J. Phys. B: Atom. Molec. Phys., Vol. 12, No. 3, 1979. Printed in Great Britain

Correspondence principle in multiphoton inverse bremsstrahlung

L Friedland†

Racah Institute of Physics, Hebrew University of Jerusalem, Jerusalem, Israel

Received 20 June 1978, in final form 7 August 1978

Abstract. Multiphoton electron–ion inverse bremsstrahlung is studied using the correspondence principle. It is shown that the existing quantum-mechanical theory of this phenomenon is only valid in the low-frequency limit $\omega\tau \ll 1$. A different approach is suggested which enables us to obtain the multiphoton cross sections for all values of $\omega\tau$. Using these cross sections, the absorption coefficient of laser radiation in an isotropic plasma is found.

1. Introduction

The emission and absorption of electromagnetic radiation by electrons in collisions with heavy particles plays an important role in the creation and heating of a plasma by laser radiation. Depending on the average electron energy, the laser photon energy and the radiation flux density, these induced processes can be treated by using classical and/or quantum mechanics.

Meyerand and Haught (1963), in their first study of laser-induced gas breakdown, indicated that classical microwave theory is not suitable for optical-breakdown conditions. The reason is that the average classical energy gained by an electron in each single collision with a gas molecule in this case is much smaller than the laser photon energy $\hbar\omega$. Therefore, the electron does not absorb or emit energy in every collision as is assumed in the classical treatment. The energy exchange between the electron and the radiation field is a single quantum process. Such a single-photon approach is given by Zeldovich and Raizer (1965) in their theory of laser-induced gas breakdown. In their work, the cross sections σ_{-1}, and σ_{+1} of induced emission and absorption of the laser photon are obtained using the principle of detailed balancing for the low-frequency limit $\omega\tau \ll 1$, where τ is the duration of the collision. It is shown that for electron energies $\epsilon \gg \hbar\omega$ the total cross section for the absorption or emission of a laser photon $\hbar\omega$ by the electron in an elastic collision with the gas molecule can be expressed in the form

$$\sigma(\epsilon, \theta) = \sigma_{+1}(\epsilon, \theta) + \sigma_{-1}(\epsilon, \theta) \simeq \tfrac{4}{3}(\epsilon_0/\hbar\omega)(\epsilon/\hbar\omega)(1-\cos\theta)\sigma_{\text{el}}(\epsilon, \theta) \qquad (1)$$

where θ and $\sigma_{\text{el}}(\epsilon, \theta)$ are the scattering angle and elastic collision cross section respectively and ϵ_0 is the classical electron-oscillation energy

$$\epsilon_0 = e^2E^2/2m\omega^2 \qquad (2)$$

†Present address: Department of Computer Science, Yale University, New Haven, Connecticut 06520, USA.

where e and m are electron charge and mass and E is the average radiation electric field. The single-photon theory successfully explained the main features of laser-induced breakdown; however its application was limited to relatively weak radiation fields. In fact, it follows from equation (1) that the quantity

$$\beta = \tfrac{4}{3}(\epsilon_0/\hbar\omega)(\epsilon/\hbar\omega)(1-\cos\theta) \tag{3}$$

can be interpreted as the probability for induced emission or absorption of a laser photon per collision. Thus, one can estimate the limit of the single-photon theory, assuming $\beta \ll 1$. With the increase of the laser-radiation flux density, β increases and so does the probability of induced emission or absorption of two or more laser photons in a single collision. At a high enough radiation flux density, the multiphoton-induced processes dominate and the single-photon theory cannot be used in this case. The condition $\beta > 1$ is fulfilled, for instance, in optical gas breakdown by picosecond laser pulses (Ireland 1974) and in various experiments on plasma heating by intense laser radiation.

