Correspondence principle in free-electron lasers

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(Received 27 May 1983)

Multiphoton interaction in free-electron lasers is studied using the correspondence principle. The developed theory is applicable to both quantum-mechanical and classical regimes of operation of the laser. The suggested formalism simplifies the traditional calculation of the small-signal, single-particle gain in the system. As an example, the theory is applied to the case of the free-electron laser operating in combined transverse helical and axial guide magnetic fields.

I. INTRODUCTION

Early theoretical studies of stimulated emission from relativistic electron beams, propagating through a periodic transverse magnetic field (free-electron laser) were based on quantum-mechanical formalisms.[1-4] These theories triggered an experimental effort, which led in 1975 to the first successful operation of a free-electron laser amplifier at Stanford University.[5] The experiment verified theoretical predictions and further stimulated extensive experimental and theoretical studies of free-electron lasers. At that time, several researchers[6,7] demonstrated that the results of the Stanford experiment can be also explained by using purely classical theory, which provided similar expressions for the gain, as much more tidy quantum-mechanical calculations. Since then, the classical approach was commonly adopted and widely exploited in studying various aspects of free-electron lasers.[8]

Thus, at present, with few exceptions,[9] the application of the quantum-mechanical formalism to the theory of free-electron lasers is widely thought of as being unnecessary, and having, because of its complexity, only academic interest. It is usually forgotten, however, that in some situations the classical theory, in principle, cannot be applied. This is the case, for example, in a free-electron laser oscillator start-up from noise, when the initial radiation level in the system is so low that the classical energy exchange with the radiation in one electron pass through the laser is less than the photon energy $\hbar\omega$. Quantitatively, if G is the classical small signal gain in a free-electron laser, operating in the single-particle regime,[10] then the average energy loss by an electron in a single path is $\delta\epsilon = GS/J$, where $S = cE_0{}^2/8\pi$ is the radiation energy flux, and $J = N_e u$ is the electron flux, N_e and u ($\simeq c$) being the electron beam density and axial velocity, respectively. Thus, the classical formalism can only be applied when

$$\xi = \frac{\delta\epsilon}{\hbar\omega} \simeq \frac{GE_0^2}{8\pi N_e} \gg 1 \ . \tag{1}$$

For example, with a gain of 10% at the wavelength, $\lambda = 10^{-3}$ cm, and the current flux of 1 A/cm^2, (1) is valid only as long as the radiation flux exceeds 1 W/cm^2. Such a radiation level is obviously much higher than that expected initially from the beam noise in this example. Therefore, in the initial stage of the radiation buildup in such an oscillator, $\xi \ll 1$, and only the quantum-mechanical description is appropriate. Later, when the intensity of radiation increases, ξ becomes large and the problem can be approached classically.

The purpose of this work is to exploit the close interrelation between the seemingly different quantum and classical descriptions of the interaction in free-electron lasers. The approach will be similar to that used recently[11] in studying the electron-ion inverse bremsstrahlung. It will be based on the fact that soft photons ($\hbar\omega/\epsilon \ll 1$, ϵ being the electron energy) are emitted or absorbed by an electron statistically independently.[12] Consequently, the process of multiple emission or absorption of soft photons can be described by the Poisson distribution. For example, the probability $P(n)$ of stimulated emission of n photons during the interaction is given by

$$P(n) = \frac{\bar{n}^n}{n!}\exp(-\bar{n}) \ , \tag{2}$$

where $\bar{n}$ is the average number of emitted quanta. A similar expression describes also the probability of absorption of a number of photons. It is the value of the parameter $\bar{n}$ which makes the difference between the quantum ($\bar{n} \lesssim 1$) and the classical ($\bar{n} \gg 1$) limits. In Sec. II, we will derive the classical analog to the multiphoton interaction in a free-electron laser and then turn to the quantum description by using the correspondence principle. We will derive an expression for $\bar{n}$ and use this parameter in developing a formalism for the calculation of the gain in the system. The formalism will exploit the principle of detailed balancing in connecting the emission and absorption probabilities and will be applicable uniformly in the quantum, intermediate, and classical regimes of operation. In contrast to the usual classical method of calculating the small-signal single-particle gain from the second-order change (in terms of the strength of the radiation field) in the electron energy, the detailed balancing arguments, adopted here, require the knowledge of only the first-order change in the energy, thus simplifying the calculations considerably. Finally, at the end of Sec. III, we will apply the developed theory to the case of a free-electron laser operating in combined helical transverse and axial guide magnetic fields and consider the problem of thermal spread in the beam.

