THE NONLINEAR DYNAMICS OF DENSE ELECTRON BEAMS IN THE AUTORESONANCE LASER ACCELERATOR

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The axial evolution of the electromagnetic fields is solved self-consistently with the nonlinear dynamics of the electron beam in the autoresonance laser accelerator (ALA). A 50 kA/cm^2 electron beam can be accelerated over 100 m from an initial energy of 25 MeV to 4.6 GeV by a CO_2 laser with a peak initial intensity of 2.8×10^{14} W/cm^2 along a 100 kG guide magnetic field.

Different approaches to the acceleration of charged particles to high energies by high power lasers have received much attention recently [1]. Within the far field approach two schemes were suggested, namely, the autoresonance laser accelerator (ALA) [2–4] and the inverse free electron laser (IFEL) [5]. The ALA scheme is based on a self-sustained cyclotron resonance between the particles and a circularly (or a linearly) polarized laser radiation propagating along an axisymmetric guide magnetostatic field. The resonance condition is $\Omega'/\gamma=k_0v_z-\omega'_0$, where k_0 and ω'_0 are the laser wavevector and frequency, and γ, v_z and Ω'/γ are the relativistic factor, the axial velocity and the relativistic cyclotron frequency of the particles. The detailed nonlinear single particle dynamics in the ALA fields configuration was analyzed recently [2]. It was shown that the acceleration of electrons to high energies with low radiation losses is possible in the ALA scheme for homogeneous electromagnetic fields. Therefore, it was suggested that the acceleration of high current beams might be feasible. It was also shown that the accelerated beam can be launched into the desired autoresonance regime through transition regions. The use of spontaneously generated megagauss magnetic fields in laser produced plasmas for autoresonance acceleration was also discussed [3]. The simultaneous autoresonance acceleration of both electrons and positrons by a linearly polarized laser radiation presents a novel possibility of accelerating high current "quasi-neutral" beams to high energies [4].

In this Letter we shall discuss the collective acceleration of dense electron beams to high energies by high power lasers within the ALA scheme. Consider a circularly polarized laser radiation field with frequency ω'_0 propagating together with a cold electron beam of density N along a homogeneous axial magnetic field B_0 in the z direction. The acceleration of the electrons in the system is achieved by satisfying the cyclotron resonance condition at $z=0$ [2], and is accompanied by a self-consistent decrease of the laser field amplitude along the interaction region. We assume for the sake of simplicity a 1D model in which the system parameters have spatial dependence only along the z axis. We define the following set of orthonormal base vectors,

$$\hat{e}_1 = -\hat{e}_x \sin\theta + \hat{e}_y \cos\theta ,$$

$$\hat{e}_2 = -\hat{e}_x \cos\theta - \hat{e}_y \sin\theta ,$$

$$\hat{e}_3 = \hat{e}_z , \tag{1}$$

where $\hat{e}_x$, $\hat{e}_y$, $\hat{e}_z$ are unit vectors in the x, y, z directions, respectively,

$$\theta = \omega_0'(z/c - t) \tag{2}$$

and c is the light velocity.

The electromagnetic fields $\boldsymbol{E},\boldsymbol{B}$ as well as the density N and velocity $\boldsymbol{v}$ of the cold electron beam can be written as

$$\boldsymbol{E} = E_1(z)\hat{e}_1 + E_2(z)\hat{e}_2 = \boldsymbol{E}_\perp(z) , \tag{3}$$

$$\boldsymbol{B} = B_1(z)\hat{e}_1 + B_2(z)\hat{e}_2 + B_3(z)\hat{e}_3 , \tag{4}$$

$$\boldsymbol{V} = V_1(z)\hat{e}_1 + V_2(z)\hat{e}_2 + V_3(z)\hat{e}_3 , \tag{5}$$

$$N = N(z) . \tag{6}$$

The electrostatic field has been neglected in eq. (3), assuming that this field have small effect on the electrons dynamics compared to that of the intense laser electromagnetic fields at frequency ω_0'. As far as the radial effects are considered, this assumption is justified according to the Poisson equation if the electron beam current J satisfies [2]

$$J(kA) \ll 8\omega_0' R_b \alpha/c , \tag{7}$$

where R_b is the beam radius and $\alpha = e|E_\perp|/m\omega_0' c$. Furthermore, the space charge effects can be cancelled in the simultaneous acceleration of electrons and positrons by using a linearly polarized laser radiation [4].

