag{2}$$

Correspondingly, the momenta of the waves of a given energy E involved in the scattering at the interface between the two regions with different α are determined as follows:

$$p_{\mathrm{SO}}^{\pm} = m[\sqrt{2(E-B)/m + \alpha^2} \mp \alpha] = mv_F(\sqrt{1+\tilde{\alpha}^2} \mp \tilde{\alpha}). \tag{3}$$

Here $v_F = \sqrt{2(E-B)/m}$, and we introduce a small dimensionless parameter $\tilde{\alpha} = \alpha/v_F$ which we will use throughout the paper. Notice that at a given energy the velocity of electrons of both chiralities is the same: $v = \partial E^{\pm}/\partial p = v_F\sqrt{1+\tilde{\alpha}^2}$.

Let us discuss the kinematical aspects of the electron scattering at the lateral interface. An incident (nonpolarized) beam comes at angle φ from the region without or with a suppressed spin-orbit term, denoted as the N region, and when transmitted into the SO region splits into two beams of different chirality that propagate at different angles $\theta^{\pm}$. Figure 1 illustrates the scattering for the simplest case when $\alpha(x<0)=0$. The conservation of the projection of the momentum on the interface together with Eq. (3) determine the angles of the transmitted and reflected beams (Snell's law):

$$p_{\mathrm{N}} \sin\varphi = p_{\mathrm{SO}}^{\pm} \sin\theta^{\pm}, \tag{4}$$

where p_{N} is a momentum of an electron in the N region and $p_{\mathrm{SO}}^{\pm}$ are the momenta in the SO region after passing through the interface.

From Eqs. (3) and (4) it follows that the SO region is more "optically" dense for the + mode (i.e., it has a smaller wave vector) and less dense for the − mode. Correspondingly, the + mode is refracted to larger angles than the − one, and therefore the + mode exhibits a total internal reflection with a critical angle φ^c. As to the − mode it has a limited aperture in the SO region for outgoing angles: $\theta < \theta^c < \pi/2$.

Remarkably, the angle interval where only the + mode can penetrate is not narrow as its width has a square root dependence on $\tilde{\alpha}$. It follows from Snell's law that $(\pi/2 - \theta^c) \approx (\pi/2 - \varphi^c) \approx \sqrt{2\tilde{\alpha}}$. Actually, one can reduce θ^c even further. With the gates acting selectively on the different regions of the electron gas, $\delta B = B(-\infty) - B(+\infty) \neq 0$, one can alter the position of the bands relative to the Fermi level in the N and SO regions. A simple analysis shows that with an increase of δB (i.e., lowering p_F in the normal region) the angle interval $(\pi/2 - \theta_c)$ grows and reaches $2\sqrt{\tilde{\alpha}}$. Starting from this moment the angle interval suitable for spin filtration narrows and eventually becomes $\sim\tilde{\alpha}$, instead of $\sim\sqrt{\tilde{\alpha}}$.

The problem of scattering of electrons at a lateral interface between the two regions with different magnitudes of the Bychkov-Rashba term has been considered in Ref. 1 for the two limiting cases of a sharp, $\lambda/d \gtrsim 1$, and smooth, $\lambda/d \ll 1$, interface, where λ is an electron wavelength and d is an effective width of the interface. Here a qualitative description of the analysis of the spin-dependent scattering at the interface will be presented only.

A scattering state of an electron coming from the N region in the state $e^{i(p_x x + p_z z)}\chi_{\mathrm{N}}^{+}$ is given by

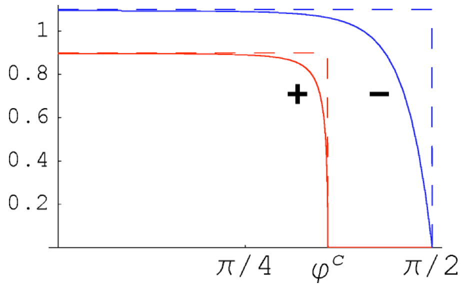

FIG. 2. (Color online) The intensities per unit outgoing angle of the electrons transmitted without change of their chirality $\sim(d\theta^{+}/d\varphi)^{-1}|t_{++}|^2$ and $\sim(d\theta^{-}/d\varphi)^{-1}|t_{--}|^2$ per unit angle as a function of an angle of incidence for sharp (solid line) and smooth (dashed line) interfaces (Ref. 13).

$$\Psi^{+}=e^{ip_z z}\begin{cases} e^{ip_x x}\chi_{\mathrm{N}}^{+}+e^{-ip_x x}\chi_{\mathrm{N}}^{+}r_{++}+e^{-ip_x x}\chi_{\mathrm{N}}^{-}r_{-+}, & x<0 \\ e^{ip_x^{+}x}\chi_{\mathrm{SO}}^{+}t_{++}+e^{ip_x^{-}x}\chi_{\mathrm{SO}}^{-}t_{-+}, & x>0 \end{cases}, \tag{5}$$

where $\chi^{\pm}_{\mathrm{N/SO}}$ are spinors corresponding to the $\pm$ chiral modes in the N and SO regions, and r and t are the amplitudes of the reflected and the transmitted waves. In Eq. (5) the interface is at $x=0$ and for simplicity we limit ourselves to the case of the interface between the N and SO regions. A similar expression holds also for Ψ^{-} which evolves from the incident state χ_{N}^{-}.

The flux of particles impinging on the interface at a given angle φ can be defined as $I_{\epsilon}d\epsilon d\varphi ds$, where ds is the cross-sectional area (width) of a beam. The intensity $I_{\epsilon}^{\mathrm{SO}}$ of the transmitted flux of a given chirality can be found from the relation $|\mathcal{T}_{\mathrm{SO}\leftarrow\mathrm{N}}(\varphi)|^2 I_{\epsilon}^{\mathrm{N}}\cos\varphi d\varphi = I_{\epsilon}^{\mathrm{SO}}\cos\theta d\theta$. Here the cosine factors take into consideration the change of the cross-sectional width of the beam as a result of the scattering, while $|\mathcal{T}_{\mathrm{SO}\leftarrow\mathrm{N}}(\varphi)|^2=|t|^2(v_x^{\mathrm{SO}}/v_x^{\mathrm{N}})$ (we use the fact that for the two chiralities the velocities are the same). Finally, the intensity of the refracted flux relative to the incident one is equal to

$$I_{\epsilon}^{\mathrm{SO}}/I_{\epsilon}^{\mathrm{N}}=(d\theta/d\varphi)^{-1}(v^{\mathrm{SO}}/v^{\mathrm{N}})|t|^2. \tag{6}$$

For a sharp interface the amplitudes r,t can be found from the continuity conditions, while for a smooth interface one can conduct the analysis of the refraction using a small parameter $\eta=(d\alpha/dx)/\alpha p_F\sim\lambda/d\ll 1$. In the latter case the electron spin adjusts itself adiabatically to the momentum keeping its polarization in the direction perpendicular to the momentum.

It has been shown in Ref. 1 that in the course of refraction at the interface with $\tilde{\alpha}\ll 1$ transitions between states of different chirality are strongly suppressed. For that reason, the drop of the intensities of the transmitted electrons without change of their chirality t_{++} and t_{--}, see Fig. 2, occurs practically only due to the reflection which becomes decisive only for $\varphi\gtrsim\varphi^c$. In particular, for a smooth interface the probability of the reflection outside the region of the total internal reflection is almost entirely suppressed because the matrix elements describing reflection are integrals of the rapidly oscillating functions. Consequently, the transmission amplitudes presented in Fig. 2 have almost rectangular shape for a smooth interface.

