Robust Autoresonant Excitation in the Plasma Beat-Wave Accelerator

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A modified version of the plasma beat-wave accelerator scheme is proposed, based on autoresonant phase locking of the Langmuir wave to the slowly chirped beat frequency of the driving lasers by passage through resonance. Peak electric fields above standard detuning limits seem readily attainable, and the plasma wave excitation is robust to large variations in plasma density or chirp rate. This scheme might be implemented in existing chirped pulse amplification or CO_2 laser systems.

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The plasma beat-wave accelerator (PBWA) was first proposed by Tajima and Dawson [1] and has been subsequently studied by many groups [2–9] (for a review, see [10]). In the basic scheme, two lasers copropagating in plasma are detuned from each other by the electron plasma frequency $\omega_p \equiv (4\pi n_0 e^2/m)^{1/2}$. The beating lasers act ponderomotively on the plasma to resonantly excite a large-amplitude, high-phase-velocity plasma wave suitable for particle acceleration. The primary advantage of the PBWA over conventional accelerators is the ability of plasma to sustain accelerating fields far in excess of metallic structure breakdown limits.

In the original PBWA, the plasma wave is driven with a fixed beat frequency. In this case, relativistic detuning limits the longitudinal field E_z to the Rosenbluth-Liu (RL) limit [11]: $E_z \le E_{\rm RL} = E_0[\frac{16}{3}\frac{\omega_p^2}{\omega_1\omega_2}(E_1E_2)/E_0^2]^{1/3}$, where $E_0 \equiv mc\omega_p/e$ is the cold, nonrelativistic wave-breaking field, and $\omega_{1,2}$ and $E_{1,2}$ are the frequency and peak electric field of the two drive lasers. Deutsch, Meerson, and Golub [12] (DMG) proposed to overcome this detuning effect by chirping the lasers to compensate for the change in the nonlinear plasma frequency.

We propose a novel variant of the chirped PBWA, exploiting a robust transition to nonlinear phase locking (autoresonance) by adiabatic passage through resonance [13]. Rather than starting on resonance (as did DMG), we chirp from above resonance, slowly sweeping the beat frequency through resonance and below. Under certain conditions (which we calculate), the plasma wave frequency locks to that of the drive. Since the nonlinear Langmuir frequency is a function of energy, frequency locking implies that changing the beat frequency correspondingly changes the plasma wave amplitude. Our excitation scheme is relatively insensitive to the exact chirp history and precise plasma characteristics and is discussed further in [14]. Furthermore, while the previous analysis of RL and DMG use expansions assuming weakly relativistic electron motion, we employ a reduced but fully nonlinear Eulerian fluid model [5,15,16], thereby accounting for arbitrarily relativistic electron motion.

We model the plasma as a cold, fully relativistic electron fluid in a neutralizing, stationary ionic background and restrict our analysis to one dimension, where all dynamical quantities depend only on the longitudinal coordinate z and the time t. We define a dimensionless time $\tau \equiv \omega_p t$, comoving position $\xi \equiv \omega_p(t - z/v_p)$, longitudinal velocity $\beta \equiv v_z/c$, Langmuir phase velocity $\beta_p \equiv v_p/c$, vector potential $\boldsymbol{a} = \boldsymbol{a}_\perp \equiv \frac{e}{mc^2}\boldsymbol{A}$, and electrostatic potential $\phi \equiv \frac{e}{mc^2}\Phi$. Our analysis is based on the quasistatic approximation, wherein we assume that the dynamics are independent of τ in the comoving frame, and the plasma wave evolves without dispersion. In this approximation, the electron continuity and longitudinal momentum equations can be integrated, yielding algebraic equations, and the Poisson equation becomes, in the underdense limit, $\beta_p \to 1$ [16],

$$\frac{\partial^2}{\partial\xi^2}\phi = \frac{1}{2}\left[\frac{1 + a^2}{(1+\phi)^2} - 1\right]. \qquad (1)$$

We ignore any change in the laser envelope, taking $\boldsymbol{a}(\xi, \tau)$ to be a prescribed function traveling at the group velocity $\bar{v}_g = v_p$. This approximation does impose some limitations on the laser parameters—pulse duration, spot size, and intensity—so that diffraction, self-focusing, and Raman instabilities, etc., do not become appreciable.

