Autoresonant (Nonstationary) Excitation of the Diocotron Mode in Non-neutral Plasmas

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We report on the autoresonant (nonlinear phase locking) manipulation of the diocotron mode in a non-neutral plasma. Autoresonance is a very general phenomena in driven nonlinear oscillator and wave systems, and allows us to control the amplitude of a nonlinear wave without the use of feedback. These are the first controlled laboratory studies of autoresonance in a collective plasma system. [S0031-9007(99)09167-X]

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An important goal of nonlinear dynamics is the excitation and control of nonlinear waves and oscillations. Autoresonance (nonlinear phase locking) is one method of achieving this goal. Autoresonance is the natural tendency of a weakly driven nonlinear system, under certain conditions, to stay in resonance with its drive even if the parameters of the system vary in time and/or space. For example, consider a system in which the oscillation frequency increases with oscillation amplitude. Assume that the system is initially phase locked to its drive. In autoresonance, increasing the drive frequency will cause a corresponding increase in the oscillation amplitude, while decreasing the drive frequency will cause the oscillation amplitude to decrease. In some cases, the system need not start phase locked to become phase locked; if the drive frequency is swept slowly through the linear resonant frequency, the system will phase lock automatically. The occurrence of automatic phase locking is not self-evident, as the system, in a Hamiltonian picture, has to cross the separatrix between streaming and trapped orbits.

The autoresonance concept dates back to McMillan [1] and Veksler [2] and was further developed by Bohm and Foldy [3] for particle accelerators. The term "phase stability principle" was used to describe the phenomenon in these early studies. The synchrotron, synchrocyclotron [4], and other, later acceleration schemes [5,6] all are based on autoresonance. Recently, the effect has been studied theoretically in atomic and molecular physics [7,8], nonlinear dynamics [9], and nonlinear waves [10].

"Jumps" have long been studied in nonlinear dynamics [11]. The swept, or nonstationary excitation of oscillators has also been studied. The linear case was solved exactly [12], and Mitropolskii [13] has studied the nonlinear case. None of these studies uncover the threshold and scaling effects discussed here. Entrainment in self-excited systems like van der Pol oscillators [14] bears some resemblance to the results discussed here, as do effects noted in computer modeling of planetary systems [15].

Here, we discuss the first direct experimental observations of autoresonant manipulation of the off-axis rotation state of a pure-electron plasma column interacting with its image charge (the diocotron mode) [16]. The diocotron mode results from the collective action of the plasma on itself; previous observations of autoresonance have been in single particle systems. As there exists a one-to-one correspondence [17] between the relevant plasma dynamics and two-dimensional, inviscid incompressible fluid dynamics, autoresonance should also occur in vortex dynamics.

The pure-electron plasma is confined in a Penning-Malmberg trap [18]. The plasma forms a cylindrical column centered inside a metallic, cylindrical trap wall (see Fig. 1). Longitudinal confinement is provided by appropriately biasing wall segments at the trap ends. Radial confinement is provided by an axial magnetic field **B**. The **E** × **B** drifts which result from the plasma's self-electric

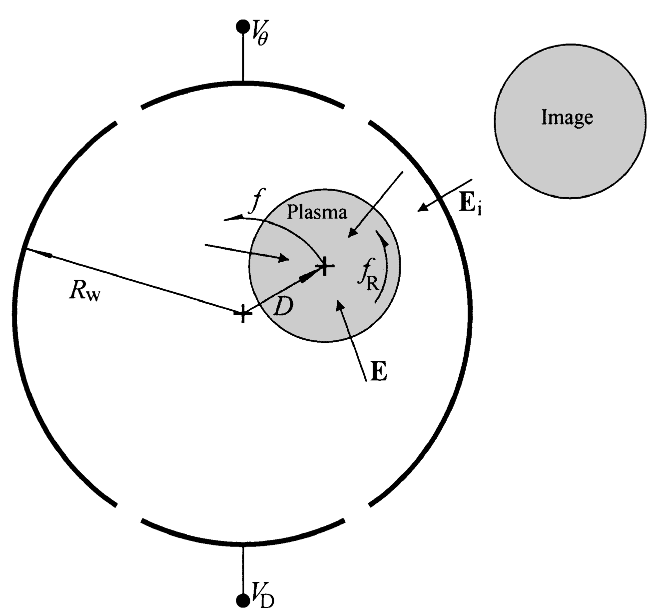

FIG. 1. End view of the trap showing the confining wall at R_w, the pickup V_θ and drive V_D sectors, the plasma a distance D from the trap center, the self-electric field **E**, the self-rotation drift f_R, the plasma image, the image electric field $\mathbf{E}_i$, and the diocotron drift at frequency f.