A quantum theory of multiphoton electron free–free transitions was developed by Bunkin and Fedorov (1966). The cross sections σ_n ($n = 0, \pm 1, \pm 2, \ldots$) for induced emission or absorption of n photons were derived using first-order perturbation theory with the field of a nucleus as the perturbation. The electromagnetic field, however, was treated classically. Since no direct measurements of the multiphoton cross sections exist, an obvious check of the multiphoton theory is to see if, in the classical limit when $\hbar\omega \to 0$, it gives the well known classical results. This was done by Seely and Harris (1973), who used the multiphoton cross sections σ_n, derived by Bunkin and Fedorov to find the absorption coefficient of laser radiation in a plasma and to compare it with the results of the classical theory (Silin 1965). However, in the case of a weak radiation field, the calculations were made in the single-photon approximation and the limit $h \to 0$ was taken. This procedure is inconsistent since when $\hbar\omega \to 0$ one always, sooner or later, moves into a multiphoton case ($\beta \gg 1$). Use of the multiphoton cross sections of Bunkin and Fedorov raises another difficulty. For $\beta \ll 1$ the cross sections σ_{+1} and σ_{-1} of Bunkin and Fedorov satisfy equation (1), which was derived by Zeldovich and Raizer (1965) for the low-frequency limit. Integrating this equation over all possible scattering angles, the following total cross section is obtained

$$\sigma_t = \frac{4}{3}\frac{\epsilon_0}{\hbar\omega}\frac{\epsilon}{\hbar\omega}2\pi\int_0^{\pi}(1-\cos\theta)\sigma_{el}(\epsilon,\theta)\sin\theta\,d\theta.$$

For Rutherford scattering, $\sigma_{el} \propto (\sin\theta/2)^{-4}$ and the total cross section σ_t for absorption and emission of a single photon $\hbar\omega$ becomes infinite. The infinite integral in σ_t also appears in the expression for the absorption coefficient derived by Seely and Harris (1973), where the divergence was artificially removed by introducing the Debye cut-off. Assuming now that Bunkin and Fedorov's cross sections are correct for large values of θ ($\omega\tau \ll 1$) (this is confirmed by the analysis of Zeldovich and Raizer), we come to the conclusion that the divergence in σ_t is due to an incorrect dependence of the probability β on angle θ for small values of θ ($\omega\tau > 1$). Thus, the cross sections of Bunkin and Fedorov are only valid in the low-frequency limit $\omega\tau \ll 1$. In contrast to electron collisions with neutral atoms considered by Zeldovich and Raizer, this condition in electron–ion collisions strongly restricts the possible values of impact parameter.

This paper presents a theoretical study of multiphoton electron free–free transitions without the limitation on values of $\omega\tau$. The investigation is based on an approach different from the one used by Bunkin and Fedorov (1966) and Seely and Harris (1973). First we find the classical analogue of the multiphoton transitions and then we turn to the quantum picture using the correspondence principle. The expressions for the multiphoton cross sections are found and compared with the results of Bunkin and Fedorov. Finally the absorption coefficient for an isotropic plasma is calculated. This work is a generalisation of previous results (Friedland 1975), where a similar approach was used to study the multiphoton electron free–free transition in collisions with neutral atoms.

2. The classical analogue

Let us consider classically an electron moving in an alternating electric field $\boldsymbol{E} = \boldsymbol{k}E\cos(\omega t)$ with a constant polarisation vector $\boldsymbol{k}$. The classical description is valid provided the amplitude of oscillations in electron velocity $\Delta v = eE/m\omega$ and position $\Delta x = eE/m\omega^2$ satisfy the inequality $m\Delta v\Delta x \gg \hbar$. It follows from this condition that $\epsilon_o = e^2E^2/2m\omega^2 \gg \hbar\omega$. Then according to (3), when $\epsilon > \hbar\omega$ the parameter β is much greater than one. Therefore, a multiphoton case is considered here, as would be expected in a classical approach.

Let us now find the change in average electron energy in the alternating field resulting from a single collision between electron and ion. The influence of the polarisability of the ion on the electron motion is negligible due to the long range of the Coulomb interaction. Therefore, the collision process will be considered classically as scattering in an electrostatic field $\boldsymbol{H}(\boldsymbol{r})$. During the collision the electron energy changes in the following way

$$\epsilon(t) = \epsilon_1 + \int_{t_1}^{t} \boldsymbol{F}(t)\,\mathrm{d}\boldsymbol{r}(t) \tag{4}$$

where ϵ_1 is the electron energy at the time t_1 (*before* the collision), $\boldsymbol{r}(t)$ is the electron radius vector and

$$\boldsymbol{F}(t) = \boldsymbol{k}eE\cos(\omega t) + e\boldsymbol{H}(\boldsymbol{r}(t)). \tag{5}$$