Summarizing the results obtained in Ref. 1 for the cases of the sharp and smooth interface one can state that for both the discussed cases an electron in a state of a definite chirality propagates along the classically allowed trajectory for this chirality, while a change of the chirality is very ineffective.

The case of the N-SO interface described so far was taken mostly for illustration. Actually, any interface with a change of α results in splitting of the trajectories that can be used for the purposes of spin polarization and filtration. In the analysis above, one should replace α by the difference of the strength of the spin-orbit interaction $\delta\alpha_{\mathrm{SO}}$ across the interface.

III. KINETIC EQUATION, ELECTRONEUTRALITY, AND BOUNDARY CONDITIONS

In this section we discuss the kinetics of the two-dimensional (2D) electron gas near the boundary between the diffusive and ballistic regions. We show that within the linear response approximation the equation determining the current distribution function and the equation determining the density and potential profiles are decoupled, except for boundary conditions. This analysis justifies the concept introduced by Landauer:[14] to determine the transport properties it is enough to consider noninteracting quasiparticles that propagate without interaction, i.e., ignoring any effects related to the redistribution of the potential due to the Coulomb interaction of electrons. The problem requires, however, fixing up the boundary conditions, which we perform below for a particular geometry.

The kinetic equation describing a stationary flow of electrons by a distribution function $n_p(x)$ is

$$\frac{\partial\epsilon_p(\mathbf{r})}{\partial\mathbf{p}}\frac{\partial n_p(\mathbf{r})}{\partial\mathbf{r}}-\frac{\partial\epsilon_p(\mathbf{r})}{\partial\mathbf{r}}\frac{\partial n_p(\mathbf{r})}{\partial\mathbf{p}}=\mathrm{St}\{n_p(\mathbf{r})\}. \tag{7}$$

The electron flow is forced by the electric field, $\partial\epsilon_p(\mathbf{r})/\partial\mathbf{r}=e\nabla\Phi(\mathbf{r})$. Correspondingly, Eq. (7) should be supplied by the Poisson equation which when limited to the 2D plane is

$$\Delta\Phi=-4\pi e\rho_{2D}\delta(y). \tag{8}$$

Here the Laplacian acts in 3D space, while ρ_{2D} is the deviation of density of the 2D electron gas from the equilibrium value. Therefore Eq. (8) should be supplemented with the 3D conditions on Φ that will take care of the 3D environment of the 2D electron gas. This introduces an element of nonuniversality into this problem. Fortunately, in the linear response approximation the situation is much more tractable and the problem of the current flow of the 2D gas becomes self-contained (see, e.g., Ref. 15).

In the linear response approximation one keeps terms linear in $\nabla\Phi(\mathbf{r})$ and $\delta n_p(\mathbf{r})$ only:

$$\mathbf{v}_F \frac{\partial \delta \overline{n_p}(\mathbf{r})}{\partial \mathbf{r}} - e \nabla \Phi(\mathbf{r}) \mathbf{v}_F \frac{\partial n_F^0}{\partial \epsilon} = \mathrm{St}\{\delta n\}, \tag{9}$$

where $\delta n_p(\mathbf{r}) = n_p(\mathbf{r}) - n_F^0$ and n_F^0 is the Fermi-Dirac equilibrium distribution. (We assume throughout the paper that the spatial variation of the Fermi-energy level as well as other parameters of the electron liquid such as the density of states and the screening length occurs on distances greatly exceeding the wavelength and the mean free path. For that reason, the spatial variation of the parameters characterizing the electron liquid is ignored below.) In the presence of the electron-electron interaction there is an additional force that originates from the interaction of the quasiparticles. This effect has been accounted for by a substitution of the local equilibrium distribution $\delta \overline{n_p}(\mathbf{r})$ in place of $\delta n_p(\mathbf{r})$ in Eq. (9), see Ref. 16.

Following Refs. 17 and 15, one can shift the distribution function by a local value of the potential $\Phi(\mathbf{r})$ introducing a displacement function $f(\mathbf{r}, \varphi)$

$$\delta \overline{n_p}(\mathbf{r}) = \frac{\partial n_F^0}{\partial \epsilon} e[\Phi(\mathbf{r}) - f(\mathbf{r}, \varphi)], \tag{10}$$

where the direction of the momentum $\mathbf{p}$ is given by the angle φ. In terms of the displacement function $f(\mathbf{r}, \varphi)$ the system of Eqs. (9) and (8) acquires the form

$$\mathbf{v}_F \frac{\partial f(\mathbf{r}, \varphi)}{\partial \mathbf{r}} = \mathrm{St}\{f\}, \tag{11}$$

$$\Delta \Phi - 2\kappa_{2\mathrm{D}} \Phi(\mathbf{r}) \delta(y) = -2\kappa_{2\mathrm{D}} \langle f(\mathbf{r}, \varphi) \rangle_\varphi \delta(y), \tag{12}$$

where $\kappa_{2\mathrm{D}} = 2\pi e^2 \partial n / \partial \mu$ is the inverse screening length of the 2D electron gas, which in the Thomas-Fermi approximation is equal to $2e^2 m$; $\langle f(\mathbf{r}, \varphi) \rangle_\varphi = \int (d\varphi / 2\pi) f(\mathbf{r}, \varphi)$ is $f(\mathbf{r}, \varphi)$ averaged over directions of momentum. Thus, although Eq. (9) together with the Poisson equation (8) constitutes a system of two coupled equations, the potential $\Phi(\mathbf{r})$ drops out from Eq. (11) governing the function $f(\mathbf{r}, \varphi)$. Given boundary conditions, the displacement function $f(\mathbf{r}, \varphi)$ determines the current distribution without any feedback from Eq. (12):

$$\mathbf{j}(\mathbf{r}) = 2e \int \frac{d^2 p}{(2\pi)^2} \mathbf{v}_F \delta \overline{n_p}(\mathbf{r}) = 2e^2 \nu_{2\mathrm{D}} \langle \mathbf{v}_F(\varphi) f(\mathbf{r}, \varphi) \rangle_\varphi, \tag{13}$$

where $\nu_{2\mathrm{D}}$ is the density of states per one spin species, $\nu_{2D} = m^* / 2\pi$.