For circular polarization, the vector potential $\boldsymbol{a} = \frac{1}{2}[\hat{e}_+ a_1 e^{i\psi_1} + \hat{e}_+ a_2 e^{i\psi_2} + \text{c.c.}]$, with $\hat{e}_+ \equiv \frac{1}{2}(\hat{x} + i\hat{y})$. The laser frequencies $\omega_j(z,t) \equiv -\frac{\partial}{\partial t}\psi_j$ and wave numbers $k_j(x,t) \equiv \frac{\partial}{\partial z}\psi_j$ satisfy the dispersion relation $\omega^2 = c^2k^2 + \omega_p^2$. Allowing for a slow, weak frequency chirp, we define $\bar{\omega}_j \equiv \frac{1}{T}\int_0^T dt'\omega_j(t')$ to be the average carrier frequency and take our reference group velocity $\bar{v}_g$ to be the group velocity $v_g \equiv \frac{d}{dk}\omega(k)$ evaluated at the average frequency $\bar{\omega} \equiv \frac{1}{2}(\bar{\omega}_0 + \bar{\omega}_1)$. Using these definitions, linear laser propagation results in the beat phase

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$$\psi_2 - \psi_1 = \int_0^\xi dt'[\omega_2(t') - \omega_1(t')] + \mathcal{O}\left(\frac{\omega_p^3}{\bar{\omega}^3}\frac{\omega_p L}{c}\right). \quad (2)$$

The neglected terms limit the validity of the constant group velocity approximation to lengths L less than the dispersion length: $L \leq L_{\rm disp} \sim (\bar{\omega}/\omega_p)^3(c/\omega_p)$. Since the useful accelerating length is already limited by the dephasing length, which is a factor $\sim\bar{\omega}/\omega_p \gg 1$ shorter than $L_{\rm disp}$, this adds no additional constraints.

Defining the normalized beat frequency $\Delta\omega(\xi) = (\omega_2 - \omega_1)/\omega_p$, beat phase $\psi(\xi) = \psi_2 - \psi_1$, drive strength $\epsilon = a_1 a_2$, and average intensity $\bar{a}^2 = \frac{1}{2}(a_1^2 + a_2^2)$, the slow part of the ponderomotive drive may be written as a function of ξ only: $a^2(\xi) \approx [\bar{a}^2 + \epsilon\cos\psi(\xi)]$. Thus, the nonlinear response of the plasma is described by a second-order ordinary differential equation in ξ:

$$\frac{d^2}{d\xi^2}\phi = \frac{1}{2}\left[\frac{1 + \bar{a}^2 + \epsilon\cos\psi(\xi)}{(1+\phi)^2} - 1\right]. \quad (3)$$

To study autoresonance, we express (3) in canonical action-angle variables. We note that (3) is Hamiltonian, with $(\phi, p \equiv \frac{d}{d\xi}\phi)$ the canonical position-momentum conjugates, and make a canonical transformation to the action-angle variables of the free oscillator, $\phi = \phi(I, \theta)$, $p = p(I, \theta)$. Then, the transformed Hamiltonian becomes

$$\mathcal{H}(I, \theta; \xi) = \mathcal{H}_0(I) + \frac{\bar{a}^2 + \epsilon\cos\psi(\xi)}{2[1 + \phi(I, \theta)]}. \quad (4)$$

Here, $\mathcal{H}_0(I)$ describes the free nonlinear oscillator in the absence of forcing, and the action I is defined by the phase space area contained within an unperturbed orbit of energy H: $I \equiv \frac{1}{2\pi}\oint p d\phi$. An explicit calculation of $I(H)$ is given in [14]. The nonlinear frequency $\Omega(H)$ of the unforced oscillator is given by

$$\Omega(H) = \frac{\partial\mathcal{H}_0}{\partial I} = \frac{\pi}{2}\frac{[1 + H - \sqrt{H^2 + 2H}]^{1/2}}{E(\kappa)}. \quad (5)$$