II. THE MODEL

Consider a relativistic electron beam propagating along the z axis of a periodic transverse magnetic field structure of length L in the presence of N uncorrelated, plane electromagnetic waves with random relative phases and polarizations. We neglect to the lowest order the influence of the beam on the waves and assume that the electric fields of the waves are given by

$$\vec{E}_i=\vec{p}_i E_0\cos(kz-\omega t+\phi_i),\quad i=1,2,\ldots,N \tag{3}$$

where $k=\omega/c$ and $\vec{p}_i$ and ϕ_i are the random polarization vectors and phases, respectively. We now consider a single electron of the beam and assume that the transverse periodic magnetostatic field is that which exists on the axis of the conventional magnetic wiggler and thus is described by

$$\vec{B}_w=B_\perp[\hat{e}_x\cos(k_0z)+\hat{e}_y\sin(k_0z)]\ . \tag{4}$$

We will also assume that the electromagnetic fields are weak, so that they only slightly perturb the motion along the wiggler.

Now, we use the energy balance equation for the electron,

$$\frac{d\epsilon}{dt}=-e\vec{E}\cdot\vec{V},\quad \vec{E}=\sum_{i=1}^{N}\vec{E}_i\ , \tag{5}$$

where $\epsilon=mc^2\gamma(t)$, t is the time measured along the electron trajectory in the assigned magnetostatic and electromagnetic fields, and $\vec{V}(t)$ is the velocity vector of the electron. Let, in the presence of the perturbing electric field, $\vec{V}=\vec{V}_0+\vec{v}$, where $\vec{V}_0$ is the unperturbed velocity. Then, to the first order in terms of the strength of the electromagnetic field E_0, the change in the electron energy after passing the wiggler is given by

$$s=-e\int_{t_1}^{t_2}\vec{E}'\cdot\vec{V}_0\,dt'\ , \tag{6}$$

where $\vec{E}'$ is the electric field vector at a time t along the unperturbed electron trajectory and t_1,t_2 are the time moments the electron enters and leaves the interaction region. As is well known, inside the wiggler

$$\vec{V}_0=-w[\hat{e}_x\cos(k_0z)+\hat{e}_y\sin(k_0z)]+\hat{e}_z u\ , \tag{7}$$

where u is a constant,

$$w=\Omega_\perp/k_0\gamma_0\ , \tag{8}$$

$$\gamma_0=[1-(u/c)^2-(w/c)^2]^{-1/2}\ , \tag{9}$$

and $\Omega_\perp=eB_\perp/mc$. Thus in Eq. (6)

$$\vec{E}'(t')=E_0\sum_{i=1}^{N}\vec{p}_i\cos[(ku-\omega)t'+\phi_i']\ , \tag{10}$$

where t' $(-\tau/2\le t'\le+\tau/2)$ is the time measured with respect to the moment t_0 when the unperturbed electron passes the center $(z=z_0)$ of the wiggler, and

$$\phi_i'=\phi_i+k(z_0-L/2)+ku\tau/2-\omega t_0$$

with L being the length of the wiggler and $\tau=L/u$. We write $z=z_0-L/2+u(\tau/2+t')$ in (7) and substitute the resulting expression for $\vec{V}_0$ and Eq. (10) into (6). This yields for the energy change

$$s=eE_0w/2\int_{-\tau/2}^{+\tau/2}dt'\sum_{i=1}^{N}(\cos\{[(k+k_0)u-\omega]t'+\alpha_i'+\phi_i'\}+\cos\{[(k-k_0)u-\omega]t'+\alpha_i'-\phi_i'\})\ , \tag{11}$$

where $\alpha_i'=k_0(z_0-L/2+u\tau/2)-\alpha_i$ and α_i is the random angle between the polarization vector p_i and the x axis. As usual, we will focus our attention on the case when

$$\beta=(k+k_0)u-\omega=k_0u-\omega(1-u/c)\simeq 0\ , \tag{12}$$

which describes the double-Doppler-upshifted frequency $\omega\simeq k_0u/(1-u/c)\simeq 2k_0u\gamma_0^2$, characteristic to free-electron lasers. Accordingly, we will neglect the rapidly varying second terms in Eq. (11). Then

