Direct Excitation of High-Amplitude Chirped Bucket-BGK Modes

W. Bertsche, J. Fajans,* and L. Friedland†

Department of Physics, University of California–Berkeley, Berkeley California 94720, USA
(Received 26 September 2003; published 29 December 2003)

For the first time, high amplitude ($\Delta n/n \approx 40\%$), high Q (up to 100 000) Bernstein, Greene, and Kruskal modes have been controllably excited in a plasma. The modes are created by sweeping an excitation voltage downwards in frequency, thereby dragging a phase space "bucket" of low density into the bulk of the plasma velocity distribution. The modes have no linear limit and differ markedly from plasma waves and Trivelpiece-Gould modes.

Plasma waves [Trivelpiece-Gould (TG) waves in finite geometry [1]] are easy to generate and ubiquitous in nature. However, plasma waves are Landau damped, often quickly. At first glance this damping seems unavoidable, so it was quite surprising when Bernstein, Greene, and Kruskal (BGK) predicted that there exists a broad class of waves that do not damp. These BGK modes [2] are undamped because the distribution of the particles in the wave is already in the Landau relaxed form.

BGK modes underpin much of kinetic wave theory, but experimental verification of the existence of undamped BGK modes has proved difficult. It is easy to create transient large-amplitude waves or structures, but the waves are typically short-lived or unstable. For instance, waves created by Wharton, Malmberg, and O'Neil [3] were unstable due to a sideband instability. More recent work has not been much more successful [4,5]. Long-lasting structures can be created by continuous drives; driven double layers, which are closely related to BGK modes, have been observed in the earth's auroral zone [6]. Danielson [7] recently reported that plasma waves eventually decay into low amplitude, but long-lasting, BGK modes.

Here we report that we can excite very high amplitude BGK modes that differ markedly from the more common TG modes. The modes are excited by an oscillating voltage applied to one end of a pure-electron plasma column confined in a standard Penning-Malmberg trap [8] [see Fig. 1(a)]. The resulting density fluctuations are detected by monitoring the image charge on another cylinder, typically at the opposite end of the trap. When the plasma is cold, TG waves are observed as expected. A typical spectrum is shown in Fig. 2. But as the plasma temperature T is increased, the TG waves become so heavily damped that they essentially disappear. Nonetheless, undamped waves can be excited in hot plasmas by low amplitude drives that sweep downward from some frequency f_s to some lower frequency f_e. Typical response curves are shown in Fig. 3. Large-amplitude waves are excited for a very broad range of f_s and f_e.

We believe that these new waves are BGK modes. They differ markedly from TG modes. For instance, TG modes exist only when the plasmas are sufficiently cold, while our BGK modes exist only when the plasmas are sufficiently hot; there is only a small overlap region. The TG modes occur at distinct frequencies; even when thermally broadened, the TG modes possess a well-defined linear limit. In contrast, the BGK modes have *no* linear limit. As can be seen in Fig. 3, they can be excited over a broad range of frequencies. The TG modes are typically excited
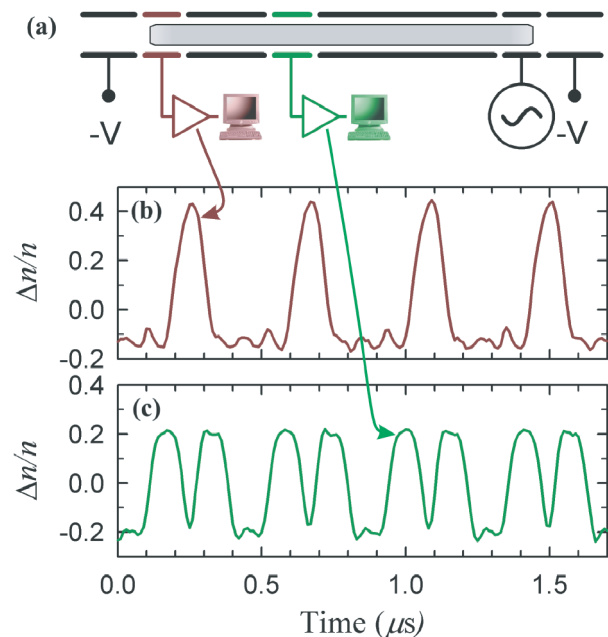