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field cause the plasma to rotate around itself. In global thermal equilibrium, the plasma rotates rigidly with frequency f_R; in the partially equilibrated plasmas used in this work the plasma's self-rotation rate is only approximately constant. If the plasma is moved off center, it undergoes an additional $\mathbf{E}_i \times \mathbf{B}$ drift from the electric field of its image. As this drift always points azimuthally, the plasma orbits around the trap center. This motion, at frequency f, is called the diocotron mode and is very stable and can last for hundreds of thousands of rotations.

Assuming that the plasma column's charge per unit length is λ, the electric field of its image, E_i, is approximately radial and constant across the plasma, $E_i \approx 2\lambda D/[R_w^2(1 - D^2/R_w^2)]$ (cgs Gaussian units). Here R_w is the wall radius, and D is the distance that the plasma column is off center, i.e., the mode amplitude. The diocotron mode frequency f follows by equating $2\pi f D$ to $c\mathbf{E}_i \times \mathbf{B}/B^2$, giving

$$f = f_0\left(\frac{1}{1 - D^2/R_w^2}\right). \tag{1}$$

Here, $f_0 = \omega_0/2\pi \equiv c\lambda/\pi B R_w^2$ is the linear resonant frequency. Note that the mode frequency increases with mode amplitude [19,20]. Experimentally, we can determine both the mode frequency and amplitude by measuring the image charge at a particular angle on the trap wall as a function of time. More precisely, we measure the time dependence of the surface charge on an azimuthal sector like the one labeled V_θ in Fig. 1. The mode can be driven by applying a signal to a second, driving sector V_D [21]. This driving signal creates electric fields which induce additional drifts. As we generally use weak driving signals, these drifts are much smaller than the rotation and diocotron drifts. Nevertheless, because of phase locking, these drifts are sufficient for efficient control of the diocotron mode.

The experiments reported here were done at $B = 1485$ G in a trap with wall radius $R_w = 1.905$ cm. The plasma density was approximately 2×10^7 cm^{-3}, temperature $T = 1$ eV, and plasma radius 0.6 cm. The measured linear diocotron frequency was approximately 28 kHz. The plasma was confined within negatively biased cylinders separated by 10.25 cm. Finite length and radius effects, discussed in Ref. [22], increase the linear frequency from that given by Eq. (1) by approximately 50% and also modify the dependence on D. We have obtained similar autoresonance effects for plasmas of different lengths and radii, confined by magnetic fields of different strengths.

The mode can be autoresonantly excited to high amplitude by applying a swept oscillating signal to the driving sector V_D. A typical result is shown in Fig. 2, where a sinusoidal signal of amplitude 0.8 V p.-p. is swept from 20 kHz (well below the linear resonant frequency) to 45 kHz (well above the linear resonant frequency) in 0.05 s. In the beginning, the mode amplitude is small and has frequency components at both the drive frequency and the linear diocotron mode frequency. The mode is not well phase locked to the drive. As the drive frequency increases, the amplitude grows slowly and the phase locking improves. After the drive frequency passes the linear resonant frequency, the amplitude grows autoresonantly. The system is phase locked and only one frequency is present. Finally, the amplitude grows large enough to send the plasma into the wall, the mode frequency drops precipitously, and phase locking fails abruptly.