$$s\simeq eE_0w/2\sum_{i=1}^{N}\int_{-\tau/2}^{+\tau/2}\cos(\beta t'+\psi_i)dt'$$

$$=eE_0w\frac{\sin(\beta\tau/2)}{\beta}\sum_{i=1}^{N}\cos\psi_i\ , \tag{13}$$

where $\psi_i=\alpha_i'+\phi_i'$. On defining $\theta=\beta L/2u$, we finally rewrite Eq. (13) as

$$s=\sqrt{2\pi}Ax\ , \tag{14}$$

where

$$A=\frac{eE_0wL}{2\sqrt{2\pi}u}\frac{\sin\theta}{\theta} \tag{15}$$

and

$$x=\sum_{i=1}^{N}\cos\psi_i\ . \tag{16}$$

The phases in (16) are random and distributed uniformly in the interval $(0,2\pi)$. Therefore, s in Eq. (14) is also a random quantity. Its mean value is zero and the standard deviation D is given by

$$D^2=\langle s^2\rangle_{av}=N\pi A^2\ . \tag{17}$$

Moreover, according to the central-limit theorem, the distribution function $f(s)$ of the values of s, for large N, asymptotically becomes normal,

$$f(s)=\frac{1}{D\sqrt{2\pi}}\exp\left[-\frac{s^2}{2D^2}\right]\ . \tag{18}$$

At this point we give a quantum-mechanical interpretation of Eq. (18). From the quantum-mechanical point of view, the electron in the interaction region exchanges energy with the radiation field by means of a certain number of the quanta $\hbar\omega$. As was already mentioned in the Introduction, the process of emission or absorption of a soft photon during the scattering process is statistically independent of the number of photons exchanged with the radiation at prior times. Thus, we describe the probability $P(n)$ of the emission of n photons by the Poisson distribution (2). We also assume that the average number $\bar{m}$ of elementary absorptions, during the interaction, is to the lowest significant order equal to that of the elementary emissions ($\bar{m} \simeq \bar{n}$). This yields equal probabilities for the absorption or emission of equal number of photons. On using the probabilities $P(n)$, we can find the probability $R(l)$ of the exchange with the radiation of a total number of l photons ($l=0,\pm1,\pm2,\pm3,\ldots$) during the interaction:

$$R(|l|)=\sum_{n>0} P(n)P(|l|+n)$$
$$=\bar{n}^{|l|}\exp(-2\bar{n})\sum_{n}\frac{\bar{n}^{2n}}{n!(|l|+n)!}\,,$$
$$R(-|l|)=R|l|)\,. \quad (19)$$

This distribution also becomes normal, for large $\bar{n}$, with the standard deviation given by

$$(\langle l^2\rangle_{\rm av})^{1/2}=(2\bar{n})^{1/2}\,. \quad (20)$$

Thus, the distribution of the total change in the electron energy after the interaction with the laser will be normal and characterized by standard deviation

$$D=\hbar\omega(2\bar{n})^{1/2}\,. \quad (21)$$

Since the case of large values of $\bar{n}$ corresponds to the classical limit, we compare (21) and (17) and get

$$\bar{n}=\frac{Q\pi}{(\hbar\omega)^2}\left[\frac{ewL}{u}\right]^2 F(\theta)\,, \quad (22)$$

where

$$F(\theta)=(\sin\theta/\theta)^2 \quad (23)$$

and

$$Q=NE_0^2/8\pi \quad (24)$$

is the average radiation density. Note that $\bar{n}$ in (22) is directly proportional to the radiation quanta density $Q/\hbar\omega$ in the interaction region. When the density of radiation decreases, so does $\bar{n}$, until formally, $\bar{n}$ becomes less than one. According to (2), we interpret $\bar{n}$ for the case $\bar{n} \ll 1$ as a probability of stimulated emission (or absorption) of a single photon $\hbar\omega$ in the interaction region. The probability of the multiple exchange of quanta with the radiation becomes negligible in this case. An expression, similar to Eq. (22), for the probability $\bar{n}(\bar{n} \ll 1)$ for the stimulated single-quantum transition can be also found by using the quantum-mechanical perturbation theory.[3] Thus, we conclude that the interpretation of $\bar{n}$ in (22) as the average number of elementary emissions by a single electron during the passage through the laser is valid in both the quantum-mechanical and the classical limits. We will show in the next section that this "universal" applicability of (22) leads to similar quantum-mechanical and classical expressions for the small signal gain in the laser, as was indeed found in a number of previous calculations.[1–7]