When one is interested in the connection of the current density with the distribution of the potential and density in the 2D electron gas, Eq. (12) should to be involved along with the relation connecting $\rho_{2\mathrm{D}}$ with $\Phi(\mathbf{r})$ and $f(\mathbf{r}, \varphi)$:

$$\delta \rho_{2\mathrm{D}}(\mathbf{r}) = 2 \int \frac{d^2 p}{(2\pi)^2} \delta n_p(\mathbf{r}) = (\kappa_{2\mathrm{D}} / 2\pi e)[\langle f(\mathbf{r}, \varphi) \rangle_\varphi - \Phi(\mathbf{r})]. \tag{14}$$

For good enough conductors the Poisson equation reduces to the condition of the electroneutrality[18] (which is valid in any dimension). Under these circumstances typical distances on which the potential Φ in the left-hand side of Eq. (12) changes are much longer than the screening length $\kappa_{2\mathrm{D}}^{-1}$. Correspondingly, $|\nabla \Phi| \ll |\kappa_{2\mathrm{D}} \langle f(\mathbf{r}, \varphi) \rangle_\varphi|$, $|\kappa_{2\mathrm{D}} \Phi(\mathbf{r})|$, and the Poisson equation reduces to the condition of the electroneutrality:

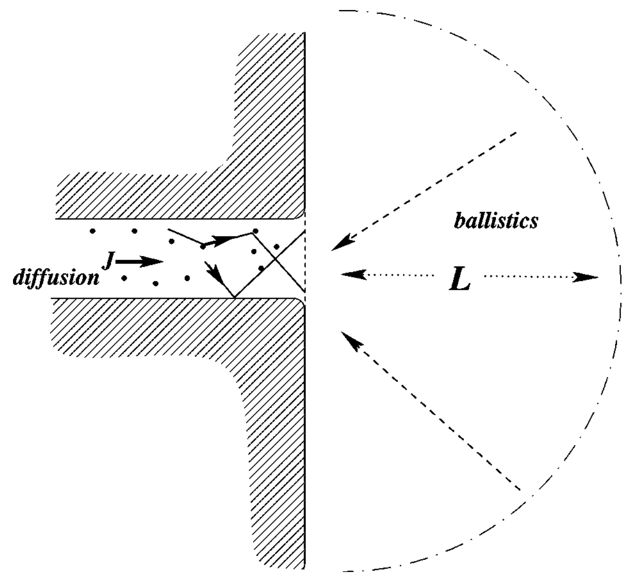

FIG. 3. The physical setup which leads to Eq. (24) as a boundary condition for the kinetic equation. The diffusive region has a geometry of a stripe that runs into a ballistic basin. The current density vanishes inside the ballistic region at a typical distance $L \ll l_{in}$. Trajectories that start from a terminal on the ballistic side of the device and reach the diffusive stripe are shown by dashed lines.

$$\delta \rho_{2\mathrm{D}}(\mathbf{r}) = 0; \quad \langle f(\mathbf{r}, \varphi) \rangle_\varphi = \Phi(\mathbf{r}). \tag{15}$$

Now we turn to the discussion of the boundary conditions. Let us assume that the diffusive region has a stripe geometry with the x axis directed along the stripe (see Fig. 3). In the diffusive region the collision term is controlled by the elastic relaxation time τ_{el}, and the kinetic equation acquires the form

$$\mathbf{v}_F \frac{\partial f(\mathbf{r}, \varphi)}{\partial \mathbf{r}} = \mathrm{St}\{f\}_{\mathrm{elastic}} = -\frac{f(\mathbf{r}, \varphi) - \langle f(\mathbf{r}, \varphi) \rangle_\varphi}{\tau_{el}}. \tag{16}$$

Here we assume that the impurity scattering is of short range nature. (The angular dependence of scattering of electrons by the impurities does not influence the distribution function of electrons deep inside the diffusive region, but is important for the angular profile of the diffuse emission.) With the elastic mean free path $l = v_F \tau_{el}$ used as a unit of length, the kinetic equation can be rewritten in terms of the dimensionless variables $\zeta = x / l$:

$$-\cos \varphi \frac{\partial}{\partial \zeta} f(\zeta, \varphi) + f(\zeta, \varphi) = \langle f(\mathbf{r}, \varphi) \rangle_\varphi. \tag{17}$$

Here the minus sign in the first term containing $\partial / \partial \zeta$ appears because φ is chosen as an angle formed by a momentum with the direction $-\hat{x}$ (note that $-\hat{x}$ directs inwards the diffusive region).

In the current carrying state electrons deep inside the diffusive region are distributed according to the Drude form. Correspondingly, at a distance (counted from the interface) exceeding few l Eq. (17) has a solution:

$$f_{\rm Dr}(\zeta,\varphi) = -J\pi^{-1}(\cos\varphi + \zeta). \tag{18}$$

Here the factors are fixed in such a way that the current $J(\zeta)$ defined as

$$J(\zeta) = -2\pi\langle\cos\varphi f(\zeta,\varphi)\rangle_\varphi \tag{19}$$

is equal to J for the distribution function $f_{\rm Dr}(\zeta,\varphi)$. With the use of Eq. (13) the physical current $j\equiv \mathbf{j}_x = -2e^2\nu_{2D}v_F\langle\cos\varphi f(\zeta,\varphi)\rangle_\varphi$ can be related to the current J defined in Eq. (19) as follows:

$$j = p_F(e^2/2\pi^2)J. \tag{20}$$

To obtain the relation between the current j and the electric field let us apply a gradient to the both sides of Eq. (14) with the distribution function $f_{\rm Dr}$ used for $f(\mathbf{r},\varphi)$. Then, together with the fact that the electric field $E=-\nabla\Phi$, one gets the relation:

$$j = \sigma E - D_{ch}\nabla(e\,\delta\rho_{2D}), \tag{21}$$

where the conductivity $\sigma=2e^2\nu_{2D}(l^2/2\tau)$ while the diffusion coefficient of charge density, D_{ch}, corresponds to the Einstein relation, $D_{\rm ch}=(\partial\mu/\partial n)\sigma/e^2$. Under the condition of the electroneutrality $\delta\rho_{2D}(\mathbf{r})=0$ and $j=\sigma E$. [Actually Eq. (21) does not require the Drude form, $f_{\rm Dr}$, of the distribution. It holds in the diffusion approximation when the distribution function has only two first harmonics $f(x,\varphi)=f^{(0)}(x)-f^{(1)}(x)\cos\varphi$ and the functions $f^{(0)}(x)$ and $f^{(1)}(x)$ vary on scales much exceeding the mean free path l.]

To proceed further with Eq. (11), one has to specify the distribution of particles incident from the terminals located on the ballistic side of the device under discussion. Generally, this distribution is not universal as it depends on a particular geometry. For definiteness, we consider the case when the diffusive region that has stripe geometry runs into a ballistic basin, see Fig. 3 (a stripe is wide enough allowing the transverse quantization of electrons to be ignored). Following Ref. 15, we use the method of characteristics to determine a distribution of particles that impinge onto the diffusive stripe from the ballistic region (corresponding family of angles φ will be denoted as $\varphi_{b\to d}$).

In the ballistic region the collision term is controlled by the inelastic relaxation time τ_{in}, and the kinetic equation acquires the form

$$\mathbf{v}_F\frac{\partial f(\mathbf{r},\varphi)}{\partial\mathbf{r}} = {\rm St}\{f\}_{\rm inelastic} = -\frac{f(\mathbf{r},\varphi)-\Phi(\mathbf{r})}{\tau_{in}}. \tag{22}$$

The solution of Eq. (22) for the directions $\varphi_{b\to d}$ is given by

$$f(\mathbf{r},\varphi) = \int_{-\infty}^{0} dt\frac{e^{t/\tau_{in}}}{\tau_{in}}\Phi[\tilde{\mathbf{r}}(t)_{\mathbf{r},\varphi}], \tag{23}$$

where $\tilde{\mathbf{r}}(t)_{\mathbf{r},\varphi}$ is the trajectory (the characteristics) of an electron that starts at the remote past from a terminal on the ballistic side of the device and reaches the point $\mathbf{r}$ with the momentum directed along φ at the moment $t=0$; see dashed lines in Fig. 3. We assume that the current carrying area widens sharply on the ballistic side of the setup. Correspondingly, the current density vanishes inside the ballistic region at a typical distance L. Under the condition $L\ll l_{in}=v_F\tau_{in}$, the integral in Eq. (23) is accumulated at distances where electrons are at the equilibrium and the potential $\Phi(\mathbf{r})$ is equal to its equilibrium value at a terminal deep inside the ballistic region, $\Phi(+\infty)$. Then, it follows from Eq. (23) that for incoming directions $f(\mathbf{r},\varphi_{b\to d})=\Phi(+\infty)$. This provides us with the boundary condition to be imposed at the interface, $x=0$, on the incoming part of the function:

$$f(x=0,\varphi_{b\to d}) = \Phi(+\infty). \tag{24}$$

The obtained boundary condition is isotropic. This remarkable feature is a consequence of the choice of the proper geometry.