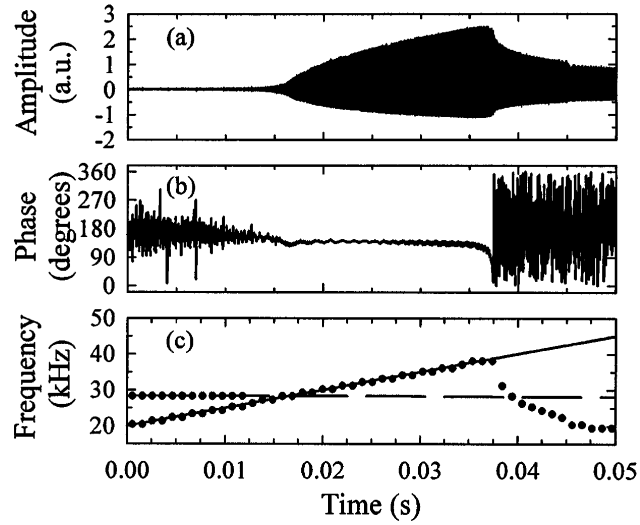

FIG. 2. Autoresonant response to a swept drive. (a) Signal received on the pickup sector. (b) Relative phase between the mode and the drive. The phase is found by correlating the received signal with the drive signal in a small window slid over the data. At the very beginning and end the relative phase probably oscillates between 0° and 360°, however, the oscillations are too fast to be fully detected by the correlation routine. See Fig. 4(b) for a more accurate result. (c) Drive frequency (solid line), measured linear resonant frequency (dashed line), and measured excitation frequencies (•).

As illustrated in Fig. 3, autoresonance can also be used to decrease the mode amplitude by sweeping the drive frequency downwards. Here the 0.5 V p.-p. drive is repeatedly swept up and down between 23 and 31 kHz. The mode amplitude grows when the drive frequency is swept upwards and damps when the drive frequency is swept downwards. Notice that only the drive frequency is present in the autoresonant region when the drive frequency is above the linear frequency, while the drive and the linear mode frequencies are present when the drive frequency is below the linear frequency.

The experiments shown in Figs. 2 and 3 occur over a short enough time period that changes in the system parameters are relatively unimportant; autoresonance occurs because the drive frequency varies. Autoresonance also occurs when the drive frequency is constant but the system parameters change. For example, in our experiment plasma expansion causes the linear diocotron frequency to drop [22] by about 14% in 0.5 s. If the system is driven

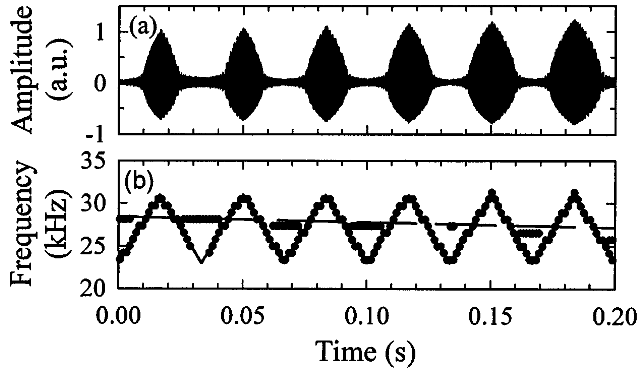

FIG. 3. Autoresonant response to a sawtooth swept drive. (a) Signal received on the pickup sector. (b) Drive frequency (solid line), measured linear resonant frequency (dashed line), and measured excitation frequencies (•).

by a constant frequency which is initially below linear resonance, the mode grows autoresonantly when its linear resonant frequency drops through the drive frequency. A typical example is shown in Fig. 4, where the drive frequency is 27.4 kHz and the drive amplitude is 0.04 V p.-p. The initial linear diocotron frequency is 28.4 kHz. Autoresonant growth occurs only after the linear mode frequency has dropped to the drive frequency, at $t = 0.11$ s. The envelope of the phase locking curve, as well as many details hidden in Figs. 2–4, is well understood within the associated Hamiltonian picture, described briefly below, and will be elucidated in a future paper.

