Erratum: Parametric amplification in Josephson junction embedded transmission lines [Phys. Rev. B 87, 144301 (2013)]

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In our derivation of Eq. (A13) in our original paper, there was an error of a factor of 2.[1] This error affected our subsequent analysis, resulting in a wrong definition of the parameter β in our original paper after Eq. (39). The correct expression should be

$$\beta = \frac{\mu}{2} B_{p,0}^2. \tag{1}$$

This correction yields the following modifications in the paper. In particular, an exponential scaling of the gain with length is incorrect and a quadratic dependence is expected. A design modification to engineer the dispersion relation may yield an exponential scaling of the gain when phase matching conditions are met; such work lies outside the scope of the design presented in the original manuscript and is not discussed here. We substitute Eq. (42) into the gain formula, Eq. (51), in our original paper,

$$G_s = \left| \cos(\sqrt{\Delta_p} x) + i \frac{\beta^2 + \sigma^2}{\beta^2 - \sigma^2} \sin(\sqrt{\Delta_p} x) \right|^2, \tag{2}$$

where, by definition [Eqs. (43) and (47) in our original paper],

$$\Delta_p = \left(\frac{\alpha_s + \alpha_i}{2} \right)^2 - \beta^2 \tag{3}$$

and

$$\sigma^2 = \left(\frac{\alpha_s + \alpha_i}{2} \right)^2 + \Delta_p + (\alpha_s + \alpha_i)\sqrt{\Delta_p}. \tag{4}$$

Then, assuming $\Delta_p \geqslant 0$ (to be seen later),

$$G_s = 1 + \frac{4\beta^2 \sigma^2}{(\beta^2 - \sigma^2)^2} \sin^2(\sqrt{\Delta_p} x). \tag{5}$$

Here, as will be seen later, the corrected value of β changes the sign of Δ_p and makes it non-negative. Consequently, the gain factor becomes quasiquadratic instead of asymptotically exponential. Consequently, Fig. 4 in our original paper should be corrected and replaced by Fig. 1 presented here, in which the signal gain is calculated using Eqs. (1), (3), and (5). All the parameters of the computation are the same as in the first paragraph of page 5 in our original paper except the range of the signal frequencies.

In the following, we derive an approximate expression for G_s, assuming that the signal frequency ω_s mismatch $\Delta\omega = \omega_s - \omega_p$ is small, i.e., $|\Delta\omega|/\omega_p \ll 1$. We focus on the case when $\sqrt{\Delta_p} \ll |\alpha_s + \alpha_i|/2$ (we will see later that $\Delta_p \to 0$ if $|\Delta\omega| \to 0$). Equations (3) and (4) in this case yield

$$\sigma^2 \approx \frac{(\alpha_s + \alpha_i)^2}{4}, \tag{6}$$

and

$$(\beta^2 - \sigma^2)^2 \approx (\alpha_s + \alpha_i)^2 \, \Delta_p. \tag{7}$$

Substituting Eqs. (6) and (7) into Eq. (5), we obtain the following approximation for the gain:

$$G_s = 1 + \left[\tfrac{3}{8}\gamma \tilde{k}_p (\tilde{k}_s \tilde{k}_i)^{1/2} B_{p,0}^2\right]^2 x^2 \,\mathrm{sinc}^2(\sqrt{\Delta_p} x). \tag{8}$$

Next, we write Eq. (3) for Δ_p explicitly, using Eq. (1) and the definitions of α_s and α_i in our original paper, and neglect the term with $\Delta\tilde{k}/\tilde{k}_p \ll 1$:

$$\begin{aligned} \Delta_p &\approx [H + \Delta k/2]^2 - H^2 \widetilde{k}_s \widetilde{k}_i \widetilde{k}_p^{-2} \\ &\approx H^2\big(1 - \widetilde{k}_s \widetilde{k}_i \widetilde{k}_p^{-2}\big) + H \Delta k, \end{aligned} \tag{9}$$

where $H \equiv 3\gamma \widetilde{k}_p^2 B_{p,0}^2/8$. Here, by definition in our original paper, $\widetilde{k}_m = \sqrt{\rho} F(\omega_m)$, where $F(\omega_m) \equiv \omega_m (1 - \omega_m^2)^{-3/2}$. Consequently, we can write

$$\begin{aligned} 1 - \widetilde{k}_s \widetilde{k}_i \widetilde{k}_p^{-2} &= 1 - F(\omega_p + \Delta\omega) F(\omega_p - \Delta\omega) F^{-2}(\omega_p) \\ &= F^{-2}[(F')^2 - F F''](\Delta\omega)^2 + O[(\Delta\omega)^3], \end{aligned}$$

where F, F', and F'' are evaluated at ω_p. If the system is weakly dispersive ($\omega_p \ll 1$), then $F = \omega_p + \text{h.o.}$, $F' = 1 + \text{h.o.}$, $F'' = 9\omega_p + \text{h.o.}$, where "h.o." stands for higher order terms of ω_p. Hence, $1 - \widetilde{k}_s \widetilde{k}_i \widetilde{k}_p^{-2} \approx (\Delta\omega/\omega_p)^2$ and

$$\begin{aligned} \Delta_p &\approx H^2 (\Delta\omega/\omega_p)^2 + H \Delta k \\ &\approx \big(H^2 \omega_p^{-2} + 3k_p H\big)(\Delta\omega)^2 \geqslant 0, \end{aligned} \tag{10}$$

where we substituted $\Delta k \approx 3k_p(\Delta\omega)^2$ [Eq. (A32) from our original paper]. Within the same weak dispersion approximation, we can replace $\tilde{k}_m$ by k_m to rewrite Eq. (8) as

$$G_s = 1 + H^2 \frac{k_s k_i}{k_p^2} x^2 \,\mathrm{sinc}^2(\sqrt{\Delta_p} x), \tag{11}$$

where $\mathrm{sinc}(\sqrt{\Delta_p} x) \equiv \sin(\sqrt{\Delta_p} x)/(\sqrt{\Delta_p} x)$. Thus, the maximal gain scales as $G_s \sim B_{p,0}^4 x^2$, and the bandwidth in which the gain is about 40% of the local maximum gain ($\sqrt{\Delta_p} x = \pi/2$) is obtained at

$$(\Delta\omega)_{0.4} = \frac{\pi}{2x\big(H^2 \omega_p^{-2} + 3k_p H\big)^{1/2}}. \tag{12}$$

