Autoresonant Phase-Space Holes in Plasmas

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Electron phase-space holes are formed and controlled in a plasma by adiabatic nonlinear phase locking (autoresonance) with a chirped frequency driving wave. The process has a threshold on the driving amplitude and involves dragging a void region in phase space into the bulk of the distribution via persistent Cherenkov-type resonance.

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Waves in collisionless plasmas are frequently Landau damped [1] via Cherenkov-type resonant particles, having velocities $\mathbf{v}$ such that $\mathbf{k} \cdot \mathbf{v} \approx \omega$ ($\mathbf{k}$ and ω being the wave vector and frequency of the wave). Nevertheless, nearly 50 years ago, Bernstein, Green, and Kruskal [2] discovered a class of *dissipationless* nonlinear waves (BGK modes) in collisionless plasmas. Resonant particles played a major role in BGK modes as well, requiring delicate shaping of the velocity distribution in resonant phase-space regions, making implementation of pure BGK modes difficult. Following the prediction that Landau damped waves evolve into a BGK equilibrium [3], a natural path to excitation of BGK waves used a strong perturbation-relaxation approach [4–6]. Alternatively, BGK modes may emerge due to instabilities [7,8]. This *passive* self-organization is sensitive to initial conditions and may involve a violent stage, leading to poor controllability of the excited BGK structures.

On the other hand, autoresonance is a relatively recent approach to *active* formation and control of nonlinear waves [9–11]. It involves an external wave passing through a resonance in the system, resulting in a continuing nonlinear phase locking (autoresonance) and subsequent excitation of a large amplitude wave controlled by variation of external parameters. Can we excite BGK modes in plasmas by autoresonance? Recently a similar question was addressed in application to magnetized electron clouds in a Penning trap [12,13]. BGK-type waveforms were created in this system by passage through electron bounce resonance in the trap. In the present work we apply similar ideas to formation of autoresonant electron phase-space holes in a plasma using Cherenkov-type resonance. Studying *free* (undriven) phase-space hole equilibria in plasmas is a subject of active research [14]. In earlier studies, these equilibria emerged in simulations by using special initial conditions or as the result of relaxation of two-stream instability [7]. This Letter describes an autoresonant approach to the formation and control of phase-space holes in plasmas and, for the first time, analyzes dynamics of the whole excitation process from the initial fluid-type phase locking in the driven system to the emergence of an autoresonant BGK mode.

We proceed by illustrating formation of an autoresonant electron hole in phase space in simulations. Consider a one-dimensional distribution of plasma electrons driven by electrostatic wave $E_d = \varepsilon \sin\Psi_d$ of amplitude ε, phase $\Psi_d = kx - \psi(t)$, having wave vector $k = 2\pi/L$ and *chirped* frequency $\psi_t = \omega_0 - \alpha t \equiv \omega_d(t)$, $\alpha =$ const. Such a wave may represent a ponderomotive force due to counterpropagating chirped frequency laser beams. We shall assume uniform *initial* electron density n_0 and an immobile, neutralizing ion background. The electron density is perturbed by E_d, creating self-field $E(x, t)$ at later times. We model our plasma by the one-dimensional, L-periodic Vlasov-Poisson system