FIG. 1 (color online). (a) Penning-Malmberg trap geometry. The pure-electron plasma column is confined axially by the large negative end potentials, and radially by a 1500 G magnetic field. Typical plasma densities are $\sim 10^7$ cm^{-3}, lengths ~ 27 cm, and radii ~ 1 cm, and the plasma is confined in cylinders with radius 1.905 cm. (b) Density fluctuations due to the BGK mode at the column end, and (c) near the column center. The positive fluctuations correspond to electron holes. Because the BGK mode must pass through the center twice (approaching and leaving) each time the pulse reaches the end, there are twice as many pulses in (c) as in (b). As expected for an open-ended reflection, the mode is larger at the end (b) than near the center (c).
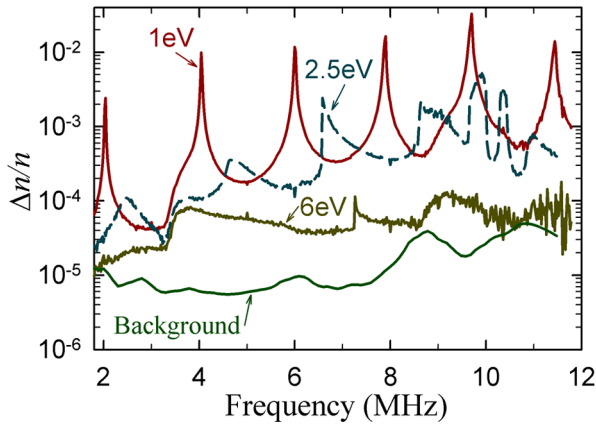
FIG. 2 (color online).   Observed response from a pure 50 mV drive, for three different plasma temperatures. Also shown is the background noise from external sources. The spectrum follows the TG dispersion relation, $\omega_{mH}^2 = \omega_m^2 + 3v_{th}^2 k^2$, where $\omega_m = [a^2 k^2 \omega_p^2 / (1 + a^2 k^2)]^{1/2}$ are the cold TG mode frequencies, a is the radial geometric constant, $k = (m + 1)\pi/L$ is the wave number, m is a non-negative integer, L is the plasma length, ω_p is the plasma frequency, and v_{th} is the electron thermal velocity.

by single frequency drive. The BGK modes can only be excited by a swept frequency drive. Using a swept frequency drive, Yamazawa and Michishita [9] determined that TG modes possess a hard nonlinearity: as their amplitude increases, their frequency increases slightly ($<5\%$). BGK modes have a soft nonlinearity: as their amplitude increases, their frequency decreases by nearly 50%. Even for cold plasmas, the Q's (ω/γ, where γ is the damping rate) for undriven, low amplitude TG modes are never much higher than 600, and the Q's get very low for hotter plasmas. Once excited by a swept frequency drive,
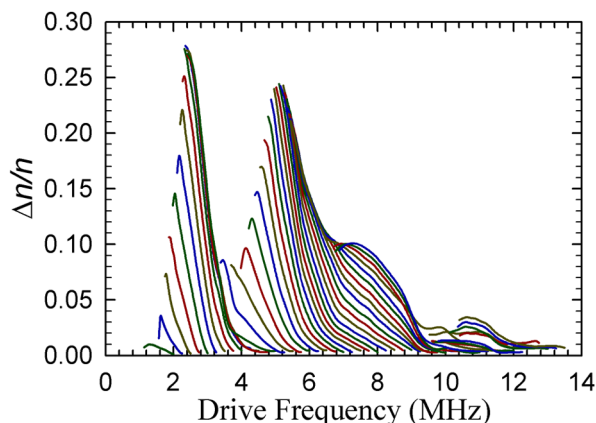

FIG. 3 (color online).   Response to 50 mV, 1 GHz/s swept frequency drives. Each curve plots the envelope of the response as a function of time, and, hence, frequency, as the sweep progresses downward from each plot's individual start frequency f_s. Along each curve, the response is phase locked to the drive. As the drive is swept past the peaks at 2.4 and 5 MHz, phase locking is lost and the mode amplitude collapses.

the Q's for the undriven BGK modes are remarkably high: typically around 6000, but occasionally as high as 100 000. Indeed, Q is often a poor measure of the lifetime of these modes as their amplitude does not decay exponentially. For example, the very-high-Q mode amplitudes are often nearly flat, except for small fluctuations, followed by a sudden collapse. High amplitude TG modes are unstable; Hart and Peterson [5] and Yamazawa and Michishita [9] demonstrated that they mode convert quickly. As one might expect from their high Q's, the BGK modes are very stable even when their density fluctuations exceed 40%. Finally, low amplitude TG modes have little harmonic content, while the harmonic content of the BGK modes is rich even at low amplitude.