Together with the current distribution deep in the diffusive region given by Eq. (18), the relation (24) constitutes the full set of the boundary conditions needed for the solution of Eq. (16). Two remarks are now in order to complete the discussion of the boundary conditions.

(i) Notice that $\Phi(x=0)\neq\Phi(+\infty)$. The point is that due to the abrupt change in scattering the solution $f(\mathbf{r},\varphi)$ of Eq. (16) has a singular derivative near the interface. Under these circumstances, one cannot neglect the term $\Delta\Phi$ in Eq. (12), and therefore the condition of the electroneutrality is violated in the vicinity of the interface in a strip of a width $\propto\kappa_{2D}^{-1}$. The deviation of the density distribution $\delta\rho_{2D}(\mathbf{r})$ from the equilibrium at the interface leads to the variation of the potential, and hence $\Phi(x=0)$ differs from $\Phi(+\infty)$.

(ii) As Eq. (16) can be satisfied by $f(\mathbf{r},\varphi)=$const, any solution of Eq. (16) can be shifted by a constant with no consequences for the physical quantities [at any measurement of the current one registers the difference between fluxes of incoming and outgoing particles and the isotropic part of $f(\mathbf{r},\varphi)$ is cancelled out]. It follows from Eq. (10) that the distribution of particles impinging onto the diffusive region at the interface, $\overline{\delta n_{\varphi_{b\to d}}}(x=0)$, is not affected by a global shift of the potential: $\overline{\delta n_{\varphi_{b\to d}}}(x=0)=(\partial n_F^0/\partial\epsilon)e[\Phi(x=0)-\Phi(+\infty)]$. This is in full accord with the fact that one is free to shift the potential Φ by a constant.

In Sec. IV and V we choose $\Phi(+\infty)=0$ and correspondingly use $f(x=0,\varphi_{b\to d})=0$ as the boundary condition for the function $f(\mathbf{r},\varphi)$ at the interface between the diffusive and ballistic regions.

IV. DIFFUSE EMISSION PROBLEM

In this section we present a solution of the classical problem of the diffuse emission (Milne problem[11,12]) (see Fig. 4) in the form convenient for the subsequent analysis of Sec. V. It is given for an arbitrary dimension d, but we are interested in the particular case of $d=2$.

Let us specify the notation for angle φ used throughout Sec. IV and V: for each of the two regions, diffusive and ballistic, φ is chosen as an angle formed by the momentum of an electron with a normal to the interface directed inwards

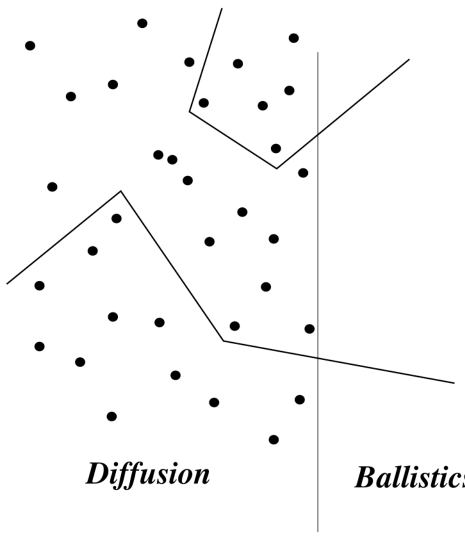

FIG. 4. (Color online) Milne problem. The diffusive region is to the left of the interface, $\zeta<0$. The ballistic region is to the right, $\zeta>0$.

to the corresponding region. Correspondingly, for $\mu=\cos\varphi$ we adopt the convention[19] that for electrons propagating away from the interface $\mu>0$ (denoted as $+\mu$ in what follows), while $\mu<0$ (denoted $-\mu$) corresponds to electrons propagating towards the interface.

The general solution of Eq. (17) can be written as a sum of the current carrying Drude flow [see Eq. (18)] and the currentless "counterflow" $g(\zeta,\mu)$, i.e., $f(\zeta,\mu)=f_{\mathrm{Dr}}(\zeta,\mu)+g(\zeta,\mu)$. At a distance about a few mean free paths l the counterflow $g(\zeta,\mu)$ approaches an isotropic distribution (generally, a nonzero constant). For currentless counterflow particles injected into the diffusive region eventually return back to the interface. Hence the distribution function of particles emitted by the diffusive region, $g(-\mu)$, should be determined completely by the distribution function of the injected particles, $g(+\mu)$. Due to the linear nature of Eq. (17), the $g(\pm\mu)$ parts of the distribution function are related linearly[11] by the angular Green's function $S(\mu,\mu')$:

$$g(\zeta,-\mu)=\frac{1}{\mu}\int_{+}d\Omega' S(\mu,\mu')g(\zeta,+\mu'). \qquad (25)$$

Here $d\Omega'$ stands for the angular integration in d dimensions, while the subscript $+$ in the integral indicates that the integration is limited to the directions for which $\mu'>0$. With the use of Eq. (25) one can reexpress the density $\rho_g(\zeta)$ corresponding to $g(\zeta,\mu)$ through the incoming part of the distribution only:

$$\rho_g(\zeta)=\int_{+}d\Omega H(\mu)g(\zeta,+\mu) \qquad (26)$$

where

$$H(\mu)=\left[1+\int_{+}d\Omega'\frac{S(\mu,\mu')}{\mu'}\right]. \qquad (27)$$

The Green's function $S(\mu,\mu')$ satisfies a nonlinear integral equation which can be derived as follows. Differentiate Eq. (25) with respect to ζ, express the derivatives $\partial g/\partial\zeta$ through the kinetic equation (17), and use Eq. (25) to eliminate $g(\zeta,-\mu)$ in favor of $g(\zeta,+\mu)$. In result one gets

$$\int_{+}d\Omega' S(\mu,\mu')\left(\frac{1}{\mu'}+\frac{1}{\mu}\right)g(\zeta,+\mu')$$
$$=\frac{1}{S_d}H(\mu)\int_{+}d\Omega' H(\mu')g(\zeta,+\mu'), \qquad (28)$$

where S_d is a total solid angle in d-dimensions. As this relation holds for an arbitrary incident distribution the equation for the Green's function $S(\mu,\mu')$ follows:

$$S(\mu,\mu')\left(\frac{1}{\mu'}+\frac{1}{\mu}\right)=\frac{1}{S_d}H(\mu)H(\mu'). \qquad (29)$$

The counterflow does not carry current and therefore it satisfies

$$\int_{+}d\Omega[\mu g(+\mu)-\mu g(-\mu)]=0. \qquad (30)$$

This implies that Green's function $S(\mu,\mu')$ should satisfy the condition

$$\mu=\int_{+}d\Omega' S(\mu,\mu') \qquad (31)$$

that can be verified with the use of Eq. (29) along with the relation $\int_{+}d\Omega H(\mu)=S_d$.