$$f_t + uf_x - (E + E_d)f_u = 0, \qquad E_x = 1 - \int f du, \quad (1)$$

where $f(u, x, t)$ is the electron distribution function. We use dimensionless variables in Eq. (1), i.e., express u, x, and t in units of thermal velocity $u_{\rm th}$, Debye length $\lambda_D = u_{\rm th}/\omega_p$, and inverse plasma frequency ω_p, respectively. The distribution function f is rescaled by $n_0/u_{\rm th}$, while dimensionless fields are in units of $mu_{\rm th}\omega_p/e$. We have solved our system numerically by using a standard pseudospectral method [15] in the case of spatially uniform flattop initial distribution, $f \approx 1/2$ or 0 for $|u| < 1$ or $|u| > 1$, respectively. This distribution was modeled via $f(u, x, 0) = C\exp(-u^{2n}) \equiv f_0(u)$, where $n \gg 1$ ($n = 250$ in simulations below) and $C \approx 1/2$ is the normalization constant. For avoiding numerical difficulties characteristic of Vlasov codes for distributions having large phase-space gradients, we have introduced artificial high frequency filters at grid scales in x and u. The accuracy of our code was tested by both comparing its results with those of the Lagrangian code [13] neglecting the self-field and, with the self-field, by varying the number of spatial harmonics and discretization steps in u. We used parameters $\varepsilon = 0.01$, $\alpha = 0.01$, and $k = 3$ as an example and the initial phase velocity $u_d(0) = \omega_d(0)/k = 1.3$ was outside the initial electron distribution. Therefore, only a negligible fraction of initial electrons experienced Cherenkov resonance. The emergence of a phase-space hole in this example is illustrated in Fig. 1, showing electron distribu-

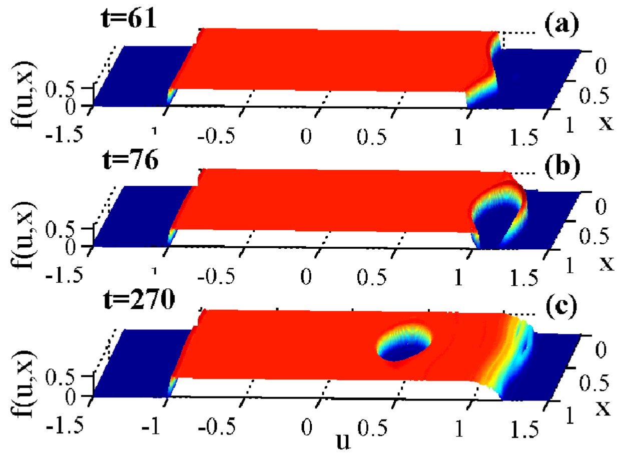

FIG. 1 (color online). The formation of autoresonant electron phase-space hole: (a) a surface wave in the phase-space fluid, (b) emergence of a void, and (c) a fully developed autoresonant hole.

tions at different times: a wave on the surface of the phase fluid at $t = 61$ [Fig. 1(a)] in the initial excitation stage, transition through the phase-fluid boundary at $t = 76$ [Fig. 1(b)], and a fully developed, phase-locked electron hole at $t = 270$ [Fig. 1(c)]. The evolution of the amplitudes A_i ($i = 1, 2, 3$) of the first three spatial harmonics of the self-field and the phase mismatch Φ between the first harmonic and the driving wave are shown in Fig. 2 by solid lines, while the dashed lines represent our theory below. Note that the self-field of the electron hole is mainly composed of the first spatial harmonic. The initial phase-locking stage ($t < 100$) in Fig. 2 is followed by adiabatic autoresonant stage, where the phase mismatch Φ remains bounded, indicating continuing phase locking in the system as the hole passes region $|u| < 1$ in phase space. When the driving phase velocity $u_d(t)$ is comparable to the ion acoustic speed, one may expect excitation of the acoustic waves. This effect was neglected assuming fast passage though the ion acoustic region, i.e., (dimensional) $\alpha \gg (ku^i_{\rm th})^2$, $u^i_{\rm th}$ being the ion thermal velocity.

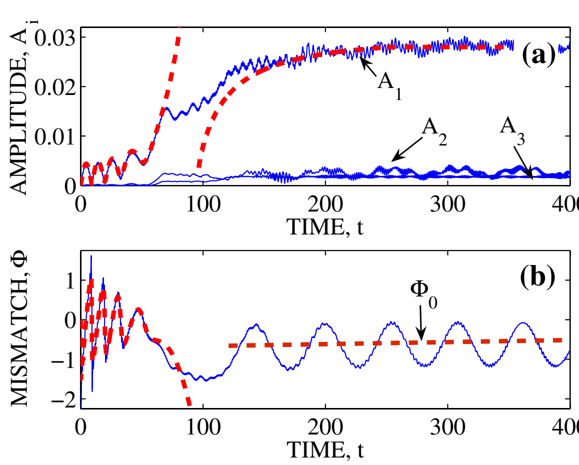

FIG. 2 (color online). The evolution of autoresonant BGK mode, simulations (solid lines) and theory (dashed lines). (a) The amplitudes A_i of the first three self-field harmonics and (b) the phase mismatch of the driven BGK mode versus time.