Thus at time t_2 *after* the collision the electron energy is

$$\epsilon_2 = \epsilon_1 + eE\boldsymbol{k}\int_{t_1}^{t_2} \boldsymbol{v}(t)\cos(\omega t)\,\mathrm{d}t \tag{6}$$

where $\boldsymbol{v}(t)$ is the electron velocity vector. Let us integrate equation (6) by parts and choose times t_1 and t_2 so that $\sin(\omega t_1) = \sin(\omega t_2) = 0$. Then

$$\epsilon_2 = \epsilon_1 - \frac{eE}{m\omega}\boldsymbol{k}\int_{t_1}^{t_2} \boldsymbol{F}(t)\sin(\omega t)\,\mathrm{d}t. \tag{7}$$

Substituting equation (5) into the last expression we have:

$$Z = \epsilon_2 - \epsilon_1 = -\frac{eE}{\omega}\boldsymbol{k}\int_{t_1}^{t_2} \frac{e\boldsymbol{H}(t)}{m}\sin(\omega t)\,\mathrm{d}t. \tag{8}$$

Since the average electron energies before and after the collision are $\bar{\epsilon}_1 = \epsilon_1 + \frac{1}{2}(e^2E^2/m\omega^2)$ and $\bar{\epsilon}_2 = \epsilon_2 + \frac{1}{2}(e^2E^2/m\omega^2)$ respectively, the quantity Z in equation (8) represents the change in the average electron energy resulting from a single collision in the alternating field. This quantity can be positive or negative according to the parameters of the scattering and corresponds (depending on the sign) to induced emission or absorption of the energy of the field during the collision.

The simplest expression for Z is obtained in the low-frequency limit, when $\omega\tau \ll 1$ (τ is the duration of the scattering). In this case

$$Z = -(eE/\omega)\boldsymbol{k}\Delta\boldsymbol{v}_c \sin(\omega t_c) \tag{9}$$

where t_c is the time of the collision and $\Delta\boldsymbol{v}_c$ is the change in the velocity vector caused by the collision. As already mentioned in § 1, the condition $\omega\tau \ll 1$ for collisions with neutral atoms is fulfilled even at optical frequencies. For electron–ion scattering however (in this case $\tau \simeq b/v$, where v and b are the electron velocity and impact parameter) we cannot restrict our treatment by the low-frequency limit. Equation (8) can be considerably simplified even in the case

$$\omega\tau = \omega(b/v) \gtrsim 1 \tag{10}$$

if 'hot' electrons, whose velocity v is much larger than the amplitude of the velocity oscillations

$$v \gg eE/m\omega \tag{11}$$

are considered. It follows from equation (10) that in this case

$$eE/bm\omega^2 = \Delta x/b \ll 1 \tag{12}$$

where Δx is the amplitude of the electron oscillations in the electromagnetic field. The last inequality shows that the function $e\boldsymbol{H}(t)/m$ in equation (8) is only slightly perturbed by the radiation field and is approximately equal to the electron acceleration in *free* scattering. Let us assume that the scattering potential is spherically symmetrical. Then the function $e\boldsymbol{H}(t)/m$ is symmetrical about time t_c. Therefore, substituting $t' = t - t_c$ into equation (8) we have

$$Z = -\frac{eE}{\omega}\boldsymbol{k}\int_{-\tau/2}^{\tau/2} \frac{e\boldsymbol{H}'(t')}{m} \sin[\omega(t'+t_c)]\,dt' \tag{13}$$

where $\boldsymbol{H}'(t') = \boldsymbol{H}(t'+t_c)$ is symmetrical with respect to $t'=0$. Finally, after simple trigonometric transformations in equation (13) we get

$$Z = -(eE/\omega)|C| \sin(\omega t_c + \phi) \tag{14}$$

where $|C|$ and ϕ are the amplitude and phase of

$$C = \boldsymbol{k}\int_{-\tau/2}^{\tau/2} \frac{e\boldsymbol{H}'(t)}{m} \exp(i\omega t)\,dt. \tag{15}$$