III. THE GAIN

Consider now the ensemble of the electrons in the beam and assume that the beam is initially cold and stationary. Define $a_n(\epsilon)$ and $b_n(\epsilon)$ as the rates of the absorption and the stimulated emission of n radiation quanta per electron in the interaction region. Then the energy gain by the radiation in such a cold beam case can be computed from

$$\Gamma_c=(cQ)^{-1}N_e\sum_{n>0} n\hbar\omega[b_n(\epsilon)-a_n(\epsilon)]\,. \quad (25)$$

To the lowest significant order,

$$b_n \simeq a_n \simeq uR(n)\,, \quad (26)$$

where $R(n)$ is the probability already found in Sec. II [Eq. (19)]. At this point, however, we have to take into account the small, but important difference between the absorption and emission probabilities. The classical counterpart for this is the extension of the calculation of the energy change of the electrons to the second order in terms of the strength of the radiation field. Here, we apply a different approach and use the connection between b_n and a_n given by the principle of detailed balancing,[13] according to which

$$b_n(\epsilon+n\hbar\omega)=a_n(\epsilon)\,. \quad (27)$$

Assuming that in the case of interest $n\hbar\omega \ll \epsilon$, Eq. (27) can be written approximately as

$$b_n(\epsilon)\simeq a_n(\epsilon)-n\hbar\omega\, da_n(\epsilon)/d\epsilon\,. \quad (28)$$

Therefore

$$\Gamma_c \simeq -\frac{(\hbar\omega)^2N_e}{cQ}\frac{d}{d\epsilon}\left[u\sum_{n>0} n^2R(n)\right]. \quad (29)$$

In the last equation

$$\sum_{n>0} n^2R(n)=\tfrac{1}{2}\sum_{m>0}\sum_{l>0}(m-l)^2P(m)P(l)=\bar{n}\,. \quad (30)$$

Thus, finally, (29) reduces to

$$\Gamma_c=-\frac{(\hbar\omega)^2N_e}{Qc}\frac{d}{d\epsilon}(u\bar{n})\,. \quad (31)$$

Note, that this expression describes both the quantum and the classical limits characterized by either small or large values of $\bar{n}$.

Let us discuss now the obtained result for the gain. On substituting Eq. (22) into Eq. (31), we get

$$\Gamma_c=-\frac{\omega_p^2 m}{8c}L^2\frac{d}{d\epsilon}\left[\frac{w^2}{u}F(\theta)\right], \quad (32)$$

where $\omega_p^2=4\pi N_e e^2/m$ is the plasma frequency, characterizing the beam. The main contribution to the gain in (32) comes from the fast variation of

$$\theta=[(k+k_0)u-\omega]L/2u$$

with energy. Indeed,

$$\frac{d\theta}{d\epsilon}=\frac{\omega L}{2u^2}\frac{du}{d\epsilon}\simeq\frac{k_0 L\gamma_0^2}{u}\frac{du}{d\epsilon} . \tag{33}$$

On the other hand, we have

$$1-(u/c)^2-(w/c)^2=1-(u/c)^2-\left[\frac{\Omega_\perp}{k_0 c}\right]^2\frac{m^2c^4}{\epsilon^2} = \frac{m^2c^4}{\epsilon^2} , \tag{34}$$

which yields on differentiation

$$\frac{du}{d\epsilon}=\frac{1+(\Omega_\perp/k_0 c)^2}{mu\gamma_0^3} . \tag{35}$$

Thus (assuming $\Omega_\perp/k_0 c \ll 1$), we get the well-known single particle, cold beam gain formula in free-electron lasers:

$$\Gamma_c \simeq \frac{\omega_p m u}{8c}\left[\frac{wL}{u}\right]^2\frac{dF(\theta)}{d\theta}\frac{d\theta}{d\epsilon}\simeq\left[\frac{\omega_p\Omega_\perp}{c^2k_0^2}\right]^2\left[\frac{k_0L}{2\gamma_0}\right]^3\frac{dF}{d\theta} , \tag{36}$$

which, obviously now, is applicable to the quantum as well as to the classical regimes of operation.