Our theory of autoresonant holes proceeds from a simplified physical picture of manipulation of the electron phase space. Assume that the self-field E is sufficiently small and one can neglect its effect on electrons. Electron trajectories in this approximation are governed by $x_{tt} = -\varepsilon\sin[kx(t) - \psi(t)]$. The phase $\theta(t) = kx(t) - \psi(t)$ of the driving field along a trajectory satisfies $\theta_{tt} = -\varepsilon_0 \sin\theta + \alpha$, $\varepsilon_0 = \varepsilon k$. This equation describes a quasiparticle in a *tilted* cosine potential $V_{\rm eff} = -\varepsilon_0\cos\theta - \alpha\theta$ (see Fig. 3). For $\varepsilon_0 > \alpha$, the potential has minima at periodically spaced $\theta = \theta_m$ satisfying $\sin\theta_m = \alpha/\varepsilon_0$. Then there exist trapped regions of widths $\delta\theta_t \approx 4\varepsilon_0^{1/2}$, $\delta\theta \approx 2\pi$ in (θ_t, θ) space, where both θ and θ_t are bounded. Since $\theta_t = kx_t - \omega_d(t)$, *in average*, all initially trapped electrons move with the phase velocity of the wave $x_t \approx u_d(t)$, *despite* variation of the driving frequency. In other words, the trapped region (even if empty) is dragged through phase space, staying in a continuing Cherenkov autoresonance. The autoresonance phenomenon has been known since the early days of particle accelerators, where the persistent phase locking of particles trapped by oscillations with slow parameters was referred to as the phase stability principle [16]. In this Letter we propose to use autoresonance for slowly dragging a low density region in phase space into the bulk of the distribution, forming a phase-space electron hole and the associated BGK mode. Note that the general picture of manipulating phase-locked particles in phase space remains valid if the driving amplitude ε and/or the driving frequency chirp rate α are *slow* functions of time, provided the trapped electrons are sufficiently far from the separatrix in (θ_t, θ) space. In this context, inclusion of the adiabatically growing but *phase-locked* self-field can be viewed as adding a slow variation of the amplitude of the driving field (see below) without destroying the overall picture of dy-

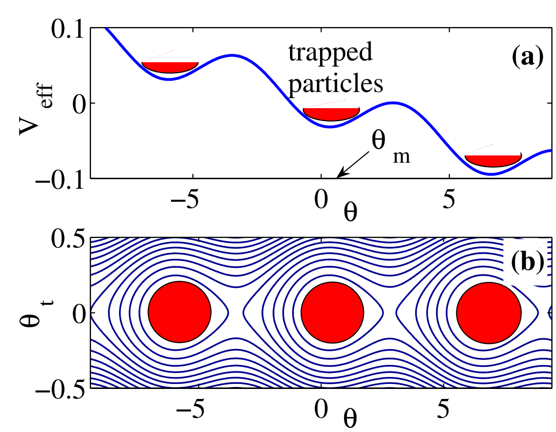

FIG. 3 (color online). Driven electron dynamics: (a) the effective potential; (b) the phase-space portrait of dynamics.

namics. We discuss self-consistent development of autoresonant BGK modes next.