Since the frequency ω_0' was already included in the basis vectors $\{\hat{e}_i\}_{i=1,2,3}$ we shall assume that any variable in eqs. (3)–(6) satisfies

$$|\psi^{-1} d\psi/dz| \ll \omega_0'/c , \tag{8}$$

where $\psi \equiv \{E_i, B_i, V_i, N\}_{i=1,2,3}$.

The Maxwell equations yield

$$c\hat{e}_3 \times \partial \boldsymbol{B}_\perp/\partial z = \partial \boldsymbol{E}_\perp/\partial t - 4\pi e N \boldsymbol{V}_\perp , \tag{9}$$

$$-c\hat{e}_3 \times \partial \boldsymbol{E}_\perp/\partial z = \partial \boldsymbol{B}_\perp/\partial t , \tag{10}$$

$$\partial B_3/\partial z = 0 , \tag{11}$$

where $\boldsymbol{\psi}_\perp = \psi_1\hat{e}_1 + \psi_2\hat{e}_2$. According to eq. (11),

$$B_3 = B_0 = \text{const} \tag{12}$$

and the wave equation is derived from eqs. (9) and (10),

$$c^2 \frac{\partial^2 \boldsymbol{E}_\perp}{\partial z^2} = \frac{\partial^2 \boldsymbol{E}_\perp}{\partial t^2} - 4\pi e N \frac{\partial \boldsymbol{V}_\perp}{\partial t} . \tag{13}$$

By using the identities

$$\frac{\partial \hat{e}_1}{\partial t} = -\omega_0' \hat{e}_2 , \quad \frac{\partial \hat{e}_2}{\partial t} = \omega_0' \hat{e}_1 ,$$

$$\frac{\partial \hat{e}_1}{\partial z} = \frac{\omega_0'}{c}\hat{e}_2 , \quad \frac{\partial \hat{e}_2}{\partial z} = -\frac{\omega_0'}{c}\hat{e}_1 , \tag{14}$$

we obtain in components

$$\frac{d^2 E_1}{dz^2} = \frac{2\omega_0'}{c}\frac{dE_2}{dz} - \frac{\omega_0'}{c^2} 4\pi e N V_2 , \tag{15}$$

$$\frac{d^2 E_2}{dz^2} = -\frac{2\omega_0'}{c}\frac{dE_1}{dz} + \frac{\omega_0'}{c^2} 4\pi e N V_1 . \tag{16}$$

For $N \to 0$ eqs. (15) and (16) yield the trivial solution $E_i = \text{const}$ $(i=1,2)$, which was assumed in previous discussions of the single particle dynamics in the ALA scheme [2–4]. In accordance with condition (8) we neglect the left-hand side of eqs. (15) and (16) so that

$$\frac{dE_j}{dz} = \frac{2\pi e N}{c} V_j , \quad j = 1,2 . \tag{17}$$

Assuming condition (8), eq. (10) yields

$$B_1 = \frac{c}{\omega_0'}\left(\frac{dE_1}{dz} - \frac{\omega_0'}{c}E_2\right) = \frac{2\pi e N}{\omega_0'} V_1 - E_2 , \tag{18}$$

$$B_2 = \frac{c}{\omega_0'}\left(\frac{dE_2}{dz} + \frac{\omega_0'}{c}E_1\right) = \frac{2\pi e N}{\omega_0'} V_2 + E_1 . \tag{19}$$