$E(\kappa)$ is the complete elliptic integral of the second kind, and $\kappa = [2(1 + H)\sqrt{H^2 + 2H} - 2H(2 + H)]^{1/2}$. A similar expression to (5) was derived in [5], and its first order expansion in H agrees with the result of RL. Since the electrostatic potential $\phi(I, \theta)$ is a periodic function of θ, we express the drive term of (4) in a Fourier series:

$$\frac{\bar{a}^2 + \epsilon\cos\psi(\xi)}{2[1 + \phi(I, \theta)]} = [\bar{a}^2 + \epsilon\cos\psi(\xi)]\sum_{n=-\infty}^{\infty} b_n(I)e^{in\theta}. \quad (6)$$

By appropriate definitions, the Fourier coefficients $b_n = b_{-n}^*$ can be made purely real, which is our convention.

Now, we further assume that the rapidly oscillating terms of the Hamiltonian (4) average to zero and contribute negligibly to the dynamics. Under certain conditions derived below, the dynamical frequency adjusts itself to match that of the driving laser beats, so that $\Omega(\xi) \approx \Delta\omega(\xi)$. In this case, the only slowly varying terms in (6) are the constant $\bar{a}^2 b_0$ and those whose phase varies as $\Psi \equiv \theta - \psi(\xi)$. Neglecting all other terms as rapidly oscillating (single resonance approximation, see, e.g., [17]), the action and slow phase Ψ are governed by

$$\frac{d}{d\xi}\Psi = \Omega(I) - \Delta\omega(\xi) + \epsilon\frac{\partial b_1}{\partial I}\cos\Psi + \bar{a}^2\frac{\partial b_0}{\partial I}, \quad (7)$$

$$\frac{d}{d\xi}I = \epsilon b_1(I)\sin\Psi. \quad (8)$$

In what follows, we seek solutions to (7) and (8) for which we have (i) an initially unperturbed plasma, (ii) an initial tuning of the beat frequency above resonance, and (iii) a subsequent slow downward frequency chirping through resonance. Under (i)–(iii), we will find autoresonant excitation with $\Delta\omega(\xi) - \Omega(\xi) \ll 1$.

When the drive is applied above resonance, the plasma wave is small and we can linearize (7) and (8), obtaining simple harmonic oscillator equations. Initially, the response has two components: one ringing at the resonant frequency, the other at the driving frequency, both of small amplitude. The singular term $\partial b_1/\partial I \sim I^{-1/2}$ in (7) permits a large change in phase at small amplitude, and the response at the driven frequency phase locks to the drive [13]. As the frequency is swept toward resonance, these driven, phase-locked oscillations grow, while the response at the resonant frequency remains small. In this way, we excite one growing, phase-locked plasma wave.

As the system approaches resonance, we include weak nonlinearity by expanding (5) to first order in I: $\Omega(I) = 1 - \frac{3}{8}I$. We linearize the frequency chirp around the effective plasma frequency $\Delta\omega(\xi) = 1 - \bar{a}^2/2 - \alpha\xi$, where $\alpha \equiv -\frac{d}{d\xi}\Delta\omega$ is the chirp rate parameter. Using the small amplitude relations $b_0 = -\frac{1}{2}I$, $b_1 = \frac{1}{4}\sqrt{2I}$, and making the change of variable $\mathcal{A} \equiv 4\sqrt{2I}$, yields

$$\frac{d}{d\xi}\mathcal{A} = \epsilon\sin\Psi, \quad (9)$$

$$\frac{d}{d\xi}\Psi = \alpha\xi - \frac{3}{256}\mathcal{A}^2 + \frac{\epsilon}{\mathcal{A}}\cos\Psi. \quad (10)$$