We identify these modes as BGK modes because of the method by which they are excited. When the drive is first applied, electrons whose end-to-end bounce frequencies are close to the drive frequency will be captured into a trapping bucket if they have the right phase relative to the drive; approximately half the resonant electrons are so captured (see Fig. 4). The untrapped electrons are perturbed and may form a TG mode, but this TG mode damps out quickly because of the high plasma temperature.

As the drive frequency is swept downward to frequency f_e, the resonant electron velocity $\bar{v} = 2Lf$ decreases. To the first approximation, few electrons can cross the separatrix between trapped and untrapped electrons if the sweep is slow; few new electrons enter the trapping bucket, and few initially trapped electrons leave it. Thus, the trapped electrons are dragged to lower velocities, and the density of electrons in the bucket remains fixed. But for typical distribution functions, the initial density of electrons is lower at velocity $\bar{v}_s$ (the velocity corresponding to the starting frequency f_s) than at $\bar{v}_e$ (the velocity corresponding to the ending frequency f_e), and dragging the bucket downward creates a hole in phase space. Since the spatially localized bucket bounces from end to end in phase with the drive, the phase space hole
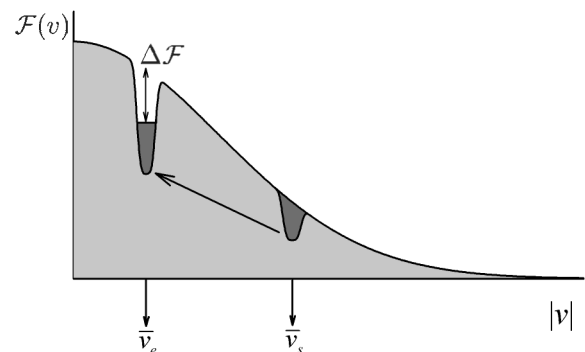

FIG. 4.   Cartoon of the spatially averaged distribution function $\mathcal{F}(v)$ showing the evolution of the trapping bucket as the frequency is swept from f_s to f_e. The dark and light regions indicate the trapped and untrapped electrons, respectively.

oscillates as well. The hole creates an electrostatic perturbation, which constitutes the postulated BGK modes. Because the excitation process involves dragging a bucket through velocity space, we call the generated waves "chirped bucket-BGK" modes.

Strong evidence for this model comes from experiments which perturb the distribution function. For example, a strong, fixed frequency drive at f_f will phase mix the velocities in the region centered around the velocity $\bar{v}_f$ resonant with the drive, effectively flattening the distribution function there. The flat in the distribution function increases the number of particles with velocity slightly greater than $\bar{v}_f$ and decreases the number of particles with velocity slightly less than $\bar{v}_f$. If a sweeping drive is then initiated, the difference between the flattening frequency f_f and the swept drive start frequency f_s will affect the final amplitude of the BGK mode (see Fig. 5). If $f_f > f_s$, the density will have been diminished at $\bar{v}_s$, so fewer particles will be trapped into the sweeping bucket, the phase space hole will be proportionally larger, and the final amplitude will increase. In contrast, if $f_f < f_s$, the density of particles near $\bar{v}_s$ will have been increased, and the final amplitude will decrease.

These flattening experiments clearly demonstrate that the system response depends on the shape of the initial distribution function. Other experiments, not shown, show that we can transport particles in velocity space; for instance, an upwardly sweeping drive will carry particles to a higher velocity, leaving a hole behind in the distribution function at the drive's initial resonant velocity $\bar{v}_s$. If a downward drive is then initiated starting from the same initial frequency, the resulting BGK mode will be larger. Repeating the upward-going drive several times before initiating the downward drive will dig an ever deeper hole, resulting in ever larger BGK modes. We can also observe trapping oscillations, proving that particles are trapped in the potential of the wave.

We have analyzed the excitation process using a Vlasov-Poisson approach. Our model is similar to the multibeam model presented in Stix [10] and elsewhere, but takes into account the axial bounce motion of the electrons in the trap. We express the distribution function $\mathcal{F}(I, \theta, t)$ in terms of the action (I)-angle (θ) variables of the bounce motion and expand the distribution in a Fourier series in θ: $\mathcal{F} = \sum \mathcal{F}_m(I, t) \exp(im\theta)$. To find the bounce-averaged part of the distribution function, $\mathcal{F}_0(I, t)$, we use a nonperturbative approach based on simulations of driven electron dynamics, which yield distributions like those in Fig. 4. These averaged distributions are then used to derive the lowest order Vlasov-Poisson equations to find the amplitude ε_m of the mth harmonic, phase-locked electric field perturbation. The details of the analysis will be given in a later paper. Here, we present the main result; the excited mode amplitude scales with frequency as