The continuity equation for the electron beam,

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial z}(N V_3) = 0 , \tag{20}$$

gives

$$N V_3 = N_0 V_{30} = \text{const} . \tag{21}$$

The electrons momentum equation is given by

$$\left(\frac{\partial}{\partial t} + V_3\frac{\partial}{\partial z}\right)(\gamma \boldsymbol{V}) = -\frac{e}{m}\left(\frac{\boldsymbol{v}}{c} \times \boldsymbol{B} + \boldsymbol{E}\right) , \tag{22}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$ and the radiation losses are neglected [2]. The last equation and eqs. (17)–(21) written in components can be combined to give the following set of seven first order ordinary differential equations:

$$\dot{u}_1 = \frac{\omega_0}{\gamma}\left(\alpha_1(u_1^2+u_3-1)+\alpha_2 u_1 u_2 - \frac{\Omega_0}{\omega_0}u_2 - \frac{\Omega_0}{\omega_0}u_2 - (1-u_3)\gamma u_2 + \frac{N}{2N_c}u_2 u_3\right), \quad (23)$$

$$\dot{u}_2 = \frac{\omega_0}{\gamma}\left(\alpha_1 u_1 u_2 + \alpha_2(u_2^2+u_3-1) + \frac{\Omega_0}{\omega_0}u_1 + (1-u_3)\gamma u_1 - \frac{N}{2N_c}u_2 u_3\right), \quad (24)$$

$$u_3 = \frac{\omega_0}{\gamma}(\alpha_1 u_1 + \alpha_2 u_2)(u_3-1), \quad (25)$$

$$\dot{N} = -N\dot{u}_3/u_3, \quad (26)$$

$$\dot{\gamma} = -\omega_0(\alpha_1 u_1 + \alpha_2 u_2), \quad (27)$$

$$\dot{\alpha}_1 = (N/2N_c)\omega_0 u_1 u_3, \quad (28)$$

$$\dot{\alpha}_2 = (N/2N_c)\omega_0 u_2 u_3, \quad (29)$$

where $\dot{\psi} \equiv c^{-1}\partial\psi/\partial t$, $\omega_0 = \omega_0'/c$, $\boldsymbol{u} = \boldsymbol{v}/c$, $\Omega_0 = eB_0/mc^2$, $\gamma = (1-u_1^2-u_2^2-u_3^2)^{-1/2}$, $\boldsymbol{\alpha} = e\boldsymbol{E}/mc\omega_0'$ and $N_c = (m\omega_0'^2/4\pi e^2)$ is the critical plasma density for the frequency ω_0'. Eqs. (23)–(29) describe the evolution of the electromagnetic fields in the system coupled to the nonlinear dynamics of the electron beam. These equations were solved numerically for different examples. It should be noted that the peak initial intensity of the laser radiation is related to α_{10} via the expression, $I_0 = (2.8\times10^{18}\ \mathrm{W/cm^2})(\alpha_{10}/\lambda_0[\mu\mathrm{m}])^2$, where $\lambda_0 = 2\pi/\omega_0$.

Figs. 1 and 2 describe two examples of an efficient conversion of electromagnetic energy into the energy of the electron beam. These figures show the axial dependence of the electrons energy and the electric field amplitude α_1 assuming that at $z=0$ the cyclotron resonance condition: $\Omega_0 = \omega_0\gamma_0(u_{30}-1)$ is satisfied and $\alpha_{20} = u_{10} = u_{20} = 0$. Fig. 1 shows the acceleration of a cold electron beam with an initial energy of 1 MeV and current density of 250 kA/cm² by a circularly polarized laser radiation with $\lambda_0 = 0.5$ mm and $I_0 = 1.1$ GW/cm², along a homogeneous axial magnetic field of 55 kG. In this case almost all of the electromagnetic flux is converted into kinetic energy flux of the electrons in 75 cm. Fig. 2 shows the acceleration of a 50 kA/cm² electron beam with an ini-