This set of equations, which is similar to the weakly nonlinear analysis of [11,12], can be reduced to a single first-order ordinary differential equation by defining the complex variable $Z \equiv -\sqrt{256/3\alpha}\mathcal{A}e^{i\Psi}$, rescaling $\zeta \equiv \sqrt{\alpha}\xi$, and defining $\mu \equiv \epsilon\sqrt{3/(256\alpha^{3/2})}$, obtaining

$$i\frac{d}{d\zeta}Z + (\zeta - |Z|^2)Z = \mu. \quad (11)$$

The weakly nonlinear problem is thus reduced to an equation with the single parameter μ. This equation is similar to the system of real Eqs. (12) and (13) in [18], where it was numerically found that the solution to (11) bifurcates at the critical value $\mu = \mu_c \approx 0.411$. For $\mu < \mu_c$, the plasma wave response quickly dephases from the drive, resulting in small excitations. For $\mu > \mu_c$, phase locking occurs and the plasma wave can grow to a large amplitude. This critical behavior in μ brings about a relation between the drive strength ϵ and chirp rate α for the plasma wave to be autoresonantly excited:

$$\alpha \leq \left(\frac{3\epsilon^2}{256\mu_c^2}\right)^{2/3} \approx 0.15\epsilon^{4/3}. \tag{12}$$

For a given laser intensity, (12) gives the maximum laser chirp rate consistent with high amplitude plasma waves. The sensitivity of this critical behavior is shown for $\epsilon = 0.005$ in Fig. 1(a). Here, a chirp rate 1% below the critical rate $\alpha_c = 1.28 \times 10^{-4}$ has strong frequency locking, attaining peak fields $E_z \approx 1.3E_0$, while the chirp 1% above α_c quickly dephases, with a final $E_z \approx 0.6E_0$.

If phase locking occurs, the amplitude continues to grow and additional nonlinearities give rise to more stringent restrictions on the chirp rate for adiabatic phase locking to persist. We calculate this condition using the second-order equation for the phase Ψ:

$$0 = \frac{d^2}{d\xi^2}\Psi + \epsilon\frac{\partial b_1}{\partial I}\sin\Psi\frac{d}{d\xi}\Psi + \frac{d}{d\xi}\Delta\omega - \epsilon b_1 \sin\Psi\left[\frac{\partial\Omega}{\partial I} + \bar{a}^2\frac{\partial^2 b_0}{\partial I^2} + \epsilon\frac{\partial^2 b_1}{\partial I^2}\cos\Psi\right]. \tag{13}$$

We assume the action can be written as $I = I_0 + \Delta I$ (see, e.g., [19]), where I_0 is the slowly varying, secularly growing action about which there are small oscillations given by ΔI. These oscillations correspond to fluctuations in Ψ about its phase-locked value $\Psi \approx \pi$, as seen in Fig. 1(a). Using $I = I_0 + \Delta I$, the lowest-order equation for Ψ is identical to (13), with $I \to I_0$. Thus, the phase obeys a nonlinear oscillator equation whose effective "potential" is dictated by the slowly evolving action I_0. For the phase to remain trapped, this potential must have a local minimum, for which the second line of (13) is zero. Since $|\sin\Psi|, |\cos\Psi| \leq 1$, the frequency chirp $\alpha(\xi)$ must be sufficiently small for phase locking to persist. As a practical limit, we set $|\cos\Psi| = |\sin\Psi| = 1$ above, to find

$$0 \leq \alpha(\xi) \leq \epsilon|b_1(I_0)|\left[\left|\frac{\partial\Omega}{\partial I_0} + \bar{a}^2\frac{\partial^2 b_0}{\partial I_0^2}\right| + \epsilon\left|\frac{\partial^2 b_1}{\partial I_0^2}\right|\right]. \tag{14}$$

For a given laser power and chirp, this inequality implicitly specifies the maximum achievable plasma wave amplitude. We show the dependence of the saturated E_z on α for a number of different drive strengths in Fig. 1(b). The solid lines show the stable branches of (14), which jump discontinuously to high E_z at the critical chirp α_c given by (12); the dotted lines correspond to solutions that cannot be accessed from zero initial amplitude and as such can be considered a form of hysteresis. Figure 1(c) compares the theoretical maximum amplitude from (14) with that found by numerically integrating Eq. (3).