If the drive frequency or system parameters are changed too quickly, autoresonance will not occur. For example, for a fixed chirp rate $\mathcal{A}$ (the change in the drive frequency

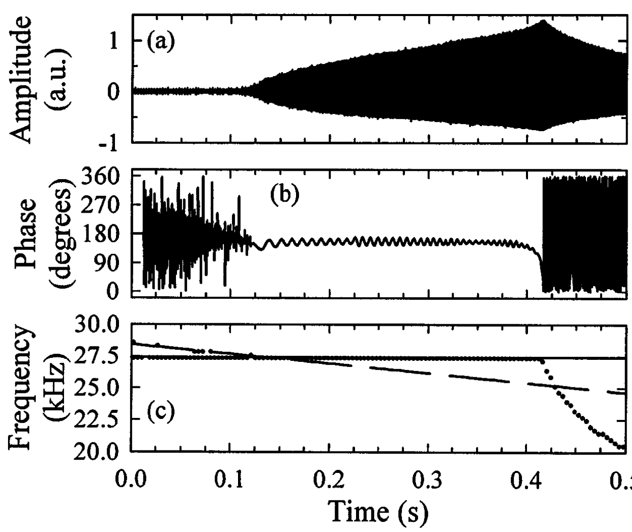

FIG. 4. Response to a constant frequency drive. Autoresonance occurs because the system's linear resonant frequency drops as the plasma expands. (a) Signal received on the pickup sector. (b) Relative phase between the mode and the drive. (c) Drive frequency (solid line), measured linear resonant frequency (dashed line), and measured excitation frequencies (•).

per second), there is a threshold drive amplitude V_a. Below this threshold, the maximum mode amplitude is relatively small and increases with the drive amplitude; above this threshold the mode amplitude follows the drive frequency to high amplitude and is independent of the drive amplitude. As shown in Fig. 5, the threshold is very sharp. Lower chirp rates have lower drive amplitude thresholds. Theoretically,

$$V_a \propto \mathcal{A}^{0.75}, \tag{2}$$

and is in excellent agreement with the data, as shown in Fig. 6. The data shown here were taken at very high Q; the same threshold and scaling phenomena were observed with Q's as low as 60.

The existence of this threshold reflects the condition in the associated Hamiltonian problem that there must exist a stable, phase locked, quasiequilibrium as the mode amplitude grows. The derivation of Eq. (2) is lengthy and the details will be presented elsewhere. A similar analysis in an analogous problem is given in Ref. [23]. Briefly, we describe the mode by an approximate, isolated-resonance Hamiltonian [24], $H(I, \theta; t) = -\omega_0(t)\ln(1 - I) + 2\epsilon I^{1/2}\cos(\theta - \phi)$. The Hamiltonian is a function of the action $I = (D/R_w)^2$, and the plasma rotation angle θ; ϵ and $\phi \equiv \int \Omega(t)\,dt$ are the normalized dipole

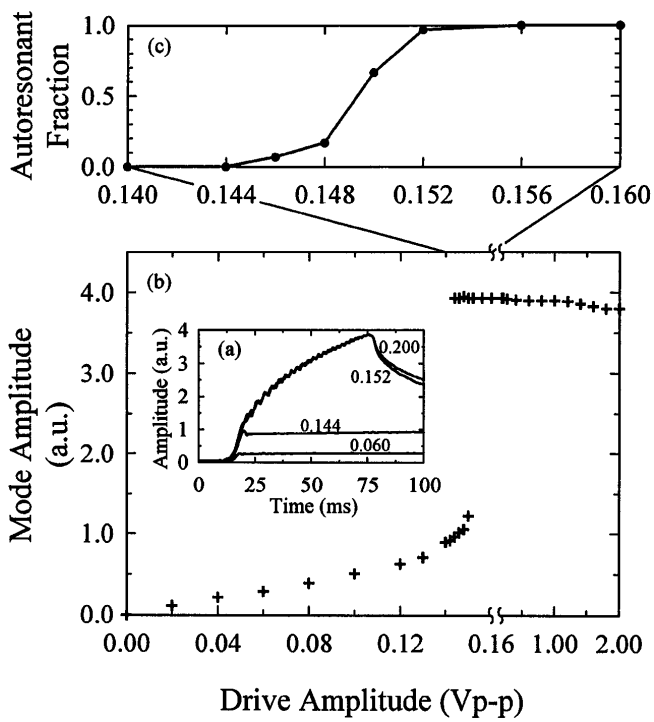

FIG. 5. Autoresonant response near threshold. (a) Mode amplitude as a function of time for drive amplitudes of 0.060, 0.144, 0.152, and 0.200 V p.-p. (b) Maximum mode amplitude as a function of drive amplitude. Near the drive threshold voltage 0.150 V p.-p., the response is bimodal; some shots stay low, while other shots go to high amplitude. (c) The fraction of shots near threshold that go to high amplitude. All data taken at a chirp rate of 1.7×10^5 Hz/s.