As it has been explained in Sec. III the geometry of the discussed setup is such that the distribution function in the ballistic region does not contain a component describing particles propagating towards the interface, i.e., $f(0,+\mu)=0$. Therefore, in the solution $f(\zeta,\mu)=f_{\mathrm{Dr}}(\zeta,\mu)+g_{\mathrm{Dr}}(\zeta,\mu)$ one has to counterbalance the part $f_{\mathrm{Dr}}(0,+\mu)$ by a proper choice of $g_{\mathrm{Dr}}(0,+\mu)=-f_{\mathrm{Dr}}(0,+\mu)$. Then the emission outside the diffusive region consists of the two contributions: $f_{\mathrm{Dr}}(0,-\mu)$ from the Drude flow, and $g_{\mathrm{Dr}}(0,-\mu)$ which originates from the compensating counterflow. As $g_{\mathrm{Dr}}(0,+\mu)$ is known, the latter contribution can be found with the help of Eq. (25). As a result, the emission in the Milne problem is given by the expression

$$f_{\mathrm{M}}(0,-\mu)=J\frac{d}{S_d}\left[\mu+\frac{1}{\mu}\int_{+}d\Omega'\mu' S(\mu,\mu')\right], \qquad (32)$$

which can be simplified using Eqs. (29) and (31) together with the relation $\int_{+}d\Omega\mu H(\mu)=S_d/\sqrt{d}$. Finally, the angular distribution of the particles emitted from the interface into the clean region is given by

$$f_{\mathrm{M}}(0,-\mu)=J\frac{\sqrt{d}}{S_d}H(\mu). \qquad (33)$$

This result coincides with the one presented in Ref. 11 for the case $d=3$.

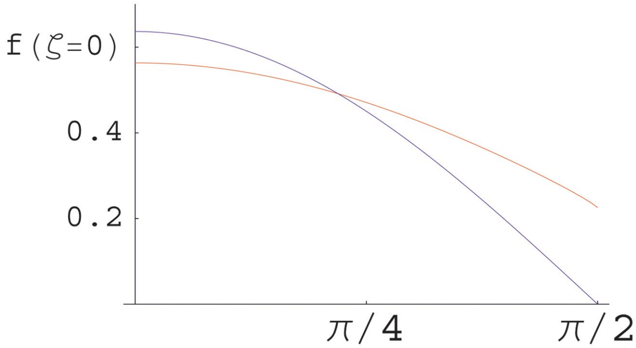

FIG. 5. (Color online) The angular distribution, $H(\cos\varphi)$, of the diffuse emission (red line) as compared to the Drude flow distribution (blue line) for $J=1$. The function $H(\cos\varphi)\neq 0$ at $\varphi=\pi/2$.

In Fig. 5 the function $H(\mu)$ and the angular dependence of the Drude flow distribution, $\sim\cos\varphi$, are plotted, both normalized to $J=1$. As compared to the Drude flow, the diffuse emission distribution in the Milne problem has a qualitatively different behavior at large angles. Namely, the distribution flattens and a considerable part of the distribution is transferred to the large angles.

V. DIFFUSE EMISSION IN THE PRESENCE OF SPIN-DEPENDENT SCATTERING AT THE N/SO INTERFACE

In this section we generalize the solution of the Milne problem to the case of spin-dependent reflection at the interface. We are mainly concerned with the influence of the strong reflection at the angles tangential to the interface on the distribution of the emitted electrons. We show that the effect of flattening of the angular distribution is the robust property of the solution of the Milne problem which becomes even stronger in the presence of such a reflection.

We now concentrate on the calculation of the diffuse emission in the presence of the spin-orbit interaction at the ballistic side of the junction. As compared to the consideration in the previous section the scattering at the interface

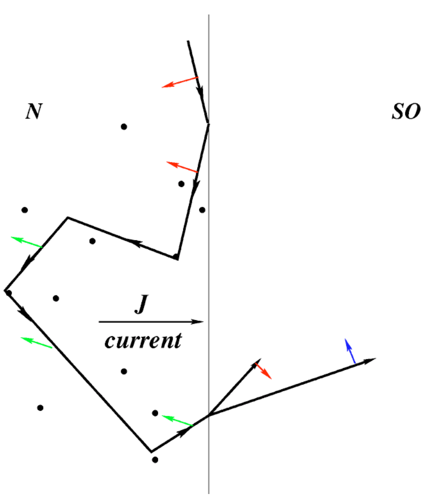

FIG. 6. (Color online) Milne problem in the presence of the spin-dependent scattering at the interface. The SO interaction in the diffusive region ($\zeta<0$) is suppressed.

now depends on the direction of spin of the incoming electron. On the other hand, electrons after a sequence of random scattering in the diffusive region return to the interface preserving their spin (see Fig. 6), as the impurity scattering of electrons in the diffusive region is assumed to be spin-independent.

To describe such scattering, one should introduce a spin density matrix $\varrho^{ss'}$. In the diffusive region each component of the spin density matrix satisfies the kinetic equation (17) because in the normal region the Hamiltonian is spin-independent. Consequently, one can apply the same angular Green's function $S(\mu,\mu')$ as in Eq. (25).

We have to describe the counterflow at the interface in the presence of the reflection. For clarification purposes, let us imagine an auxiliary line, $\zeta=-0$, separating the diffusive region from the N—SO interface. (Actually, for a real device there can be a physical, not auxiliary, interval between the diffusion region and the beginning of the interface where α and B start to vary.) Similarly to the discussion of the Milne problem it is assumed that the electron distribution is given by a sum of the Drude flow and a counterflow. The incoming part of the counterflow consists of two contributions: one, $\upsilon_{\mathrm{Dr}}(\zeta=-0,+\mu)$, counterbalances the incoming part of the Drude flow, $\upsilon_{\mathrm{Dr}}(\zeta=-0,+\mu)=-\varrho_{\mathrm{Dr}}(\zeta=-0,+\mu)$, while the other, $\upsilon_{refl}(\zeta=-0,+\mu)$, is determined by the flow incident onto the line $\zeta=-0$. As it has been explained is Sec. III and has been already used in Eq. (32) we assume that there are no electrons incident onto the N—SO interface from the SO region. In the absence of electrons incident from the SO side of the interface the flow incident onto the line $\zeta=-0$ comes only as a result of the reflection at the N—SO interface [the latter circumstance explains our choice of a subscript for $\upsilon_{refl}(\zeta=-0,+\mu)$]. Therefore the overall distribution of particles coming from the interface $\varrho_{\mathrm{overall}}(\zeta=-0,+\mu)=\upsilon_{refl}(\zeta=-0,+\mu)$.