Unfortunately, the PBWA does not have unlimited time to be excited. For the parameters of interest, Mora *et al.* [7] showed that the oscillating two-stream instability, with a growth rate of order the ion plasma frequency ω_{pi}, destroys plasma wave coherence after about five e foldings. Thus, excitation time is limited to $T \lesssim 5/\omega_{pi}$, and, for a total frequency change $\delta\omega$, the chirp rate is limited to $\alpha \gtrsim (0.2\omega_p/\omega_{pi})\delta\omega$. Below, we choose two experimental parameter sets: one for a 10 μm CO_2 laser; the other, for a 800 nm CPA Ti:sapphire laser. We show that autoresonance can robustly excite large plasma waves in times commensurate with the onset of ion instabilities.

We use UCLA parameters [8,9] for the CO_2 laser system: two 100 ps pulses at 10.27 and 10.59 μm, with intensities $a_1 = a_2 = 0.14$. In this case, $\epsilon = 0.02$, and (12) requires $\alpha < 8 \times 10^{-4}$. We choose $\alpha = 6.5 \times 10^{-4}$ over a pulse length of 100 ps, using $\Delta\omega(0) = 1.15$ so that $\Delta\omega(T) = 0.74$. For these parameters, Fig. 2(a) demonstrates a uniform accelerating field of 10 GV/m. For comparison, we include envelopes of the resonant RL case with $\Delta\omega = 1$, and the DMG chirped scheme with $\Delta\omega(0) = 1$. The resonant case demonstrates the RL limit with $E_z \leq E_{RL} = (\frac{16}{3}\epsilon)^{1/3}E_0 \approx 4.2$ GV/m; the DMG scheme does not phase lock, resulting in nearly the same accelerating field as the RL case. At UCLA, accelerating gradients ~2.8 GV/m have been inferred by

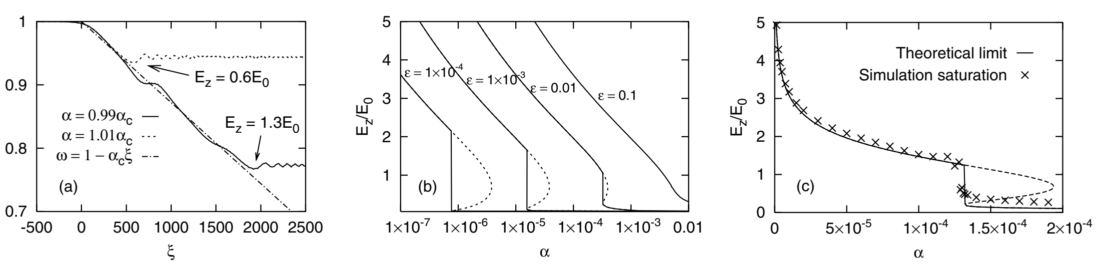

FIG. 1. Autoresonant behavior. (a) demonstrates the critical behavior for $\epsilon = 0.005$. Chirp rates 1% below critical (solid line) frequency lock to final $E_z = 1.3E_0$; chirp rates 1% above critical (dashed line) quickly dephase and saturate at $0.6E_0$. (b) shows attainable E_z before adiabaticity requirement (14) is violated as a function of α. The dotted lines indicate hysteresis at α_c given by (12). (c) compares theory (line) with numerically determined saturation for $\epsilon = 0.005$.