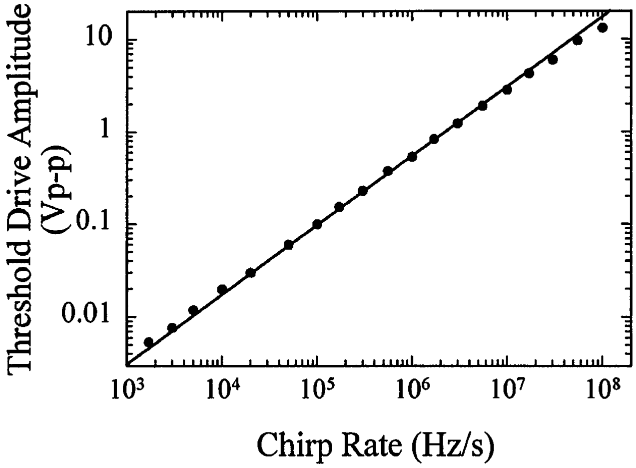

FIG. 6. Chirp rate $\mathcal{A}$ vs threshold amplitude V_a. Measured results (•), and theoretical prediction from Eq. (2) (solid line). The proportionality constant in Eq. (2) is fit to the data.

amplitude and phase of the drive. The linear oscillation frequency $\omega_0(t)$ may be a slow function of time. This Hamiltonian yields the following evolution equations for I and the phase slip $\Phi \equiv \theta - \phi$: $dI/dt = 2\epsilon I^{1/2}\sin\Phi$, and $d\Phi/dt = \omega_0(t)/(1 - I) - \Omega(t) + \epsilon I^{-1/2}\cos\Phi$. This set of evolution equations differs from those in the classical theory of nonlinear resonance by the slow variation of ω_0 and Ω with time. The essence of autoresonance is that despite this time variation, the system preserves [to $\mathcal{O}(\epsilon^{1/2})$] the approximate resonance condition $\omega_0(t)/(1 - I) \approx \Omega(t)$. To study the threshold problem, we take the small I limit of the evolution equations and expand around the quasi-steady-state, $\Phi_0 = \pi$ and $I_0(t)$ satisfying $\omega_0(1 + I_0) - \Omega - \epsilon I_0^{-1/2} = 0$. Then the evolution of a small deviation $\Delta = I - I_0$ from the steady state is given by the approximate Hamiltonian $H(\Phi, \Delta) = S\Delta^2/2 + V_{\mathrm{eff}}(\Phi)$, where $S = \omega_0 + \epsilon I^{-3/2}/2$ and $V_{\mathrm{eff}}(\Phi) = 2\epsilon I_0^{1/2}\cos\Phi + 2\pi\mathcal{A}\Phi/S$. Thus the problem reduces to that of a quasiparticle of velocity Δ and slowly varying mass S^{-1} moving in a slowly varying quasipotential V_{eff}. Phase locking corresponds to trapping of the quasiparticle in the quasipotential; trapping demands that the quasipotential has a minimum, i.e., that $\epsilon > \pi\mathcal{A}/SI_0^{1/2}$. But $SI_0^{1/2} = \omega_0 I_0^{1/2} + \epsilon/2I_0$ has a minimum at which the condition on ϵ is hardest to satisfy. This leads to Eq. (2).

In conclusion, autoresonant excitation is a practical method of controlling the amplitude of the diocotron mode in a non-neutral plasma. As autoresonance is a general property of nonlinear oscillators and waves, it should occur in many other systems. For example, we plan to search for autoresonant phenomena in higher order modes like the elliptical diocotron mode ($\ell = 2$ or Kelvin mode).

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