It can be seen that at low frequencies the quantity Z given in equation (9) is of the same form as (14) with $|C| = \boldsymbol{k}\Delta\boldsymbol{v}_c$ and $\phi = 0$. The common character of the formulae for Z is expressed in the fact that the quantity C is independent of the amplitude of the radiation field. The reason for this is that in the low-frequency limit, as well as in the case $\omega\tau \gtrsim 1$ for 'hot' electrons, the trajectory of the electron is not perturbed strongly by the radiation field during the scattering process. However this 'non-perturbation' is for a different reason in each case. For the 'hot' electrons when $\omega\tau \gtrsim 1$, the perturbation of the path due to the radiation field is small in comparison with the impact parameter. On the other hand, in the low-frequency limit, the electron trajectory and therefore also the function $\boldsymbol{H}'(t)$ in equation (15), simply have no time to be influenced by the radiation because of the short duration of the collision. Thus in the case of 'hot' electrons and $\omega\tau \gg 1$, we can take an average impact parameter of the electron in electromagnetic field as impact parameter. By contrast, in the low-frequency case the actual impact parameter is defined near the scattering centre during the period of the electric field oscillation in which the collision occurs. This impact parameter can differ greatly from the average impact parameter far from the scattering centre.

In the following, the case of 'hot' electrons will be considered. Here the scattering process is not perturbed strongly by the external radiation field, for all values of $\omega\tau$. As noted above, the impact parameter and the collision cross section have their regular meaning in this case. For 'cold' electrons whose velocities do not satisfy condition (11) a certain region of $\omega\tau$ exists where the collision process itself is influenced by the external radiation. In this region, the concepts of impact parameter and cross section have to be reconsidered. This more complicated case will not be discussed in this paper.

3. Comparison with quantum theory

In this section, the correspondence between the classical expression (14) for the energy which is emitted or absorbed in single collision and the multiphoton theory (Bunkin and Fedorov 1966) will be considered. For simplicity, consider first the low-frequency limit. According to equation (9)

$$Z = -AX \tag{16}$$

where $X = \sin(\omega t_c)$ and

$$A = eE\Delta v_\perp/\omega \tag{17}$$

where $\Delta v_\perp = (\boldsymbol{k}\Delta\boldsymbol{v}_c)$ is the projection of the change of velocity vector in the collision on the direction of the polarisation vector. For a given $\Delta v_\perp$, Z is a random quantity defined in the interval $[-A, +A]$. Since the values of ωt_c are homogeneously distributed, the distribution density of the values of Z is

$$f(Z) = [\pi A(1-(Z/A)^2)^{1/2}]^{-1}. \tag{18}$$

From the quantum-mechanical point of view, the quantity Z can have only discrete values $n\hbar\omega$ ($n = 0, \pm1, \pm2, \ldots$). The classical probability of Z being between $n\hbar\omega$ and $(n+1)\hbar\omega$ is approximately equal to $\hbar\omega f(n\hbar\omega)$. In the classical limit, this quantity, according to the correspondence principle, has to be equal to the probability P_n of absorption (or emission) of n photons:

$$P_n = \hbar\omega f(n\hbar\omega) = (\hbar\omega/\pi A)[1-(n\hbar\omega/A)^2]^{-1/2}. \tag{19}$$

Thus the cross section σ_n of induced emission or absorption of n photons can be written in the form

$$\sigma_n = P_n\sigma_{el} = (\hbar\omega/\pi A)[1-(n\hbar\omega/A)^2]^{-1/2}\sigma_{el}. \tag{20}$$

where σ_{el} is the elastic-collision cross section. Let us compare now this cross section with the one obtained in the quantum-mechanical approach by Bunkin and Fedorov (1966). Their cross section is given by

$$\sigma_n = [J_n(A/\hbar\omega)]^2\sigma_{el} \tag{21}$$

where $J_n(x)$ is a Bessel function of the nth order. Although the function $J_n^2(A/\hbar\omega)$ has a more complicated fine structure than P_n in equation (20), on average, both functions are similar for $A/\hbar\omega \gg 1$. For instance, when $n \ll A/\hbar\omega$ the function $J_n^2(A/\hbar\omega)$ can be expressed in the following asymptotic form (Abramowitz and Stegun 1964):

$$J_n^2(A/\hbar\omega) = (2\hbar\omega/\pi A)\cos^2(A/\hbar\omega - n\pi/2 - \pi/4). \tag{22}$$