As a result of scattering inside the diffusive region $\upsilon_{\mathrm{Dr}}(\zeta=-0,+\mu)$ transforms into a part of the outgoing counterflow $\upsilon_{\mathrm{Dr}}(\zeta=-0,-\mu)$. Together with the Drude flow it yields the following contribution to the outgoing distribution $\varrho_{\mathrm{M}}(\zeta=-0,-\mu)=J(\sqrt{d}/S_d)H(\mu)\sigma_0$, where σ_0 is the 2×2 unit matrix. The overall distribution of particles emitted from the line $\zeta=-0$ and incident onto the N—SO interface is given by

$$\varrho_{\mathrm{overall}}(\zeta=-0,-\mu)=J\frac{\sqrt{d}}{S_d}H(\mu)\sigma_0+\frac{1}{\mu}\int_{+}d\Omega' S(\mu,\mu')\times\upsilon_{refl}(\zeta=-0,+\mu'). \tag{34}$$

The second term in this equation is generated by the incident part of the distribution, $\upsilon_{refl}(\zeta=-0,+\mu)$, which in turn is determined by the reflection of $\varrho_{\mathrm{overall}}(\zeta=-0,-\mu)$ at the N—SO interface. The relation between the incident and reflected parts of the distribution should be found from the solution of the scattering problem at the normal side of interface

$$\upsilon_{refl}(\zeta=-0,+\mu)=\mathcal{R}(\mu)\varrho_{\mathrm{overall}}(\zeta=-0,-\mu)\mathcal{R}^{\dagger}(\mu), \tag{35}$$

where by $\mathcal{R}$ we denote a 2×2 block of the scattering matrix corresponding to the reflection at the normal side of the interface.

After substituting Eq. (35) in Eq. (34) one obtains a closed equation for $\varrho_{\text{overall}}(\mu)$:

$$\varrho_{\text{overall}}(-\mu) = J\frac{1}{\sqrt{2}\pi}H(\mu)\sigma_0 + \frac{1}{\mu}\int_+ d\Omega' S(\mu,\mu') \times \mathcal{R}(\mu')\varrho_{\text{overall}}(-\mu')\mathcal{R}^\dagger(\mu'). \quad (36)$$

To analyze the effect of the reflection at the interface on the form of the diffuse emission, we study Eq. (36) for the case of a smooth interface. In this limit the kinematical aspect of the scattering at the interface is most pronounced and not masked by unnecessary complications. Namely, we assume that for electrons of the $-$ chirality the probability of the transmission is 1 at all angles, while for electrons of the $+$ chirality the probability of the transmission is 1 for $\varphi < \varphi^c$ and 0 otherwise; see Fig. 2 and the related discussion of the smooth interface.

We will use a notation $|\chi^\pm(\pm\mu,i)\rangle$ for spinors. The sign of the first argument indicates the sign of the projection of the momentum of a scattering electron on the direction perpendicular to the interface, while $i=\pm 1$ is the sign of the momentum component along the interface; the superscript $\pm$ denotes the chirality. Also in what follows the integration over φ will be performed as $\int d\varphi = \Sigma_{i=\pm 1}\int_{-1}^{1}(d\mu/\sqrt{1-\mu^2})$. In this notation the reflection part of the scattering matrix for the smooth interface is given by

$$\begin{aligned}\mathcal{R} &= 0, \quad \varphi < \varphi^c \\ &= \sum_i |\chi^+(+\mu,i)\rangle\langle\chi^+(-\mu,i)|, \quad \varphi > \varphi^c. \end{aligned} \quad (37)$$

For each direction $-\mu$ we introduce the orthogonal basis $\mathbf{n}^\alpha(-\mu,i)$ in such a way that $\mathbf{n}^1$ is parallel to the incident momentum $\mathbf{p}$; $\mathbf{n}^2$ is perpendicular to the electron gas plane, $\mathbf{n}^2 \| \hat{\mathbf{l}}$; and $\mathbf{n}^3$ is directed along the vector of the polarization of the $+$ chirality. For the analysis of Eq. (36) it will be convenient to use the four-component Bloch vector $(s^0,\mathbf{s})$ related to the matrix ϱ as follows:

$$s^0(-\mu,i) = \frac{1}{2}\text{Tr}\,\sigma^0\varrho(-\mu,i),$$

$$s^\alpha(-\mu,i) = \frac{1}{2}\text{Tr}(\mathbf{n}^\alpha\sigma)\varrho(-\mu,i),$$

$$\varrho(-\mu,i) = s^0\sigma^0 + \sum_\alpha s^\alpha(-\mu,i)(\mathbf{n}^\alpha\sigma), \quad (38)$$

where σ^0 is the unit matrix and σ are the Pauli matrices. In terms of the $(s^0,\mathbf{s})$ matrix Eq. (36) can be rewritten as a set of coupled equations

$$s^0(-\mu,i) = J\frac{1}{\sqrt{2}\pi}H(\mu) + \frac{1}{\mu}\sum_j \int_0^{\cos\varphi^c} \frac{d\mu'}{\sqrt{1-\mu'^2}}S(\mu,\mu') \times \frac{1}{2}[s^0(-\mu',j) + s^3(-\mu',j)], \quad (39)$$

$$s^\alpha(-\mu,i) = \frac{1}{\mu}\sum_j \int_0^{\cos\varphi^c} \frac{d\mu'}{\sqrt{1-\mu'^2}}S(\mu,\mu') \times \langle\chi^+(+\mu',j)|\frac{1}{2}\sigma\cdot\mathbf{n}^\alpha(-\mu,i)|\chi^+(+\mu',j)\rangle \times [s^0(-\mu',j) + s^3(-\mu',j)]. \quad (40)$$

Remarkably, all components s^α are determined by a single combination $f_N^+(-\mu,j) = s^0(-\mu,j) + s^3(-\mu,j)$. The combinations $s^0(-\mu,j) \pm s^3(-\mu,j)$ describe the probability for an incident electron to be in the $\pm$ chiral state. The representation of $f_N^\pm$ as $s^0 \pm s^3$ is a consequence of the choice of the axis $\mathbf{n}^3$ along the direction of the spin polarization of an electron in the state of the $+$ chirality. The fact that the right-hand sides of Eqs. (39) and (40) depend solely on $f_N^+(-\mu',j)$ has a simple physical reason: electrons of this chirality only are reflected at the interface. [In Eq. (40) the component s^1 is determined by $f_N^+(-\mu',j)$ without any feedback, and s^2 vanishes identically in the chosen parametrization.]

To proceed further we have to calculate $\langle\chi^+(+\mu',j)|\frac{1}{2}\sigma\cdot\mathbf{n}^\alpha(-\mu,i)|\chi^+(+\mu',j)\rangle$ for both $i,j=\pm 1$. With the use of the relation that determines the direction of the polarization of a spinor, $\langle\chi^+(+\mu',j)|\sigma|\chi^+(+\mu',j)\rangle = \mathbf{n}_{\chi^+(+\mu',j)}$, the discussed expression is reduced to $\frac{1}{2}\mathbf{n}_{\chi^+(+\mu',j)}\cdot\mathbf{n}^\alpha(-\mu,i)$. Then Eqs. (39) and (40) can be rewritten in the form:

$$f_{\mathrm{N}}^+(-\mu,i=\pm 1) = J\frac{1}{\sqrt{2}\pi}H(\mu) + \frac{1}{\mu}\int_0^{\cos\varphi^c} \frac{d\mu'}{\sqrt{1-\mu'^2}}S(\mu,\mu') \times \frac{1}{2}\{[1-\cos(\varphi'\pm\varphi)]f_{\mathrm{N}}^+(-\mu',+1) + [1-\cos(-\varphi'\pm\varphi)]f_{\mathrm{N}}^+(-\mu',-1)\}, \quad (41)$$

where angles φ and φ' are defined in Fig. 7. For the difference $\delta f_{\mathrm{N}}^+(-\mu) = f_{\mathrm{N}}^+(-\mu,+1) - f_{\mathrm{N}}^+(-\mu,-1)$ one gets a homogeneous equation

$$\delta f_{\mathrm{N}}^+(-\mu) = \frac{1}{\mu}\int_0^{\cos\varphi^c} \frac{d\mu'}{\sqrt{1-\mu'^2}}S(\mu,\mu')\frac{1}{2}[-\cos(\varphi'+\varphi) + \cos(\varphi'-\varphi)]\delta f_{\mathrm{N}}^+(-\mu'). \quad (42)$$

