Electron-cyclotron resonance heating in plasmas with arbitrary stratification of the magnetic field

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Cyclotron resonance absorption in a plasma with arbitrary stratification of the magnetic field is considered. Using the absorption coefficient derived for perpendicular stratification, simple analytic estimates of the absorption can be made for artrary stratification.

Heating of magnetically confined plasmas near the electron-cyclotron resonance frequency is currently being investigated for both toroidal and mirror-type devices.[1-7] Theoretical studies in simple one-dimensional models have shown the possibility of significant absorption of the electromagnetic energy, even at temperatures of a few hundred electron volts.[4,5] In some of these cases, analytic approximations for the absorption coefficients for different electromagnetic modes were derived. For example, Eldridge *et al.*[6] have shown that for perpendicularly stratified plasmas, where the magnetic field **B** has a constant direction and its gradient is perpendicular to **B**, the transmission coefficient for the electromagnetic energy flux through the electron-cyclotron resonance is given by

$$T_\perp = \exp(-|A_\perp|)\,, \tag{1}$$

where

$$A_\perp = L_\perp Q_\perp\,, \tag{2}$$

and

$$Q_\perp = \frac{\omega}{c}\frac{2\pi T_e}{m_e c^2}\frac{n_\parallel^2}{\alpha^2 n_\perp}\left(\frac{(1-\alpha^2/2-n_\parallel^2)(1-\alpha^2)-n_\perp^2}{(1-\alpha^2-n_\perp^2)^2+(1-\alpha^2)n_\parallel^2}\right). \tag{3}$$

In these formulas it is assumed that the absorption takes place in a thin layer in the neighborhood of the electron-cyclotron resonance where

$$\omega=\omega_c,\quad \alpha=\omega_{pe}/\omega,\quad n_\parallel = ck_\parallel/\omega,\quad n_\perp = ck_\perp/\omega\,,$$

and $n^2 = n_\parallel^2 + n_\perp^2$. The parameter $L_\perp$ in Eq. (2) is the local scale length of variation of the cyclotron frequency at the resonance surface, namely, $L_\perp = \omega_c/|\nabla\omega_c|$.

Because of the complexity of the magnetic field geometries in realistic devices, such as tandem mirrors or bumpy torii, the use of the aforementioned simple analytic results is often not valid. The problem is usually solved by using the geometric optics approximation and numerical procedures. Such studies typically involve ray tracing, performed using the cold plasma model. The absorption of the electromagnetic energy flux is then found by computing the imaginary correction $i\nu$ to the frequency of the wave due to the thermal effects and integrating ν along the rays as they pass through the resonance surface.[7] In this note we shall show that simple geometric considerations allow one to derive an analytic expression for the absorption coefficient even for plasmas with arbitrary stratification of the magnetic field. Thus, the absorption may be calculated using the analytic results obtained previously for perpendicular stratification. This can result in considerable savings in the numerical work, as well as allowing one to obtain simple analytic estimates for the absorption.

Consider a general two-dimensional case shown schematically in Fig. 1. It can be shown[8] that, in general, consistent with Eq. (1), the transmission coefficient can be found from

$$T = \exp(-|A|)\,, \tag{4}$$

with

$$A = 2\int_{-\infty}^{+\infty} \nu(t)\,dt\,, \tag{5}$$

where the time integration is along the ray. If the resonance region, where ν makes a significant contribution in Eq. (5), is narrow enough, one can evaluate Eq. (5) by assuming that the index of refraction **n** remains constant and has the components corresponding to the point where the ray crosses the resonance surface. Now one can use the fact that the dielectric tensor for hot Maxwellian plasmas is a function of the wave vector **k**, ω and a parameter $\xi = (\omega_c - \omega)/k_\parallel v_e$, where $k_\parallel$ is the component of the k vector along the direction of the magnetic field, and $v_e = (2T_e/m_e)^{1/2}$ is the average velocity of the electrons.[9] Therefore, ξ is the only variable in ν which changes rapidly as the ray passes the resonance region. All the other variables cannot only be assumed constant in Eq. (5), but are also independent of the directions of spatial gradients of various plasma parameters. In addition, the group velocity of the wave $v_g = \partial\omega/\partial\mathbf{k}$ involves differentiation of ω with respect to the components of the k vector and thus the direction of the ray, and the absolute value of the group velocity in the resonance region are also approximately constant and independent of the type of the spatial stratification of the plasma. Then,

$$A = 2\int_{-\infty}^{+\infty} \nu(\xi)\,dt \simeq \frac{2}{|\mathbf{v}_g|}\int_{-\infty}^{+\infty}\nu(\xi)\,ds$$

$$\simeq L_g\,\frac{2k_\parallel v_e}{\omega|\mathbf{v}_g|}\int_{-\infty}^{+\infty}\nu(\xi)\,d\xi\,, \tag{6}$$