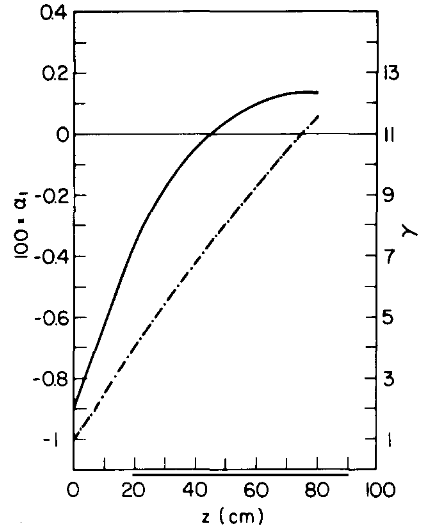

Fig. 1. Acceleration of an electron beam with $\gamma_0=2$, $N_0=5\times10^{10}$ cm^{-3} by a laser with $\omega_0=125$ cm^{-1}, $\alpha_{10}=-0.01$ along an axial magnetic field of $\Omega_0=-33$ cm^{-1}. The cyclotron resonance condition is satisfied at $z=0$ where $\alpha_{20}=u_{10}=u_{20}=0$. The spatial evolution of γ (solid line) and α_1 (dot-dashed) is shown.

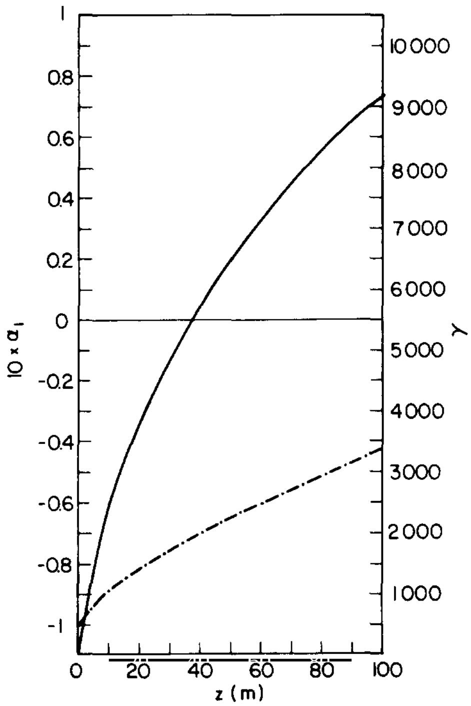

Fig. 2. The same as in fig. 1 for $\gamma_0=50$, $N_0=10^{13}$ cm^{-3}, $\omega_0=6300$ cm^{-1}, $\Omega_0=-63$ cm^{-1} and $\alpha_{10}=-0.1$.

tial energy of 25 MeV by a CO_2 laser ($\lambda_0 = 10$ μm) with a peak initial intensity of 2.8×10^{14} W/cm^2 ($\alpha_{10} = -0.1$) along a 100 kG axial magnetic field. The electron beam is accelerated to an energy of 4.6 GeV and gains 82% of the electromagnetic power at a distance of 100 m where $\alpha_1 = -0.042$ and $\alpha_2 = -0.007$. It should be noted that the guidance of a high intensity laser beam over large distances is a crucial problem common to all the schemes in the far field approach for particles acceleration to high energies [1]. Alternative methods to the intensity-limited waveguides [1] are, for example, arrays of lenses, plasma fibers [3] or electron beam guidance [6]. In the context of the ALA scheme, this issue requires detailed discussion and will be considered elsewhere.

In conclusion, it was shown that dense electron beams can be accelerated efficiently to high energies, applying the autoresonance laser acceleration concept. Collective autoresonance acceleration of electrons or positions might also occur in astrophysical regions, such as pulsars, where strong magnetostatic fields and electromagnetic waves coexist.

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