The formalism, presented here can be used in more complex free-electron laser configurations. Consider, for example, a laser which combines an axial guide magnetic field with the conventional field of the magnetic wiggler. It is known[14] that this field configuration also allows an unperturbed equilibrium of the beam with helical orbits of individual electrons. The unperturbed velocity in this equilibrium is again described by Eq. (8). The transverse component of the velocity, however, becomes more complex[14]:

$$w=\frac{\Omega_\perp u/\gamma_0}{k_0 u-\Omega_\parallel/\gamma_0} , \tag{37}$$

where

$$\Omega_\parallel=\frac{eB_\parallel}{mc}$$

is the cyclotron frequency associated with the guide field. Instead of (34) we now have

$$1-\frac{u^2}{c^2}\left[1+\frac{\Omega_\perp^2}{(\gamma_0 k_0 u-\Omega_\parallel)^2}\right]=\frac{m^2c^4}{\epsilon^2} , \tag{38}$$

which yields, on differentiation,

$$\frac{du}{d\epsilon}=\frac{\eta}{mu\gamma_0^3} , \tag{39}$$

where

$$\eta=1+\mu^{-2}\left[\frac{\Omega_\perp^2u^2}{c^2}+bdu^2\left[\gamma_0^2+\frac{c^2}{u^2}\right]\right] , \tag{40}$$

$$\mu^2=a^2-bd , \tag{41}$$

$$a=k_0u-\Omega_\parallel/\gamma_0=\frac{\Omega_\perp}{\gamma_0}\frac{u}{w} , \tag{42}$$

$$b=k_0w-\Omega_\perp/\gamma_0=\frac{\Omega_\parallel}{\gamma_0}\frac{w}{u} , \tag{43}$$

$$d=\Omega_\perp/\gamma_0 . \tag{44}$$

The quantity μ in Eq. (41) has an important physical meaning, namely it represents the natural response frequency of the electrons to a perturbation of the aforementioned helical equilibrium.[14] The use of (39) leads to the following expression for the gain in the presence of the guide field:

$$\Gamma_c=\eta\left[\frac{\omega_p\Omega_\perp}{c^2k_0^2}\right]^2\left[\frac{k_0L}{2\gamma_0}\right]^3\frac{dF}{d\theta} . \tag{45}$$

Thus, the factor η describes the enhancement of the gain, compared to that in the laser without the guide field, but with the identical helical orbits of individual electrons. Note, that when $B_\parallel=0$, we have $\mu=k_0u$, so that

$$\eta=1+\left[\frac{\Omega}{k_0c}\right]^2\simeq 1$$

and, as is expected, we restore expression (36). For the case of strong axial fields, the presence of γ_0^2 in the square brackets in (40) allows us to write approximately

$$\eta\simeq 1+\gamma_0^2bd/\mu^2 . \tag{46}$$

Almost precisely this form for the enhancement factor was found in Ref. 15. Due to the possibility of a substantial reduction of μ^2 in the presence of a guide field,[14] the gain can be significantly higher than in a laser without a guide field, even if all the remaining parameters (w,γ_0,k_0) in both lasers are the same. Detailed classical studies of the influence of the guide field on the performance of the laser were reported recently for both single-particle[15] and collective[16,17] regimes of operation. One should mention here that the single-particle gain formula (45) was derived in the present work by using, in contrast to Ref. 15, a simpler formalism, based on the parameter $\bar{n}$, the calculation of which required the knowledge of only the first-order change in the electron energy during the interaction.

In the following we address the problem of a thermal spread of the beam. Let $\rho(\epsilon)\neq\delta(\epsilon-\epsilon_0)$ be the initial energy-distribution function of the electrons. In such a hot beam case, Eq. (25) must be replaced by

$$\Gamma_h=(cQ)^{-1}\sum_{n>0}\left[\int_{n\hbar\omega}^{\infty}n\hbar\omega b_n(\epsilon)\rho(\epsilon)d\epsilon - \int_0^{\infty}n\hbar\omega a_n(\epsilon)\rho(\epsilon)d\epsilon\right] , \tag{47}$$

where, as before, b_n and a_n are the corresponding transition rates per electron. Equation (27) can now be used to rewrite (47) in the form

$$\Gamma_h=(cQ)^{-1}\sum_{n>0}\int_0^{\infty}d\epsilon\, n\hbar\omega a_n(\epsilon)[\rho(\epsilon)-\rho(\epsilon+n\hbar\omega)] . \tag{48}$$