Therefore, the quantum-mechanical cross sections σ_n for $n \ll A/\hbar\omega$ are described by an oscillating function of n, whose average value is $(\hbar\omega/\pi A)\sigma_{\mathrm{el}}$ and this is in accordance with equation (20). When n increases, both functions P_n and J_n reach their maximum at $n \simeq A/\hbar\omega$. For $n > A/\hbar\omega$, $J_n^2(A/\hbar\omega)$ rapidly goes to zero and $P_n = 0$ in all this region. In addition, P_n and J_n^2 in equations (20) and (21) are both normalised to unity. Thus we come to the conclusion that, in the low-frequency limit, the classical theory is in good agreement with the quantum-mechanical results of Bunkin and Fedorov.

Let us consider the case $\omega\tau \gtrsim 1$. Here the expression (14) for Z has the same form as for the low-frequency limit. Therefore, using the same correspondence arguments, we can express the cross sections σ_n for all values of $\omega\tau$ in the form (20), where instead of A we introduce $(eE/\omega)|C|$ (see equation (14)):

$$\sigma_n = \hbar\omega[(e^2E^2/\omega^2)|C| - (n\hbar\omega)^2]^{-1/2}\sigma_{\mathrm{el}}. \tag{23}$$

This cross section agrees in gross outline with the one derived by Bunkin and Fedorov (1966) (equation (21)), only in the low-frequency limit, when $|C| = \omega A/eE$.

4. Another quantum-mechanical interpretation

The approach used in the previous section made it possible to find the correspondence between the classical and quantum-mechanical pictures, as well as to define and expand the limits of the existing quantum theory of the multiphoton-induced electron free–free transitions. This approach, however, seems to be slightly artificial. Let us show now that another more natural quantum interpretation of the classical results can be found. With that end in view we will consider the low-frequency limit first. In the expression (16) for Z in this case, in addition to ωt_c, there is another independent random quantity, taking various values for the same scattering angle θ. This quantity is the angle α between the change of velocity vector $\Delta \boldsymbol{v}_c$ due to the collision, and the polarisation vector $\boldsymbol{k}$. Equation (16) can now be written in the following form

$$Z = -D(2\pi)^{1/2}XY \tag{24}$$

where (as above) $X = \sin(\omega t_c)$, $Y = \cos\alpha$ and

$$D^2 = (4/\pi)\epsilon\epsilon_o(1-\cos\theta). \tag{25}$$

In the last equation ϵ is the electron energy just before the collision and ϵ_o is the electron oscillation energy in the electromagnetic field (see equation (2)).

Let us find now the probability density of the values of Z, the change in the average electron energy due to a single scattering at the angle θ. We assume that all directions of the vector $\Delta \boldsymbol{v}_c$ are equally probable. Then the quantity Y in equation (24) is distributed with the density

$$\Phi(Y) = \begin{cases} \frac{1}{2} & |Y| \leq 1 \\ 0 & |Y| > 1. \end{cases} \tag{26}$$

Let us also assume, as was done in the last section, that the phase of the collision ωt_c is distributed homogeneously. Then the probability density of the values of X can be written (see equation (18)):

$$\Psi(X)=\begin{cases}(\pi)^{-1}(1-X^2)^{-1/2} & |X|\leqslant 1\\ 0 & |X|>1.\end{cases} \tag{27}$$

The values of X, Y and Z are symmetrically distributed around the zero point and we consider below only the X, $Y\geqslant 0$ region. With the help of the probability densities Φ and Ψ, the probability density $\rho(V)$ of the values of $V=XY$ can be written in the following form

$$\rho(V)=2\int_{Y>0}\Phi(Y)\Psi(V/Y)\frac{\partial(V/Y)}{\partial V}\,\mathrm{d}Y \qquad V\geqslant 0. \tag{28}$$