We consider the initial phase locking and developed autoresonant stages of interaction separately. Since $u_d(0)$ is outside the bulk of the electron distribution, we neglect resonant particles in the initial excitation stage. We write the Maxwell equation (dimensionless) $E_t + J = 0$ for the electrostatic self-field in the problem (J being the current density in units of $en_0u_{\rm th}$) and seek eikonal-type solution $E = {\rm Re}[a(t)\exp(i\Psi)]$ and, similarly, $J = {\rm Re}[j(t) \times \exp(i\Psi)]$, where $\Psi = kx - \Theta(t)$, while a, j, and frequency $\omega = \Theta_t$ are slow functions of time. Then

$$-i\omega a + a_t + j = 0. \tag{2}$$

Next, we write the eikonal form of Ohm's law [17]:

$$j = \sigma a' + i(\tfrac{1}{2}\sigma_{\omega t}a' + \sigma_\omega a'_t). \tag{3}$$

Here $\sigma(\omega, k, t)$ is the conductivity of the time independent plasma, having parameters as those in our time dependent system at time t, and $a' = a + \varepsilon\exp(-i\tilde{\Phi})$ includes phase-shifted (by $\tilde{\Phi} = \Psi - \Psi_d - \pi/2$) driving wave. We use linearized, nonresonant kinetics [18] to write $\sigma = -i\int\frac{f_{0u}u du}{\omega - ku}$, yielding $\sigma \approx i\omega/(\omega^2 - k^2)$ for our flattop distribution. Now, we define ω via the lowest order (with respect to ε and ∂_t) part of Eq. (2), i.e., $(-i\omega + \sigma)a = 0$, yielding linear dispersion of plasma waves, $\omega^2 = 1 + k^2$. To first order in Eq. (2), $[1 + i\partial\sigma/\partial\omega]a_t + \varepsilon\sigma(-i\tilde{\Phi}) = 0$, and by defining $a = A\exp(i\eta)$, ${\rm Im}(A, \eta) = 0$, we obtain

$$A_t = -\mu\sin\Phi, \qquad \Phi_t = \omega_d - \omega - (\mu/A)\cos\Phi, \tag{4}$$

where $\mu = \varepsilon/(2\omega)$ and $\Phi = \tilde{\Phi} + \eta$ is the phase mismatch between the self-field and the drive. For $\omega_d = \omega - \alpha t$, this system can be written as a single complex equation for $Z = A\exp(i\Phi)$, i.e., $iZ_t - \alpha t Z = \mu$, which has a solution in terms of Fresnel integrals [9]. This solution (dashed lines in Fig. 2 for $t < 100$) is in an excellent agreement with simulations and shows efficient trapping of Φ near zero as the driving frequency ω_d approaches ω (at $t = 74$ in the figure), provided one starts sufficiently far from the resonance. As ω_d passes below ω and approaches Cherenkov resonance $\omega_d = k$ at edge $u = 1$ of the electron distribution, the system enters a relatively short transition stage, where a "jet" of phase fluid from the edge moving along the separatrix [see Fig. 1(b)] encircles a low density region of trapped trajectories, creating a drifting void (hole) in phase space. Later, for driving amplitudes ε exceeding threshold $\varepsilon_{\rm th}$, the hole enters the bulk of the distribution [Fig. 1(c)], while the associated self-field remains phase locked [Φ remains bounded, as in Fig. 2(b)] with the drive continuously. At the same time, as the hole drifts through the electron distribution in u space, the self-field amplitude increases [Fig. 2(a)], reaches a maximum, and then decreases and nearly disappears (not shown in the figure) as the hole leaves the plasma through $u = -1$ boundary of the phase-space fluid. In contrast, below $\varepsilon_{\rm th}$, the phase locking discontinues and the self-field decouples from the drive. We have found $\varepsilon_{\rm th}$ in simulations for different k and α, the result being $\varepsilon_{\rm th} \approx 2.7\omega^{-2}\alpha^{3/4}$ for the whole range of simulation parameters ($2 \le k \le 4$ and $1.25 \times 10^{-3} \le \alpha \le 2 \times 10^{-2}$) within $\pm 15\%$. The $3/4$ power scaling of $\varepsilon_{\rm th}$ with α is interesting because it is characteristic of many other autoresonant waves [11]. The dynamics of fully developed autoresonant holes ($|u_d(t)| < 1$) is discussed next.