One can check that all eigenvalues of the right-hand side of this equation considered as a kernel of the transformation are less than 1. Therefore we conclude that $\delta f_{\mathrm{N}}^+(-\mu)=0$, i.e., $f_{\mathrm{N}}^+(-\mu,i)$ is a symmetrical function with respect to i. Finally, this yields for Eq. (41)

$$f_{\mathrm{N}}^+(-\mu) = J\frac{1}{\sqrt{2}\pi}H(\mu) + \frac{1}{\mu}\int_0^{\cos\varphi^c} \frac{d\mu'}{\sqrt{1-\mu^2}}S(\mu,\mu') \times (1-\mu\mu')f_{\mathrm{N}}^+(-\mu'). \quad (43)$$

Following the same route, it can be found from Eq. (40) the distribution of particles in the $-$ chiral state:

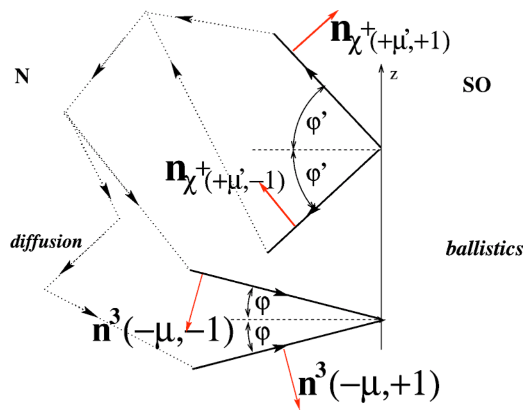

FIG. 7. (Color online) Angles and directions of spinors involved in the derivation of Eq. (41). For each angle φ and φ' there are two possible momentum directions corresponding to a different sign of the p_z component.

$$f^{-}_{\mathrm{N}}(-\mu) = J\frac{1}{\sqrt{2}\pi}H(\mu) + \frac{1}{\mu}\int_0^{\cos\varphi_c}\frac{d\mu'}{\sqrt{1-\mu^2}}S(\mu,\mu') \times(1+\mu\mu')f^{+}_{\mathrm{N}}(-\mu'). \tag{44}$$

Equations (43) and (44) can be analyzed using the smallness of $\cos\varphi^c$, but they can be easily solved numerically. In what follows we will use the results of numerical analysis for the distribution functions $f^{\pm}_{\mathrm{N}}(-\mu)$.

Ultimately, we are interested in the distributions on the SO side of the interface. To achieve this goal, we have to connect the calculated distributions $f^{\pm}_{\mathrm{N}}$ to the distributions on the SO side $f^{\pm}_{\mathrm{SO}}$. It follows straightforwardly from the Liouville's theorem that

$$f^{\pm}_{\mathrm{SO}}(\epsilon,+\mu_\theta) = f^{\pm}_{\mathrm{N}}(\epsilon,-\mu_\varphi), \tag{45}$$

where μ_θ and $-\mu_\varphi$ are two directions connected by the Snell's law, Eq. (4). In the subsequent discussion the notations $f^{\pm}_{\mathrm{SO}}(\theta)$ and $f^{\pm}_{\mathrm{N}}(\varphi)$ will be used instead of $f^{\pm}_{\mathrm{SO}}(\epsilon,+\mu_\theta)$ and $f^{\pm}_{\mathrm{N}}(\epsilon,-\mu_\varphi)$; notice that we return to the definition of angles φ and θ as it is given in Fig. 1. Finally, in Fig. 8 our result for $f^{\pm}_{\mathrm{SO}}(\theta)$ is presented.

We observe that the distribution of particles of the $+$ chirality emitted from the diffusive region into the clean one is flat even in the presence of the spin-dependent reflection at the interface. Moreover, at large angles the function $f^{+}_{\mathrm{SO}}(\theta)$ is noticeably increased.

As a result of the reflection at the interface there is a redistribution of the population of the particles of the $+$ and $-$ chiralities. Consequently, the two chiral components of the current, $j^{\pm}$, change in the vicinity of the interface, and we are faced with the question of the conservation of the total current $j=j^{+}+j^{-}$. (We assume for a moment the width of the diffusive stripe W to be unity, and until the end of this section will not distinguish between the density of current and the total current.) To check the current conservation we first calculated numerically the currents $j^{\pm}$ with the use of the distribution functions $f^{\pm}_{\mathrm{N}}(\epsilon,-\mu)$. We get that on the N side of the interface the current $j^{+}+j^{-}$ is equal to the total current j deep inside the diffusive region. Further on, it can be shown

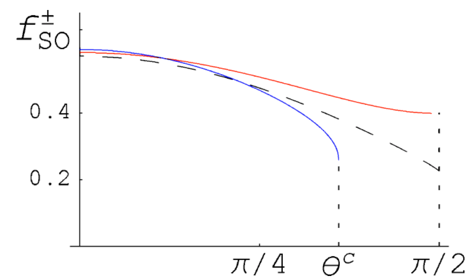

FIG. 8. (Color online) The displacement function $f^{\pm}_{\mathrm{SO}}(\theta)$ of the "$+$" (shown in red) and "$-$" (shown in blue) polarization components of the transmitted elecrrons for $\tilde{\alpha}=0.1$ as compared with the solution without reflection, $\tilde{\alpha}=0$ (dashed line).

that the two parts of the physical current $j^{\pm}$ do not change after passing the interface. To do this notice that in the SO region the chiral components of the physical current are

$$j^{\pm}_{\mathrm{SO}} = p^{\pm}_{\mathrm{SO}}\frac{e^2}{2\pi}\langle\cos\theta f^{\pm}_{\mathrm{SO}}(\theta)\rangle \tag{46}$$

[for the definition of the physical currents see Eqs. (19) and (20)]. Having in mind Eq. (45) and the Snell's law in its differential form, $p_{\mathrm{SO}}\cos\theta d\theta = p_{\mathrm{N}}\cos\varphi d\varphi$, the following relations for the $\pm$ chirality current components on the two sides of the interface can be obtained:

$$\begin{aligned} j^{+} &= p^{+}_{\mathrm{SO}}\frac{e^2}{2\pi^2}\int_0^{\pi/2} d\theta\cos\theta f^{+}_{\mathrm{SO}}(\theta) \\ &= p_{\mathrm{N}}\frac{e^2}{2\pi^2}\int_0^{\varphi^c} d\varphi\cos\varphi f^{+}_{\mathrm{N}}(\varphi) \end{aligned} \tag{47}$$

and

$$\begin{aligned} j^{-} &= p^{-}_{\mathrm{SO}}\frac{e^2}{2\pi^2}\int_0^{\theta^c} d\theta\cos\theta f^{-}_{\mathrm{SO}}(\theta) \\ &= p_{\mathrm{N}}\frac{e^2}{2\pi^2}\int_0^{\pi/2} d\varphi\cos\varphi f^{-}_{\mathrm{N}}(\varphi). \end{aligned} \tag{48}$$

For the $+$ chirality component it has to be taken into consideration that the current of the particles impinging on the interface in the interval of angles $\varphi^c<\varphi<\pi/2$ is canceled out by the flow of the reflected particles.