Substituting equations (26) and (27) for Φ and Ψ in this expression, and integrating we have:

$$\rho(V)=\pi^{-1}\ln[V^{-1}+(V^{-2}-1)^{1/2}] \qquad V\geqslant 0. \tag{29}$$

This distribution density is presented in figure 1 (full curve). In the same picture, the broken curve represents $\exp(-\pi V^2)$. It can be seen from the graphs that in the region $0<|V|<1$, both functions are quite similar. Their dispersions are also approximately equal ($1/6$ for $\rho(V)$ and $1/2\pi$ for $\exp(-\pi V^2)$). Thus the function $\rho(V)$ can be approximated by $\exp(-\pi V^2)$. Accordingly, the distribution density function for $Z=-D(2\pi)^{1/2}V$ in the region $0<|Z|<D(2\pi)^{1/2}$ may be written in the form

$$f(Z)=(1/D\sqrt{2\pi})\rho(Z/D\sqrt{2\pi})=(1/D\sqrt{2\pi})\exp(-Z^2/2D^2). \tag{30}$$

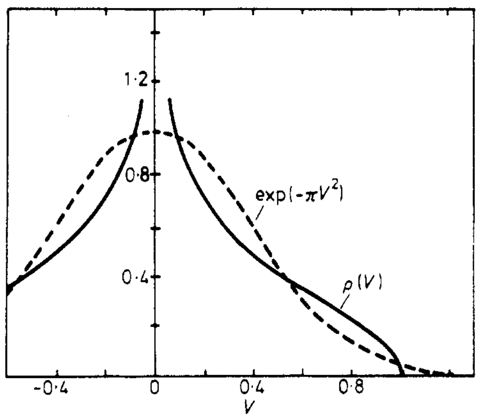

Figure 1. The distribution of values of V.

Thus the distribution of the changes in average electron energy due to elastic collision can be expressed in the Gaussian form (30). This result is not accidental and can be explained by the quantum nature of the energy exchange processes between the electron and the radiation field. From the quantum-mechanical point of view during the collision the free electron exchanges energy with the radiation field by a certain number of quanta $\hbar\omega$. Let $N(\epsilon,\theta)$ be the average number of each elementary absorptions and emissions of radiation quanta per single scattering of the electron with energy ϵ for a fixed scattering angle θ. Since during this process the energy of the 'hot' electrons does not change significantly, the electron motion and therefore the events of absorption and emission of photons during the collision will be statistically independent. Therefore the change of the electron energy in a single collision at angle θ will be described by a Gaussian distribution (30) with the dispersion D given by

$$D = \hbar\omega(N(\epsilon, \theta))^{1/2}. \tag{31}$$

We can conclude that, according to these quantum-mechanical considerations, the distribution of changes in the electron energy Z in a single collision can be expressed in Gaussian form for all values of Z, not only in the region $0<|Z|<D(2\pi)^{1/2}$ where equation (30) was derived. On the other hand, the classical theory fails to describe correctly the whole region of Z values. In fact, the difference between the classical distribution $\rho(V)$ and $\exp(-\pi V^2)$ is especially large for $V \simeq 0$ and $|V|>1$, as can be seen from figure 1. According to the classical treatment (equation (24)) an electron cannot absorb (or emit) more than $m \simeq D\sqrt{2\pi}/\hbar\omega$ laser quanta. Such a process corresponds to the region $|V|>1$ in figure 1 and therefore to the classical distribution $\rho(V)=0$ in this region. From the quantum-mechanical point of view this limitation on the number of absorbed laser quanta does not exist. Thus induced emission or absorption of $n>m$ laser quanta is a purely quantum effect, which is not described correctly by the classical distribution $\rho(V)$. The region $V \simeq 0$ in figure 1 corresponds to the emission or absorption of a small number of quanta. This case also needs quantum-mechanical treatment and therefore the classically derived distribution $\rho(V)$ is invalid in the neighbourhood of the point $V=0$. As already mentioned, in the region $0<|V|<1$, where both functions $\rho(V)$ and $\exp(-\pi V^2)$ are most significant, the classical and quantum descriptions give similar results. This fact makes it possible to use the correspondence principle, substitute the classical expression for D (equation (25)) into equation (31) and define $N(\epsilon, \theta)$:

$$N(\epsilon, \theta) = \tfrac{4}{3}(\epsilon/\hbar\omega)(\epsilon_o/\hbar\omega)(1-\cos\theta). \tag{32}$$