$$\sqrt{\varepsilon_m} \propto \frac{\Delta \mathcal{F}}{1 + (\omega_0/\omega_{th})^2 R(\xi_m)}, \qquad (1)$$

where ω_0 is the fundamental cold TG mode frequency, $\omega_{th} = \pi v_{th}/L$ is the electron bounce frequency, and $\xi_m = \omega/(\sqrt{2}m\omega_{th})$. The object $\Delta \mathcal{F} \approx [\mathcal{F}(\bar{v}) - \mathcal{F}(\bar{v}_s)]/2$ is the depth of the hole in $\mathcal{F}_0(I, t)$, and $R(\xi_m) = 1 + \xi_m[Z(\xi_m) - Z(-\xi_m)]/2$ is related to the plasma dispersion function Z (see Ref. [10].)

Equation (1) explains many aspects of the observed excitations. In the cold plasma case, $\omega_0 \gg \omega_{th}$, for $\omega > \omega_m$ one can use the large ξ limit, where $R \approx -1/2\xi^2$. This yields $\sqrt{\varepsilon} \propto \Delta \mathcal{F}/[1 - (\omega_m/\omega)^2]$. Thus, the amplitude grows as ω approaches ω_m from above, but collapses beyond ω_m where the right-hand side becomes negative. But in the cold case, the resonant bucket is in the tail of
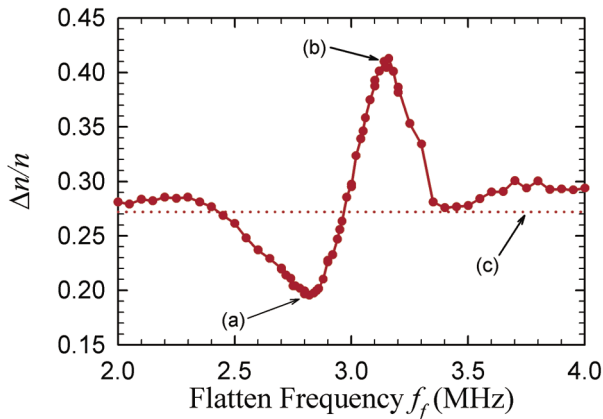

FIG. 5 (color online).   Response from a 1 GHz/s, 100 mV sweep from $f_s = 3$ MHz to $f_e = 2.2$ MHz that is preceded by a 100 μs long, 500 mV, fixed frequency f_f drive. The fixed frequency drive flattens the distribution function around the resonant velocity linked to its frequency. Near (a), the fixed frequency drive is below f_s, near (b) it is above f_s, and the dotted line (c) is the response without the fixed frequency drive.
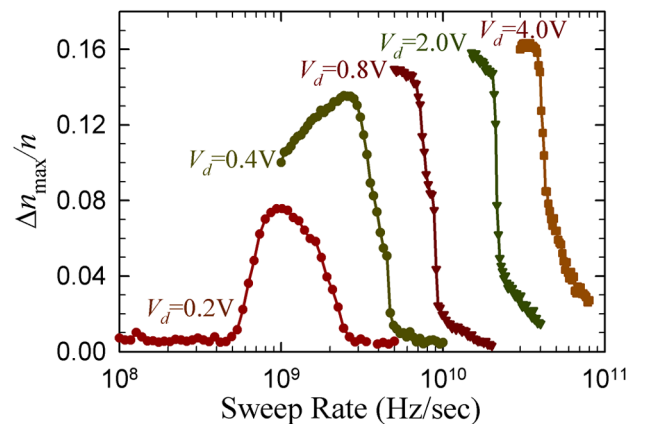

FIG. 6 (color online).   Peak signal for a $3 \rightarrow 2.2$ MHz sweep as a function of sweep rate, for several drive amplitudes.
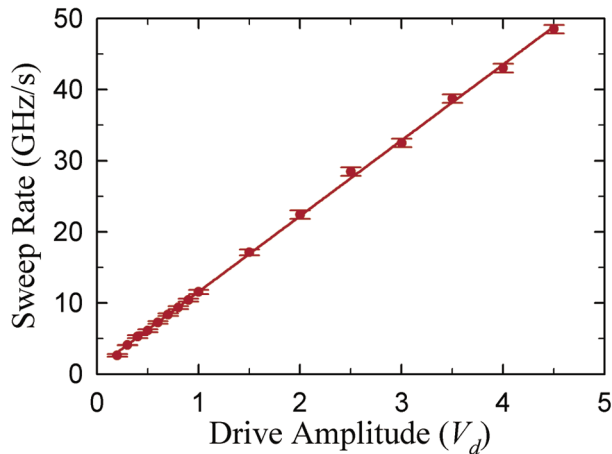
FIG. 7 (color online). Maximum sweep rate that generates bucket-BGK modes as a function of drive strength. The sweep rates extend from $3 \to 2.2$ MHz.

the distribution function, so $\Delta\mathcal{F}$ is exponentially small and only small excitations are observed. We find that this situation is characteristic of our experiments below $T \sim 1$ eV, i.e., when $\omega_{th} < 0.44 \times 10^7$ rad/sec, while $\omega_0 \approx 1.2 \times 10^7$ rad/sec.

