Velocity distribution and energy diagnostics in intense guided relativistic electron beams

P. Avivi, Ch. Cohen, and L. Friedland

Center for Plasma Physics, Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem, Israel

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A novel experimental method to measure both the velocity distribution and the relativistic factor $\gamma$ of intense guided electron beams is described. The method is based on the use of a magnetic hill to discriminate the electrons with different transverse velocity components. Analysis of the current reflected by the hill as a function of the height of the hill yields the above mentioned beam characteristics.

INTRODUCTION

Intense, guided, relativistic electron beams play a major role in the production of high-power millimeter and submillimeter waves in free electron lasers. Ideally one strives to obtain a monoenergetic cold beam in which all electrons initially have defined momenta relative to the guide field. Such a stringent requirement is almost impossible to achieve. It is, therefore, mandatory to have direct knowledge of the quality of the beam, namely its velocity distribution, as well as the relativistic factor $\gamma = (1 - v^2/c^2)^{-1/2}$ of the beam.

The presence of the guide magnetic field in typical, intense beam, free electron lasers make it impossible to apply conventional magnetic spectroscopy methods\textsuperscript{1,2} for beam diagnostics. Extraction of a small portion of the beam and subsequent diagnostics\textsuperscript{3} is also inapplicable in the presence of strong guide fields. Moreover, the conventional methods using solid state detectors have to be ruled out because of the high current density of the beams involved.

In previous communications\textsuperscript{4,5} a method is described by which the average parallel velocity can be determined, on line, without disturbing the beam. In this paper we describe an experimental method by which both the velocity distribution and $\gamma$ of the beam are measured simultaneously, thus providing the full knowledge of the beam.

I. EXPERIMENTAL SYSTEM

The experimental system (Fig. 1) comprises a Fabertron (model 706/2670) electron gun, which provides pulses of 80–100 ns width and a voltage up to 500 kV. The beam was shot through a 4-mm hole in the anode into a drift tube 80 cm long and 3.6 cm in diameter. The magnetic guide field $B_z$ could be raised up to 4 kG.

The beam was partially reflected by an adiabatic magnetic hill, which could be raised up to 15 kG. The current and density of the combined initial and reflected beams was measured by a Rogowski coil and a coaxial capacitor which measured the radial electrostatic potential induced due to the space charge of the beam.\textsuperscript{4}

Both the capacitor and the Rogowski coil were outside the fringing field of the magnetic hill. In order to provide perpendicular momenta to the electrons, a semiconductor disk “kicked” the beam on its way to the drift tube. By measuring the reflected current as a function of the height of the magnetic hill, for a given beam voltage, both the distribution function and $\gamma$ are readily obtained.

Assume that the beam current is low enough and that the time of flight of the electrons from the diode to the measuring assembly is sufficiently short compared to the typical scale of variation of the diode parameters, so that one may treat the beam as being monoenergetic at any time $t$ with an energy which is defined by the diode voltage at that time. Let $n(u_\parallel) du_\parallel$ be the density of electrons with reduced parallel velocities $u_\parallel = v_\parallel/c$ in the interval $(u_\parallel, u_\parallel + du_\parallel)$. The total density of the beam is then

$$\int_0^{(v^2/c^2)^{-1/2}} n(u_\parallel) du_\parallel = n.$$ \hfill (1)

Now, let $u_\parallel^* = \text{the maximum axial velocity of electrons which are still reflected by the hill } B_H$. Then the reflected current density is

$$j_r(t) = -ec \int_0^{u_\parallel^*} n(u_\parallel) u_\parallel du_\parallel,$$

so that the total current measured by the Rogowski coil is

$$j = j_0(t) + ec \int_0^{u_\parallel^*} n(u_\parallel) u_\parallel du_\parallel,$$ \hfill (2)

where $j_0$ is the current for $B_H = 0$ (no reflection). Differentiation of (2) with respect to $B_H$ yields

$$\frac{dj}{dB_H} = \frac{dj_r}{dB_H} = ecu_\parallel^* n(u_\parallel^*) \frac{du_\parallel^*}{dB_H}.$$ \hfill (3)

Assuming adiabaticity of the hill, the reflection condition is

$$u_\parallel^*/u_\parallel^* = \sqrt{R - 1},$$ \hfill (4)