Comparing this result with the single-photon probability (3) we find that $N \simeq \beta$. Thus, the physical meaning of N as the average number of emitted and absorbed photons per collision, is preserved even when the multiphoton processes are hardly probable. N in this case may also be interpreted as the probability of induced emission or absorption of a single photon $\hbar\omega$. Finally using the expression (32) for N, the cross section σ_n for absorption or emission of a given number of photons in a single collision can be found. In the classical limit when $N \gg 1$, according to equation (30) we have

$$\sigma_n(\epsilon, \theta) = \hbar\omega\rho(n\hbar\omega)\sigma_{el}(\epsilon, \theta) = (2\pi N)^{-1/2}\exp(-n^2/2N)\sigma_{el}(\epsilon, \theta). \tag{33}$$

Until now only the low-frequency limit has been considered. Similar ideas can be used in the general case. In order to find $N(\epsilon, \theta)$ for any value of $\omega\tau$ similar to (32), the following formula can be used

$$N(\epsilon, \theta) = \langle (Z/\hbar\omega)^2 \rangle_{av}. \tag{34}$$

Substituting equation (14) for Z into this expression and averaging it for various values of ωt_c and directions of the polarisation vector k we have

$$N(\epsilon, \theta) = \frac{e^2E^2}{\omega^2}\frac{1}{6(\hbar\omega)^2}\left|\int_{-\tau/2}^{\tau/2}\frac{e\boldsymbol{H}'(t)}{m}\exp(\mathrm{i}\omega t)\,\mathrm{d}t\right|^2. \tag{35}$$

We use the well known expression for the bremsstrahlung radiation energy $d\mathscr{E}(\epsilon, \theta)$ emitted in frequency interval $d\omega$ (Bekefi 1966, p 67)

$$\frac{d\mathscr{E}_\omega(\epsilon, \theta)}{d\omega} = \frac{2}{3}\frac{e^2}{\pi c^3}\left|\int_{-\tau/2}^{\tau/2} \frac{e\boldsymbol{H}'(t)}{m} \exp(i\omega t)\, dt\right|^2. \tag{36}$$

According to this, equation (35) for N can be rewritten in the following form

$$N(\epsilon, \theta) = \frac{\pi c^3}{4(\hbar\omega)^2}\frac{E^2}{\omega^2}\frac{d\mathscr{E}_\omega(\epsilon, \theta)}{d\omega}. \tag{37}$$

This equation is the multiphoton analogue to the relation between the Einstein coefficients for induced free–free transitions and spontaneous bremsstrahlung emission.

5. Absorption coefficient

We use in this section the multiphoton cross sections σ_n defined by formula (33) and find the absorption coefficient of laser radiation in an isotropic plasma. Let $a_n(\epsilon, \theta)$ be the frequency of absorption of n photons by electrons having energy ϵ and subsequent scattering into angle θ. We can write

$$a_n(\epsilon, \theta) = N_e\sigma_{+n}(\epsilon, \theta)(2\epsilon/m)^{1/2} \tag{38}$$

where N_e is the number density of scattering centres. In order to find the frequency $b_n(\epsilon, \theta)$ of the inverse process of induced emission of n photons, the principles of detailed balancing can be used:

$$b_n(\epsilon + n\hbar\omega) = [\epsilon/(\epsilon + n\hbar\omega)]^{1/2} a_n(\epsilon). \tag{39}$$

The absorption coefficient can now be expressed in terms of the frequencies a_n and b_n in the following way:

$$\alpha = I^{-1}N_e \sum_{n>0}\left(\int_0^\infty n\hbar\omega\langle a_n(\epsilon)\rangle_{av} f(\epsilon)\, d\epsilon - \int_{n\hbar\omega}^\infty n\hbar\omega\langle b_n(\epsilon)\rangle_{av} f(\epsilon)\, d\epsilon\right) \tag{40}$$

where $I = eE^2/8\pi$ is the average radiation-flux density, $f(\epsilon)$ is the electron-energy distribution function and

$$\begin{aligned}\langle a_n(\epsilon)\rangle_{av} &= 2\pi\int_0^\pi a_n(\epsilon, \theta)\sin\theta\, d\theta\\ \langle b_n(\epsilon)\rangle_{av} &= 2\pi\int_0^\pi b_n(\epsilon, \theta)\sin\theta\, d\theta.\end{aligned} \tag{41}$$