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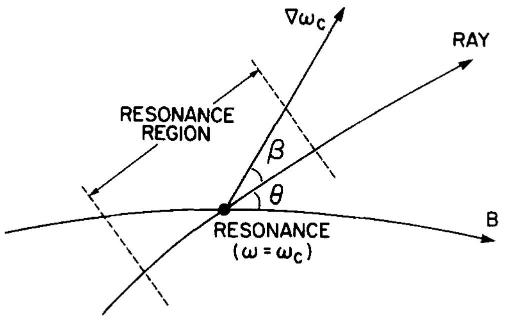

FIG. 1. Geometry of ray propagation in the case of generalized magnetic field stratification.

where we used the first order expansion of the cyclotron frequency in the direction of the ray in the resonance region

$$\omega_c=\omega(1+s/L_g)\,, \tag{7}$$

and $L_g=\omega_c/(\partial\omega_c/\partial s)$ is the scale length of variation of ω_c along the ray. Thus, one finally has

$$A=L_gF(\mathbf{k},\mathbf{r},\omega)\,. \tag{8}$$

The function F in the last equation is independent of the direction of $\nabla\omega_c$. Therefore, one can use the known result for perpendicular stratification to find this function. In fact, if at the point where the ray crosses the resonance surface the plasma would have perpendicular stratification, by comparing Eqs. (2) and (8) one would get

$$F=Q_\perp \sin\Theta, \tag{9}$$

where Θ is the angle between the direction of the ray and the direction of the magnetic field (see Fig. 1). Then, in the general case

$$A=LQ_\perp(\sin\Theta/\cos\beta)\,, \tag{10}$$

where $L=\omega_c/|\nabla\omega_c|$ is the scale length of variation of the cyclotron frequency at the resonance point and β is the angle between the group velocity and $\nabla\omega_c$.

Now consider various limiting cases described by Eq. (10). Clearly, for perpendicular stratification, $\Theta+\beta=\pi/2$ and Eq. (10) reduces to Eq. (2). In the plasma with parallel stratification, where $\nabla\omega_c$ is parallel to the direction of the magnetic field, one has $\Theta=\beta$ and therefore,

$$A_\parallel=L_\parallel Q_\perp \tan\Theta. \tag{11}$$

In cases when $\beta=\pi/2$, Eq. (10) gives infinite absorption which is the consequence of the fact that the ray is propagating parallel to the resonance surface, so that it stays in resonance for a long time and thus is heavily absorbed. It should be mentioned, however, that in such cases the strong absorption predicted by Eq. (10) must be viewed as only a qualitative indication, since the assumption of the narrowness of the absorption region becomes invalid. Thus, the absorption will be limited by the thickness of the plasma slab.

The angle Θ in Eq. (10) can easily be found. In fact, the components of the group velocity at the resonance surface can be found by differentiating the appropriate dispersion relation. Assuming that the ray can be described by the cold plasma model, at the resonance one has,

$$D=n_\perp^2(n^2-1)-(2n^2-2-\alpha^2)(1-\alpha^2)=0\,, \tag{12}$$

and, since

$$v_{g\perp,\parallel}=\frac{\partial\omega}{\partial k_{\perp,\parallel}}=\frac{(\partial D/\partial k_{\perp,\parallel})}{(\partial D/\partial\omega)}\,,$$
$$\tan\Theta=\frac{v_{g\perp}}{v_{g\parallel}}=\frac{n_\perp}{n_\parallel}\frac{(2n_\perp^2+n_\parallel^2+2\alpha^2-3)}{(n_\perp^2+2\alpha^2-2)}\,. \tag{13}$$

Thus, the direction of $\nabla\omega_c$ remains the only unknown geometric factor in Eq. (10). Note that Eq. (13) can be used directly in the case of the parallel stratification, Eq. (11), where one gets

$$A_\parallel=L_\parallel\frac{\omega}{c}\frac{2\pi T_e}{m_ec^2}\frac{n_\parallel}{\alpha^2}\frac{(2n_\perp^2+n_\parallel^2+2\alpha^2-3)}{(n_\perp^2+2\alpha^2-2)}\times\frac{[(1-\alpha^2/2-n_\parallel^2)(1-\alpha^2)-n_\perp^2]}{[(1-\alpha^2-n_\perp^2)+(1-\alpha^2)n_\parallel^2]}\,. \tag{14}$$

Note also that for small angles, Θ, one has from Eq. (13) $\sin\Theta\propto n_\perp/n_\parallel$ and, therefore, the singularity in Eq. (3) at $n_\perp\to 0$ disappears if one is using the general formula (10).

In conclusion, we have demonstrated that using simple geometric considerations with the help of the absorption coefficient obtained for perpendicular stratification, one can estimate the absorption in plasmas with arbitrary stratification of the magnetic field. Our formula can be applied in a ray tracing computer code each time the ray crosses the cyclotron resonance surface. All the parameters and geometric factors in Eq. (10) are either known from a ray tracing, or can easily be evaluated.

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