Optically Induced Polarons in Bose-Einstein Condensates: Monitoring Composite Quasiparticle Decay

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Nonresonant light scattering off atomic Bose-Einstein condensates is predicted to give rise to hitherto unexplored composite quasiparticles: unstable polarons, i.e., local "impurities" dressed by virtual phonons. Optical monitoring of their spontaneous decay can display either Zeno or anti-Zeno deviations from the golden rule, and thereby probe the temporal correlations of elementary excitations in the condensates.

The evolution of a quantum state coupled to a continuum is coherent and effectively reversible over the so-called "correlation time" t_{corr}. This time marks the duration of the nonexponential initial stage of decay of the unstable state [1–3], or, equivalently, its response time to fast (impulsive) perturbations.

Little is known about the short-time dynamics of impulsively perturbed quantum many-body systems. Atomic Bose-Einstein condensates (BECs) are especially suitable for the exploration of such effects, since their elementary excitations, which are describable as quasiparticles [4–10], may remain correlated for a long time. Here we consider the short-time dynamics of a composite quasiparticle, consisting of a moving "impurity" atom (an atom in a ground-state sublevel different from the rest of the BEC) "dressed" by a cloud of virtual phonons due to deformation of its vicinity. This quasiparticle may be viewed as the BEC analog of the solid-state polaron [11]. The time necessary for dressing an impurity atom by phonons is just the correlation time of the quasiparticle, which is shown to be exceedingly long (~ 1 ms). The polaronic effect in a BEC is a genuine example of the response of a many-body quantum system to the presence of a probe particle. Direct measurement of t_{corr} is a formidable task, since bare and dressed impurities are hardly distinguishable spectroscopically.

We therefore propose an indirect method of measuring t_{corr}: optically induced formation of polarons by Raman scattering followed by real-time optical monitoring of the products after their decay. The decay products are pair-correlated elementary excitations that are *unambiguously* detectable. Their time-resolved monitoring can reveal t_{corr}, which may be manifest by either the Zeno or the anti-Zeno effects [1–3], namely, slowdown or speedup of the decay rate compared to its golden rule value. Fluctuations of the laser fields, on time scales comparable to the very long t_{corr} of the BEC, in the millisecond range, can be used to interrogate these features more easily. Similar results are ob-tained for the Bragg process, where, in contrast to the Raman process, the atomic internal state is not changed.

We shall primarily consider polaron formation by Raman scattering off an atom in the BEC, as shown in Fig. 1(a). Two laser beams are arranged such that momentum transfer of $\hbar\mathbf{q}$ is accompanied by energy transfer of $\hbar\tilde{\Delta}$ to the atom. The effective Rabi frequency associated with the induced Raman two-photon transition is Ω. Proper choice of the laser-beam frequencies and polarizations ensures that the atom is transferred to another sublevel of the ground state, thereby leaving the condensate and forming an impurity atom [12]. The energy of such an impurity atom is given by its kinetic energy, $\hbar^2 q^2/(2m)$, plus the mean-field shift, $4\pi\hbar^2 an/m$, a being the scattering length for a collision between the impurity and condensate atoms, plus the energy difference E_D between the two (Zeeman
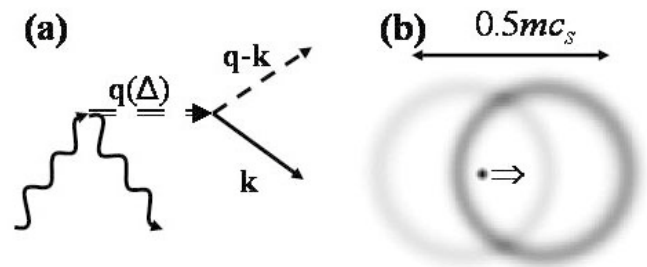

FIG. 1.   (a) The Feynman diagram for the formation and decay of the polaron includes the Raman beams (curved lines), the unstable polaron (double dashed line), and the correlated products: the bare impurity (dashed line) and BEC Bogoliubov excitation (solid line). (b) The momentum distribution (parallel and perpendicular to $\hbar\mathbf{q}$, which is indicated by $\Rightarrow$) of the impurity atoms (dark gray) and Bogoliubov excitations (light gray) of the BEC (central dark spot) produced by Raman off-resonant scattering with $\hbar\Delta = 0.66\mu$, $\hbar q = 0.14mc_s$. The momentum shells are broadened due to finite pulse duration ($120\hbar/\mu$). These shells should be observed in time-of-flight images of the excited condensate.

and/or hyperfine) sublevels involved. We can incorporate the difference between the mean-field shift and the chemical potential $\mu = 4\pi\hbar^2 a_0 n/m$, a_0 being the BEC scattering length, as well as E_D into a new definition of the Raman-transferred energy, $\hbar\Delta = \hbar\tilde{\Delta} + 4\pi\hbar^2(a_0 - a)n/m - E_D$. This enables us to identify the energy of the impurity atom with its kinetic energy. We shall assume large blue two-photon laser detuning, $\Delta > \hbar q^2/(2m)$, $\Omega \lesssim \Delta$, so as to avoid the need to deal with strongly driven atoms [13].