FIG. 1. Schematic of the experimental system. C—cathode; A—anode; D. T.—drift tube; M. K.—magnetic kicker; R. C.—Rogowski coil; C. A.—cylindrical capacitor; and S.—solenoid of the magnetic hill.
where

$$R = \frac{(B_0 + B_H)}{B_0}$$  \hspace{1cm} (5)

and $B_0$ is the strength of the guide field. Furthermore,

$$1 - u_m^* - u_0^* = 1/\gamma^2,$$  \hspace{1cm} (6)

where $\gamma$ is the relativistic factor of the beam. Substitution of (4) into (6) gives

$$u_m^* = u_m \left[ \frac{B_H}{(B_0 + B_H)} \right]^{1/2},$$  \hspace{1cm} (7)

where $u_m = (1 - 1/\gamma^2)^{1/2}$ is the maximum axial velocity of electrons in the beam for a given $\gamma$.

Now we differentiate (7) with respect to $B_H$ and substitute into (3). Then

$$n(u_m^*) = \frac{2(B_0 + B_H)^2}{ecu_m^2B_0} \frac{dj_r}{dB_H}.$$  \hspace{1cm} (8)

The derivative $dj_r/dB_H$ can be found experimentally from the current measurements (see Fig. 1).

Thus the only unknown factor in (8) is $u_m$, which can be found by using the capacitor. Indeed, the electron density of the reflected beam is

$$n_r = \int_{u_0}^{u_m} n(u) du$$

$$= \frac{1}{ecu_m} \int_{0}^{\frac{8n_{H_m}}{B_H}} \frac{dj_r}{dB_H} dB_H,$$  \hspace{1cm} (9)

where $B_{H_m}$ is the highest hill used. On the other hand

$$n_r = n_{H_m} + n_0$$  \hspace{1cm} (10)

and both the density measured with the highest hill $n_{H_m}$ and the density with zero hill $n_0$ are measurable quantities. Thus (9) and (10) yield $u_m$, namely $\gamma$ and consequently $n(u_0)$, the distribution function of the beam [see Eq. (8)].

II. MEASUREMENTS

Figure 2 gives the current from Rogowski coil measurement as a function of time for different values of the magnetic field of the hill. Each time represents a certain voltage of the beam. Similarly, Fig. 3 shows $j_0 - j_r$ as a function of $B_H$, for different times. The error bars in the figures represent the spread in the results from ten successive pulses of the diode. The curves in Fig. 3 have been used in a simple computer program which provides both $n(u_0)$ and $\gamma$. The resulting electron velocity distribution function $f(u_0) = n(u_0)/n_0$ is presented in Fig. 4 for two different times during the pulse ($t = 30$ and $60$ ns). The areas in Fig. 4 shown by the dashed lines include all electrons which were not reflected by the available magnetic hill. The size of this area is determined in order to comply with the normalization condition

$$\int_0^{\infty} f(u) du = 1.$$  \hspace{1cm} (9)

Note, that although the exact form of the part of the velocity distribution is still unknown, the absolute values of the rest of the distribution as well as of the maximum possible $u_m$ (or $u_{\gamma}$) are fully determined by the suggested diagnostics.

III. DISCUSSION

Thus, in conclusion, the described method can be useful in diagnosing beams with large values of $\alpha = u_m/u_0$. The
assumption of the constancy of $\gamma$ still allows diagnostics of beam with energies above 0.5 MeV and currents up to 1 kA since the energy spread due to space charge is less than 0.1% for such beams. Also, 1 kA is probably the upper limit on the beam current for the proposed diagnostics since at higher currents the self magnetic field of the beam and finite gyro radius can significantly influence the electron dynamics in the magnetic mirror region. Nevertheless, even at high currents there probably exists a one-to-one correspondence between the reflected current and the height of the magnetic hill. If so, then similar to the described case, an appropriate analysis would allow to recover the distribution function of the beam. This complicated task however was beyond the scope of the present work.

The proposed method, probably, gives the only direct measurement of $\gamma$ in intense relativistic, guided, electron beams. One can apply the method in order to find the $\gamma$ factor even for the multimegavolt electron beams, with initially small characteristic value of $\alpha$. By providing a sufficient disturbance of the beam by transverse magnetostatic fields (as was also done in the present work) the distribution with a sufficient value of $\langle \alpha \rangle_{\text{av}}$ can be created, providing a possibility of determining $\gamma$ by using the above mentioned method.