To first approximation, we view the charge associated with our driven hole in x space as distributed symmetrically with respect to its center of mass, $x_0(t)$. Let normalized (to unity) distribution of this charge be $F(\xi)$, an *even* function of $\xi = x - x_0$. Any point inside the hole evolves via $x_{tt} = -\varepsilon\sin[kx(t) - \psi(t)] + G(\xi)$, where acceleration $G(\xi)$ due to the self-field is an *odd* function of ξ. By averaging over the charge distribution in the last equation we have, $(x_0)_{tt} = -\varepsilon\int F(\xi)\sin[kx(t) - \psi(t)]d\xi = \tilde{\varepsilon}\sin\Phi$, where $\tilde{\varepsilon} = \varepsilon\int F(\xi)\cos(k\xi)d\xi$ and $\Phi = \psi(t) - kx_0(t)$ is the phase mismatch described by

$$\Phi_{tt} = -k\tilde{\varepsilon}\sin\Phi - \alpha. \tag{5}$$

Note that Eq. (5) yields a quasisteady state, $k\tilde{\varepsilon}\sin\Phi_0 = -\alpha$ or, for sufficiently small α, $\Phi_0 \approx -\alpha/(k\tilde{\varepsilon})$. Let $\Phi = \Phi_0 + \delta\Phi$. Then $\delta\Phi_{tt} \approx -k\tilde{\varepsilon}\sin(\delta\Phi)$, predicting oscillations of Φ around Φ_0 with characteristic frequency $\nu \approx (k\tilde{\varepsilon})^{1/2}$. These predictions agree with our simulations, particularly the relation $\Phi_0\nu^2 = -\alpha$ [$\Phi_0 \approx 0.65$, $\nu \approx 0.12$, as measured in Fig. 2(b)]. Next, by definition,

$$\Phi_t = \omega_d(t) - \Omega, \tag{6}$$

where Ω is the frequency of the self-field. Then, using the lowest order nonlinear dispersion relation, $\Omega = \Omega(A, k)$ (see below), we obtain $\Phi_{tt} = -(\partial\Omega/\partial A)A_t - \alpha$, and comparing to Eq. (5),

$$A_t = k\tilde{\varepsilon}(\partial\Omega/\partial A)^{-1}\sin\Phi. \tag{7}$$

Equations (6) and (7) comprise a complete system for amplitude A and phase $\Psi = \Psi_d + \Phi$ of the driven wave. Similarly to other autoresonant waves [10], this system predicts stable solution of form $A = A_0 + \delta A$, $\Phi = \Phi_0 + \delta\Phi$, where Φ_0 is the quasisteady state discussed above, $A_0(t)$ satisfies *autoresonance* relation $\Omega(A_0, k) = \omega_d(t)$, while $\delta\Phi$, δA are modulations oscillating at frequency ν.

Next, we seek the dispersion relation $\Omega(A, k)$. We assume *dominant* first harmonic in the self-field and use the quasistatic approximation in Eq. (2), i.e., $-i\Omega A + j = 0$, where the current density $j = \sigma A + j_h$ includes the first harmonic j_h of the self-current associated with the hole. As a simple model, we approximate the hole by an *elliptical* electron density void centered at x_0 and $u_0 = \Omega/k$ in phase space (b, and c being the semimajor axis of the ellipse in x and u). In this case, $j_h = u_0L^{-1}\int_{-L/2}^{+L/2}\exp(-ik\xi)d\xi \times \int f_h(\xi, v)dv$, where $f_h(\xi, v) = 1/2$ (*ion* charge distribution) inside the hole and zero elsewhere. This yields