In contrast, for $\omega_0 \sim \omega_{th}$ ($T \sim 6$ eV in our experiments) the denominator in Eq. (1) reaches its minimum at $\xi_0 \approx 1.5$, i.e., at $\omega \approx 2.1 m\omega_{th}$, where $\Delta\mathcal{F}$ is large because the resonant bucket has moved into the bulk of the distribution. Since $\Delta\mathcal{F}$ in Eq. (1) is a decreasing function of ξ, the excited BGK mode amplitudes reach maxima at somewhat lower frequencies. Such multi-peaked excitations are indeed seen in Fig. 3, where maxima are located at $\omega \approx 1.3 m\omega_{th}$.

A key assumption in the theory behind Eq. (1) is that particles do not escape the bucket as the driving frequency is swept. We find (Fig. 6) that this assumption is correct so long as the sweep rate is neither too slow nor too fast. If the drive is swept too slowly, particles escape out of the bucket because of collisions and/or loading in the external circuitry [7]. Preliminary evidence suggests that the time scales of these effects are related to the time scale for the decay of the undriven modes.

If the sweep rate is too high, the bucket-BGK modes disappear. Using action-angle variables, we find that the buckets disappear entirely when the sweep rate is too high; the drive strength must be increased linearly with the sweep rate for the buckets to exist. This prediction is proved by experimental measurements (Fig. 7). The analysis is similar to that for autoresonance for diocotron modes [11,12].

In conclusion, we have generated very high amplitude bucket-BGK modes. The modes have no linear limit and can be excited over a very broad range of frequencies. They are very stable, can be produced controllably, and respond appropriately to changes in the distribution function. The longevity of the modes is remarkable. Not only do the mode Q's reach 100 000, but the modes survive despite up to 60 000 reflections from the trap ends. We have developed a simple theory that explains many of the results. Our chirped bucket-BGK modes live much longer than the BGK modes found in earlier experiments. The reason may be related to the fact that most of the previous experiments used strong, or even impulsive, drives. Our gentle excitation process allows us to create seemingly perfect BGK modes.

*Electronic addresses: joel@physics.berkeley.edu
socrates.berkeley.edu/~fajans
†Permanent address: Hebrew University, Jerusalem, Israel.
[1] A. Trivelpiece and R. Gould, J. Appl. Phys. **30**, 1784 (1959).
[2] I. B. Bernstein, J. M. Greene, and M. D. Kruskal, Phys. Rev. **108**, 546 (1957).
[3] C. B. Wharton, J. Malmberg, and T. O'Neil, Phys. Fluids **11**, 1761 (1968).
[4] J. D. Moody and C. F. Driscoll, Phys. Plasmas **2**, 4482 (1995).
[5] G. Hart and B. G. Peterson, in *Non-Neutral Plasma Physics IV: Workshop on Non-Neutral Plasmas*, edited by F. Anderegg, C. F. Driscoll, and L. Schweikhard (AIP, New York, 2002), p. 341.
[6] R. E. Ergun, C. W. Carlson, J. P. McFadden, F. S. Mozer, L. Muschietti, I. Roth, and R. J. Strangeway, Phys. Rev. Lett. **81**, 826 (1998).
[7] J. Danielson, Ph.D. thesis, University of California–San Diego, 2002.
[8] *Non-Neutral Plasma Physics IV: Workshop on Non-Neutral Plasmas*, edited by F. Anderegg, C. F. Driscoll, and L. Schweikhard (AIP, New York, 2002).
[9] Y. Yamazawa and T. Michishita, Jpn. J. Appl. Phys. **40**, 5431 (2001).
[10] T. H. Stix, *Waves in Plasmas* (Springer-Verlag, Heidelberg, 1992).
[11] J. Fajans, E. Gilson, and L. Friedland, Phys. Rev. Lett. **82**, 4444 (1999).
[12] J. Fajans, E. Gilson, and L. Friedland, Phys. Plasmas **6**, 4497 (1999).