This general formula can be simplified in the case when the average electron energy change $\hbar\omega(\bar{n})^{1/2}$ [see Eq. (30)] is much less than the characteristic thermal spread in the beam. In such a case, we can expand the integrand in (48) in powers of $n\hbar\omega$ and leave only the first two terms in the expansion. This yields

$$\Gamma_h = -(cQ)^{-1}\int_0^\infty d\epsilon \frac{d\rho(\epsilon)}{d\epsilon}\left[\sum_{n>0}(n\hbar\omega)^2 a_n(\epsilon)\right] . \tag{49}$$

On using (26) and (30), Eq. (49) can be further transformed into

$$\Gamma_h = -\frac{(\hbar\omega)^2}{cQ}\int_0^\infty d\epsilon\, u\bar{n}\frac{d\rho(\epsilon)}{d\epsilon} \tag{50}$$

or, integrating by parts and using (31),

$$\Gamma_h = \int_0^\infty \Gamma_c'(\epsilon)\rho(\epsilon)d\epsilon . \tag{51}$$

Here $\Gamma_c'(\epsilon)$is the cold beam gain per electron. Note, that the last trivial result, as well as the more general gain formulas (47) and (48), remain valid in the classical (multiphoton), intermediate and quantum regimes of operation. Moreover, these results do not depend on the specifics of a concrete type of laser. In each case one can find the corresponding expression for $\bar{n}$ by following guidelines similar to those used in the present work.

ACKNOWLEDGMENTS

The author would like to thank Professor F. Dothan for his helpful comments and suggestions in the preparation of this paper. This research was supported in part by the U.S.—Israel Binational Science Foundation.

[1]J. M. J. Madey, J. Appl. Phys. 42, 1906 (1971).

[2]J. M. J. Madey, H. A. Schwettmann, and W. M. Fairbank, IEEE Trans. Nucl. Sci. 20, 980 (1973).

[3]V. P. Sukhatme and P. W. Wolff, J. App. Phys. 44, 2331 (1973).

[4]W. B. Colson, Phys. Lett. 59A, 187 (1976).

[5]L. R. Elias, W. M. Fairbank, J. M. J. Madey, H. A. Schwettmann, and T. I. Smith, Phys. Rev. Lett. 36, 717 (1976).

[6]F. A. Hopf, P. Meystre, and M. O. Scully, Opt. Commun. 18, 413 (1976).

[7]N. M. Kroll and W. A. McMullin, Phys. Rev. A 17, 300 (1978).

[8]For a review of past work, see P. Sprangle, R. A. Smith, and V. L. Granatstein, in *Infrared and Millimeter Waves,* edited by K. J. Button (Academic, New York, 1979), Vol. 1, p. 279.

[9]See, for example, M. W. Fedorov, IEEE J. Quantum Electron. QE-17, 1359 (1981); P. Bosco, W. B. Colson, and R. A. Freedman, *ibid.* QE-19, 272 (1983); S. T. Stenholm and A. Bambini, *ibid.* QE-17, 1363 (1981); W. Becker and M. S. Zubairy, Phys. Rev. A 25, 2200 (1982); W. Becker, M. O. Scully, and M. S. Zubairy, Phys. Rev. Lett. 48, (1982); G. Dattoli, A. Renieri, F. Romanelly, and R. Bonifacio, Opt. Commun. 34, 240 (1982); G. Dattoli and A. Renieri, J. Phys. (Paris) Suppl. 44, 125 (1983).

[10]W. B. Colson, *Physics of Quantum Electronics* (Addison-Wesley, Reading, Mass., 1977), Vol. 5.

[11]L. Friedland, J. Phys. B 12, 409 (1979).

[12]A. I. Akhiezer and V. R. Berestetskii, *Quantum Electrodynamics* (Interscience, New York, 1965), p. 415.

[13]E. M. Lifshitz and L. P Pitaevskii, *Physical Kinetics* (Pergamon, Oxford, 1981), p. 4.

[14]L. Friedland, Phys. Fluids 23, 2376 (1980).

[15]L. Friedland and J. L. Hirshfield, Phys. Rev. Lett. 44, (1980).

[16]L. Friedland and A. Fruchtman, Phys. Rev. A 25, 2693 (1982).

[17]I. B. Bernstein and L. Friedland, Phys. Rev. A 23, 816 (1981).