In order to simplify the practical implementation of our theoretical study to the design of experiments and their interpretation, we express our final equations in terms of the original circuit design parameters. The signal gain $G_s = |\tilde{A}_p/\tilde{A}_{p,0}|^2$ yields the following form:

$$G_s \approx 1 + H^2 \frac{f_s f_i}{f_p^2} x^2 \,\mathrm{sinc}^2(\sqrt{\Delta_p} x), \tag{13}$$

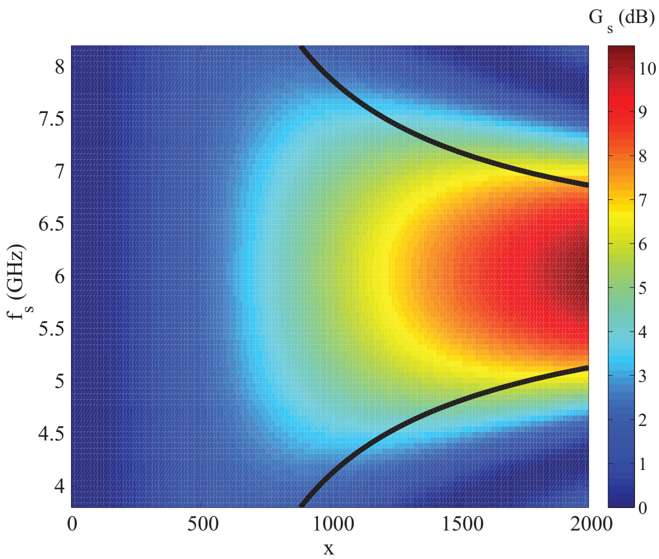

FIG. 1. (Color online) Signal power gain as a function of signal frequency and position, given by the analytical expressions Eqs. (3) and (5). All the parameters are the same as in the first paragraph of page 5 in our original paper, except from the varying signal frequency. The black line is located where the gain level is 3 dB below the local peak.

where

$$H \approx \frac{\pi^3 f_p^3 C^{3/2} |\tilde{A}_{p,0}|^2}{2\varphi_0^{1/2} I_J^{3/2}}, \tag{14}$$

$$\Delta_p \approx \left[\frac{H^2}{f_p^2} + 24\pi^3 \left(\frac{\varphi_0}{I_J}\right)^{3/2} C_J \sqrt{C} f_p H\right] (\Delta f)^2, \tag{15}$$

and $\Delta f = f_s - f_p$. The bandwidth in which the gain is about 40% of the local maximal gain is

$$(\Delta f)_{0.4} = \frac{\pi}{2x\left[\frac{H^2}{f_p^2} + 24\pi^3 \left(\frac{\varphi_0}{I_J}\right)^{3/2} C_J \sqrt{C} f_p H\right]^{1/2}}. \tag{16}$$

Finally, we would like to note that the following corrections should be made in the equations that appear in our original paper: (a) The parameter μ should be replaced by $\mu/2$ in Eqs. (29), (30), (33), (34), (36), (37), (A20), (A21), (A25), (A26), (A28), and (A29). (b) The terms that are proportional to $\exp(-i\Psi)$ should be divided by 2 in Eqs. (A13), (A14), (A16), and (A17). (c) The prefactor of the $\cos\Theta$ term in Eq. (A31) should be replaced by $\mu B_p^2 B_s B_i [(1/2)B_s^{-2} + (1/2)B_s^{-2} - 2B_p^{-2}]$. (d) The part of our original paper that starts with the words "The amplification bandwidth ..." after Eq. (51) until the end of Sec. III, including the numerical results that are presented in Figs. 2–6, is invalid.

Although the quadratic scaling of the gain with device length results in diminished amplifier performance for a given number of unit cells (compared to the performance predicted in our original paper), very practical and interesting devices can still be designed using a somewhat longer transmission line and a moderately stronger pumping condition. We have chosen to show the corrected plots for the same device parameters in this Erratum as in our original work to maintain clarity. With 4000 unit cells and a pump current just 16% larger than that used to produce Fig. 1, a peak gain of 18 dB is achieved at 6 GHz with a full bandwidth of 800 MHz.

This performance represents an improvement in bandwidth of more than an order of magnitude compared to state-of-the-art superconducting parametric amplifiers. Moreover, this level of pump power is still orders of magnitude smaller than that required for traveling wave amplifiers based on the weaker nonlinearity associated with the kinetic inductance of superconducting films. The combination of large gain and bandwidth coupled with the modest pump wave power required for amplification make this device exceptionally promising for experiments in quantum information and quantum optics at microwave frequencies.

We would like to thank S. Chaudhuri and J. Gao for drawing our attention to the error of factor 2 that fell in our derivation of Eq. (A13) in our original paper.

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[1]S. Chaudhuri and J. Gao (private communication).