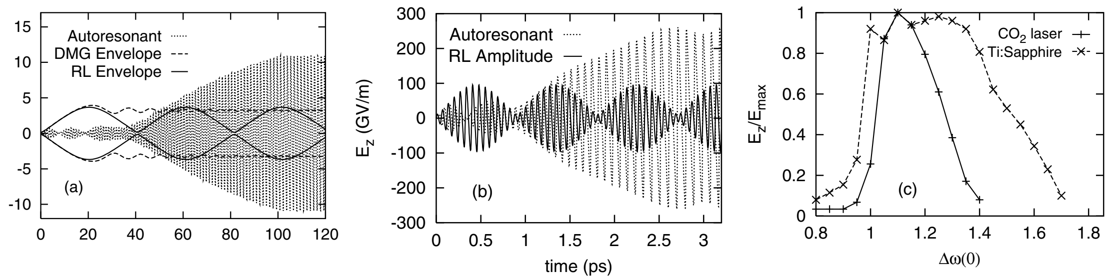

FIG. 2. Simulations solving Eq. (1). (a) uses 10 μm CO_2 lasers with intensity 2.7×10^{14} W/cm^2 ($\epsilon = 0.02$), $\Delta\omega(0) = 1.15$, and $\Delta\omega = 1$ at $t \approx 40$ ps ($\alpha = 6.5 \times 10^{-4}$). Peak fields of 11 GV/m are excited for a total chirp of $0.015\bar{\omega}$. (b) simulates an 800 nm Ti:sapphire laser with $\bar{\omega}/\omega_p = 25$, intensity 2×10^{17} W/cm^2 ($\epsilon = 0.09$), $\Delta\omega(0) = 1.2$, and $\Delta\omega = 1$ at $t \approx 0.9$ ps ($\alpha = 0.0025$), yielding 250 GV/m gradients with a total chirp of $0.03\bar{\omega}$. (c) shows the robust nature of autoresonance for different initial detuning. The CO_2 system excites fields of 8–12 GV/m with plasma density variations of $\pm 10\%$; the Ti:sapphire system is quite insensitive to $\pm 35\%$ errors in density, with fields $\gtrsim 250$ GV/m.

electron acceleration [8] and fields ~0.2–0.4 GV/m have been measured via Thomson scattering [9].

We also consider a Ti:sapphire CPA laser of duration $T = 3.2$ ps (corresponding to the ion instability limit). For two 1 J pulses compressed to 3.2 ps and focused to a 12 μm spot, we obtain intensities of 2×10^{17} W/cm^2, so that $a_1 = a_2 = 0.3$ and $\epsilon = 0.09$. Using a singly ionized He plasma with $n_0 = 2.4 \times 10^{18}$ cm^{-3} ($\omega_p/\bar{\omega} = 1/25$), we choose $\Delta\omega(0) = 1.2$, $\Delta\omega(T) = 0.5$, with $\alpha = 0.0025$. The resulting accelerating field, shown in Fig. 2(b), is ~250 GV/m, more than twice the RL limit of 100 GV/m.

Autoresonant excitation is also very robust with respect to mismatches between the beat and plasma frequency that might result from limited diagnostics or shot-to-shot fluctuations. Because one needs only to pass through the resonance at some indeterminate point, no precise matching is required. We demonstrate this robustness by plotting the normalized peak field against the initial detuning $\Delta\omega(0)$ in Fig. 2(c) for both the CO_2 and Ti:sapphire examples. In the CO_2 case, we see that lasers with an initial frequency detuning $\pm 10\%$ from the "design" of $\Delta\omega(0) = 1.15$ excite similarly large plasma waves ~10 GV/m. In the Ti:sapphire case, variations in $\omega_p \pm 35\%$ from design have little effect on the accelerating gradients ~250 GV/m achieved. Note the precipitous drop in peak field when starting near ω_p due to missing the resonance; the slow decrease at high initial detuning is caused by the limited excitation time.

We have introduced a simple modification of the DMG scheme for the chirped-pulse PBWA in a fully relativistic model. Rather than starting on the linear resonance and chirping downward at some numerically estimated rate, we sweep the beat frequency through resonance, at any chirp rate slower than a calculated critical value. This autoresonant excitation can drive plasma waves beyond traditional detuning limits and appears to be rather insensitive to uncertainties and variations in plasma and laser parameters. Furthermore, autoresonant phase locking may be used to precisely time electron injection [20,21] to accelerate monoenergetic beams. These auspicious conclusions have been derived from a simplified model, which we believe warrants a more thorough investigation to include ion motion, higher dimensional effects, and self-consistent laser evolution.

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