Thus we arrive at the conclusion that the two parts of the physical current $j^{\pm}$ do not change after passing the interface, i.e., $j^{\pm}_{\mathrm{N}}=j^{\pm}_{\mathrm{SO}}$. Together with the fact of the current conservation in the N region this implies that the total current in the ballistic SO region is equal to the current deep inside the diffusive region.

So far we limit ourselves to the diagonal elements of the density matrix. This was possible because in the N region the density matrix is diagonal in momentum space, while the current operator is diagonal in spin space. The situation is different in the SO region where the current operator acquires the spin structure, see Sec. II. In addition, the density matrix becomes nondiagonal in momentum space as an electron beam splits into two beams as a result of the refraction at the interface with the inhomogeneous spin-orbit interaction. We will not discuss this problem for the following reason. As it has been explained in Sec. III our analysis refers to the case when the current is registered far away from the near-field zone of the orifice, i.e., at a distance L much exceeding the width of the diffusive stripe. At such distances the trajectories of electrons of different chiralities diverge from each other and therefore the nondiagonal in momentum space components of the density matrix become nonlocal in space. Since the current operator is local this nonlocality of the density matrix cannot show up in the current measurements.

VI. SUMMARY

We have analyzed the transport near the interface with the inhomogeneous spin-orbit interaction in terms of the spin density matrix. The present analysis has been performed under the assumption that the SO interaction in the diffusive region is absent (suppressed completely). In fact, a much weaker condition is sufficient. It is enough that spin relaxation rate in the diffusive region is controlled by the Dyakanov-Perel mechanism:[20] $1/\tau_s \sim \Delta_{\mathrm{SO}}^2 \tau \ll 1/\tau$, where $\Delta_{\mathrm{SO}} = 2\alpha(x<0)p_F \ll 1/\tau$ is the spin splitting of the energy spectrum in the diffusive region, $x<0$. Under this condition the spin relaxation of the electrons during the propagation in the diffusive region after the reflection from the interface is negligible. (The other limit $\Delta^{\mathrm{SO}} \gtrsim 1/\tau$ will be considered elsewhere.)

We have verified that the specific property of the solution of the Milne problem—the existence of the flat distribution with a large portion of electrons emitted at small angles to the interface—still holds in the presence of the reflection at the interface. Moreover, at large angles the distribution function of electrons of the + chirality is noticeably increased, see Fig. 8. As it has already been pointed out in Sec. II, there exists an interval of the outgoing angles, $\theta^c < \theta < \pi/2$, where only the + spin chiral component can penetrate. Together, these observations call upon to exploit the diffuse emission for the purposes of spintronics.

In Fig. 9 a scheme of a device is presented which can operate as a spin filter for a current passing through the diffusive stripe confined between nonconducting areas A and B. Two additional barriers (or gates) are set at a distance L much larger than the width of the diffusive stripe W. Within this geometry each collector, $\vec{C}$ and $\overleftarrow{C}$, gets spin carriers emitted into the corresponding angular interval $\delta\theta$. When $\delta\theta < \pi/2 - \theta_c$, particles of the + chirality only can get into the collectors. As a result, the currents inside each of the two collectors are spin-polarized, dominantly along the $\pm\hat{x}$ directions.

In the setup under discussion the orifice of the stripe acts as a source of a current with an angular distribution $f_{\mathrm{SO}}^{\pm}(\theta)$. The number of electrons of a certain chirality flowing in a direction θ per unit time (i.e., the angular flux) is equal to $\mathcal{I}_{\mathrm{SO}}^{\pm}(\theta) d\epsilon d\theta$, where the intensity $\mathcal{I}_{\mathrm{SO}}^{\pm}(\theta)$ is related to $f_{\mathrm{SO}}^{\pm}(\theta)$ as $\mathcal{I}_{\mathrm{SO}}^{\pm} = [e^2/(2\pi)^2] p_{\mathrm{SO}}^{\pm} f_{\mathrm{SO}}^{\pm}(\theta) \cos\theta W$. The factor $\cos\theta W$ appears because we consider the flux of the particles outgoing from the orifice at an angle θ. The angular dependence of $\mathcal{I}_{\mathrm{SO}}^{\pm}(\theta)$ is in full accord with the expressions for the density

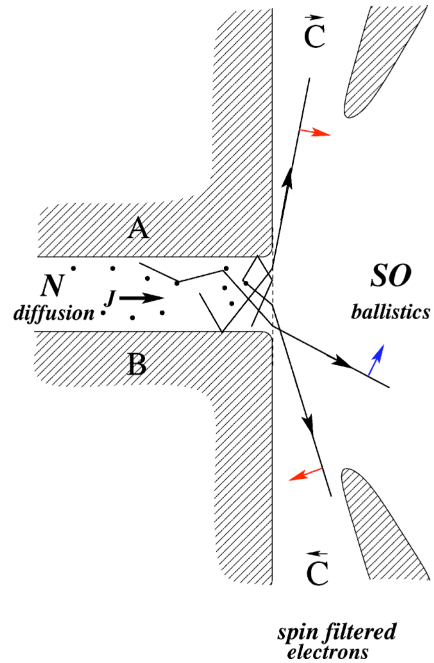

FIG. 9. (Color online) A spin filter. Electrons emitted from the diffusive stripe at small angles to the interface are spin polarized and can be collected for subsequent spin manipulations.

of the currents $j^{\pm}$; see expressions under the integrals in Eqs. (47) and (48). Assuming the angular distribution of the emitted electrons to be practically flat, the spin-polarized current that can be collected by each of the collectors is $\simeq j\delta\theta^2/8$. As it has been explained in Sec. II the width of the angular interval $\delta\theta = \pi/2 - \theta^c$ can be as much as $2\sqrt{\tilde{\alpha}}$. At this point the fraction of spin-polarized current reaches its optimal value $\simeq j\tilde{\alpha}/2$. All collected electrons have the same chirality that results in a very high level of spin polarization of the current in the collectors. A deviation from the perfect level of spin-polarization is only due to a small spread of the direction of motion of electrons within an angular interval $\delta\theta$.

In Ref. 8 a large spontaneous spin splitting has been detected in a gate controlled electron gas formed at a $\mathrm{In}_{0.75}\mathrm{Ga}_{0.25}\,\mathrm{As}/\mathrm{In}_{0.75}\mathrm{Al}_{0.25}\mathrm{As}$ heterojunction. The reported splitting corresponds to $\tilde{\alpha} \approx 0.1$. For such values of $\tilde{\alpha}$ one may expect a rather large angular interval $\delta\theta$ that can be usedfor spin filtration; $\delta\theta \approx 36^{\circ}$. Under these conditions, the device proposed in Fig. 9 has the following specifications: a fraction up to 5% of the total current is collected in $\vec{C}$ and is (almost) fully spin-polarized along the direction $\hat{x}$, while the other fraction of 5% is collected in $\overleftarrow{C}$ and is spin-polarized along the direction $-\hat{x}$.

After filtration the spin-polarized current can be manipulated similarly to the polarized light in optical devices. In particular, one can link the spin filter to the switch of the spin-polarized current discussed in Ref. 1.

ACKNOWLEDGMENT

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