According to equation (39), expression (40) for α can be written in the form

$$\alpha = I^{-1}N_e\sum_{n>0}\int_0^\infty n\hbar\omega\langle a_n(\epsilon)\rangle_{av}\left[f(\epsilon) - \left(\frac{\epsilon}{\epsilon + n\hbar\omega}\right)^{1/2} f(\epsilon + n\hbar\omega)\right] d\epsilon. \tag{42}$$

Let us expand the expression in the square brackets in the last formula in powers of $n\hbar\omega/\epsilon$ and leave only the first non-zero term in this expansion. (This approximation is justified in the case of the 'hot' electrons.) Then

$$\alpha = -I^{-1}N_e \int_0^\infty \epsilon^{1/2} \frac{d}{d\epsilon}(\epsilon^{-1/2} f(\epsilon)) \Big(\sum_{n>0} (n\hbar\omega)^2 \langle a_n(\epsilon)\rangle_{av} \Big) \, d\epsilon. \quad (43)$$

We shall consider now the sum which appears in the last integral. Using the cross sections σ_n, defined by equation (33), we get

$$\sum_{n>0} (n\hbar\omega)^2 \langle a_n(\epsilon)\rangle_{av} = \left(\frac{2\epsilon}{m}\right)^{1/2} 2\pi \int_0^\pi \sum_{n>0} (n\hbar\omega)^2 \frac{\exp(-n^2/2N)}{(2\pi N)^{1/2}} \sigma_{el}(\epsilon, \theta) \sin\theta \, d\theta. \quad (44)$$

Then in the classical limit, when $\hbar\omega \to 0$, we have

$$\sum_{n>0} (n\hbar\omega)^2 \langle a_n(\epsilon)\rangle_{av} \to \frac{1}{2}\left(\frac{2\epsilon}{m}\right)^{1/2} 2\pi \int_0^\pi (\hbar\omega)^2 N(\epsilon, \theta) \sigma_{el}(\epsilon, \theta) \sin\theta \, d\theta. \quad (45)$$

Substituting this expression into equation (43) and using the equation (37), the following final result for α is obtained

$$\alpha = -\frac{\pi^2 c^2}{\omega^2} \int_0^\infty \chi_\omega(\epsilon) \epsilon^{1/2} \frac{d}{d\epsilon}(\epsilon^{-1/2} f(\epsilon)) \, d\epsilon \quad (46)$$

where

$$\chi_\omega(\epsilon) = 2\pi \left(\frac{2\epsilon}{m}\right)^{1/2} \int_0^\pi \frac{d\mathscr{E}_\omega(\epsilon, \theta)}{d\omega} \sigma_{el}(\epsilon, \theta) \sin\theta \, d\theta \quad (47)$$

is the classical coefficient for bremsstrahlung radiation. Expression (46) is the well known classical formula for the absorption coefficient in an isotropic plasma (Bekefi 1966, p 54). Thus, the multiphoton approach used in this paper leads in the classical limit to correct classical results. It should be noticed that, as is well known (Bekefi 1966, p 87) in the case of a Maxwellian plasma, the Coulomb logarithm appears naturally in the expression for α as a result of integration in equation (46) and no cut-off is needed. This difference between our results and the ones derived by Seely and Harris (1973) is due to a more exact consideration of the induced transitions for small scattering angles, when $\omega\tau$ is greater than about one.

Acknowledgment

The author is greatly indebted to Professors J L Hirshfield, P Avivi and F Dothan for their helpful comments and suggestions in the preparation of this paper.

References

Abramowitz M and Stegun I A 1964 *Handbook of Mathematical Functions* (Washington, DC: US Govt Printing Office) p 364

Bekefi G 1966 *Radiation Processes in Plasma* (New York: Wiley)

Bunkin F V and Fedorov M V 1966 *Sov. Phys.-JETP* **22** 844–7

Friedland L 1975 *Proc. 12th Int. Conf. on Phenomena in Ionised Gases* (Eindhoven: University of Technology) p 344

Ireland C L M 1974 *J. Phys. D: Appl. Phys.* **7** L179–83

Meyerand R G and Haight A F 1963 *Phys. Rev. Lett.* **11** 401–6

Seely J F and Harris E G 1973 *Phys. Rev.* **7** 1064–7

Silin V P 1965 *Sov. Phys.-JETP* **20** 1510–6

Zeldovich J B and Raizer U P 1965 *Sov. Phys.-JETP* **20** 772–80