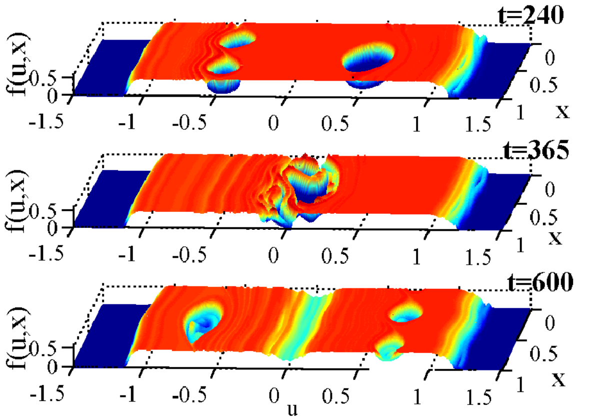

FIG. 4 (color online). Collision of two autoresonant BGK structures.

$j_h = (\Omega c/k)J_1(kb)$ (J_1 is the Bessel function of the first kind) and, for the same σ as above, the following dispersion relation

$$(k^2 - \Omega^2)^{-1} = (c/kA)J_1(kb) - 1. \quad (8)$$

On the other hand, b and c can be related to A. Indeed, under adiabatic conditions, the area $I_0 = \pi bc$ of the ellipse is conserved, while $c/b = \gamma$, where $\gamma \approx (kA)^{1/2}$ is the characteristic frequency of oscillations of the electrons trapped in the self-field. Also, for the elliptical hole, $\tilde{\varepsilon} = 2\varepsilon J_1(kb)/kb$. By replacing Ω by $\omega_d(t)$ in Eq. (8), we can calculate the slow amplitude $A_0(t)$. In comparing this theory with simulations, we measured $b = 0.55$ and $c = 0.18$ in our example in Figs. 1 and 2 at $t = 400$, calculated $A_0(t)$, and showed the result in Fig. 2(a) (dashed line for $t > 100$). Finally, frequency 0.12 of autoresonant modulations of Φ in simulations in Fig. 2 is also in an excellent agreement with theoretical value $(k\tilde{\varepsilon})^{1/2} = 0.15$ in our example, reduced by 20% to account for the finite amplitude of modulations and the tilt of the effective trapping potential.

We conclude this Letter by the following observations. First, the assumed linear chirp of the driving frequency is not a necessary condition for autoresonance in the system. Indeed, any sufficiently slow $[\alpha(t) < k\tilde{\varepsilon}]$ variation of ω_d yields continuing phase locking in the system. For instance, autoresonant holes can be moved in and out the bulk of the distribution by simply changing the direction of variation of the driving frequency. Second, autoresonant holes can be used as efficient current drives. Indeed, one can see in Fig. 1(c) that only the $u = 1$ boundary of the electron distribution is significantly perturbed and moved to higher velocities. This means asymmetry of the associated velocity distribution, i.e., emergence of a current. Third, more complex autoresonant BGK modes can be formed using a superposition of driving waves. For example, Fig. 4 shows collision of two BGK structures. The second (two-hole) structure in Fig. 4(a) was formed at the $u = -1$ boundary of the phase fluid by adding a new driving wave $E'_d = \varepsilon' \sin\Psi'_d$ on second harmonic ($\varepsilon' = 2\varepsilon$ and $\Psi'_d = -2[kx + \psi(t)]$) in the example in Fig. 1. Note that E'_d has a negative phase velocity and, therefore, the two driven structures move in opposite directions in u space. After the collision [Fig. 4(b)] one sees reemergence of counterpropagating phase-space structures (partially filled holes) in Fig. 4(c), each locked to its drive. Also, a nearly uniform depression in $f(u, x)$ at $u \approx 0$ is formed after the collision to conserve the total volume of the phase fluid. To some extent, driven BGK structures in this example exhibit properties of colliding particles.

In summary, we have suggested autoresonant approach to formation and control of coherent electron phase-space structures (BGK modes) in plasmas by chirped frequency waves and, for the first time, analyzed dynamics of the whole excitation process for a simple initial phase-space distribution. Inclusion of the self-consistent evolution of the driving wave and higher dimensionality in the model, as well as practical realizations seem to comprise interesting goals for future research.

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