The relevant Hamiltonian for a uniform BEC is

$$\hat{H} = \hat{H}_{\text{at}} + \hat{H}_{\text{int}}, \tag{1}$$

the first term describing the atomic system itself, and the second one its interaction with the laser radiation. These terms are

$$\hat{H}_{\text{at}} = \sum_{\mathbf{k}} \hbar\omega_k \hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} + \sum_{\mathbf{k}} \frac{\hbar^2 k^2}{2m} \hat{\beta}_{\mathbf{k}}^\dagger \hat{\beta}_{\mathbf{k}}$$
$$+ \frac{4\pi\hbar^2 a\sqrt{n}}{m\sqrt{V}} \sum_{\mathbf{k}} \sqrt{S_k}(\hat{\varrho}_{-\mathbf{k}}\hat{b}_{\mathbf{k}}^\dagger + \hat{\varrho}_{\mathbf{k}}\hat{b}_{\mathbf{k}}), \tag{2}$$

$$\hat{H}_{\text{int}} = \hbar\Omega\sqrt{N}(e^{-i\Delta t}\hat{\beta}_{\mathbf{q}}^\dagger + e^{i\Delta t}\hat{\beta}_{\mathbf{q}}). \tag{3}$$

Here the creation and annihilation operators of the BEC elementary Bogoliubov excitations are denoted by $\hat{b}_{\mathbf{k}}^\dagger$, $\hat{b}_{\mathbf{k}}$, and those of impurity atoms by $\hat{\beta}_{\mathbf{k}}^\dagger$, $\hat{\beta}_{\mathbf{k}}$, $\omega_k = k\sqrt{[\hbar k/(2m)]^2 + c_s^2}$ is the frequency of the BEC elementary excitation with the momentum $\hbar k$, $c_s = \sqrt{\mu/m}$ is the speed of sound in the BEC, $S_k = \hbar k^2/(2m\omega_k)$ is the BEC static structure factor, $\hat{\varrho}_{\mathbf{k}} = \Sigma_{\mathbf{k'}}\hat{\beta}_{\mathbf{k}+\mathbf{k'}}^\dagger\hat{\beta}_{\mathbf{k'}}$ is the operator of the impurity momentum shift, and V is the quantization volume.

Now we define the dressed (polaronic) states as eigenstates of the atomic-system Hamiltonian (2) and express the field-atom interaction Hamiltonian (3) in the basis of these states. We omit the intermediate calculations, which are quite involved but straightforward [11], and present the result:

$$\hat{H}_{\text{int}} = \frac{\hbar}{\sqrt{V}} \sum_{\mathbf{k}} [\mathcal{M}_{\mathbf{q},\mathbf{k}}(t)\hat{c}_{\mathbf{q}-\mathbf{k},\mathbf{k}}^\dagger \hat{c}_0 + \text{H.c.}], \tag{4}$$

where $\hat{c}_0$ is the operator of annihilation of a bosonic atom in the BEC state, and $\hat{c}_{\mathbf{q}-\mathbf{k},\mathbf{k}}^\dagger$ is the operator of creation of a correlated pair consisting of a dressed impurity atom with momentum $\hbar(\mathbf{q} - \mathbf{k})$ and a elementary Bogoliubov excitation with momentum $\hbar\mathbf{k}$. The latter operator obeys the usual bosonic commutation rules as long as the population of the state $|\mathbf{q} - \mathbf{k}, \mathbf{k}\rangle_d \equiv \hat{c}_{\mathbf{q}-\mathbf{k},\mathbf{k}}^\dagger|0\rangle$ is much less than one. The energy of the state $|\mathbf{q} - \mathbf{k}, \mathbf{k}\rangle_d$ is $\epsilon_{|\mathbf{q}-\mathbf{k}|,k} = \hbar\omega_k + \hbar^2(\mathbf{q} - \mathbf{k})^2/(2m)$, provided that the energy correction (to second order in a), which has to be calculated using the scattering-length renormalization [5], is included in the

definition of $\hbar\Delta$. The matrix element of the off-resonant two-photon transition that couples the initial vacuum state $|0\rangle$ to the state $|\mathbf{q} - \mathbf{k}, \mathbf{k}\rangle_d$ is

$$\mathcal{M}_{\mathbf{q},\mathbf{k}}(t) = \frac{4\pi\hbar a\sqrt{nS_k}\Omega e^{-i\Delta t}}{m[\omega_k + \hbar k^2/(2m) - \hbar\mathbf{q}\mathbf{k}/m]} \tag{5}$$

The relevant Feynman diagram is shown in Fig. 1(a). The operator Heisenberg equations obtainable from Eq. (4) are

$$i\frac{\partial}{\partial t}\hat{c}_0 = \sum_{\mathbf{k}} \mathcal{M}_{\mathbf{q},\mathbf{k}}^*\hat{c}_{\mathbf{q}-\mathbf{k},\mathbf{k}}, \tag{6}$$

$$i\frac{\partial}{\partial t}\hat{c}_{\mathbf{q}-\mathbf{k},\mathbf{k}} = \hbar^{-1}\epsilon_{|\mathbf{q}-\mathbf{k}|,k}\hat{c}_{\mathbf{q}-\mathbf{k},\mathbf{k}} + \mathcal{M}_{\mathbf{q},\mathbf{k}}\hat{c}_0. \tag{7}$$

Equations (6) and (7) represent the well-known model of a discrete state coupled to a continuum. Assuming a perturbative production fraction, the time-dependent rate of creation of correlated pairs, consisting of an impurity atom and a Bogoliubov excitation is

$$\Gamma(t) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} |\mathcal{M}_{\mathbf{q},\mathbf{k}}|^2 \frac{2\sin(\nu t)}{\nu}, \tag{8}$$

where $\nu = \Delta - \omega_k - \hbar(\mathbf{q} - \mathbf{k})^2/(2m)$.

In what follows we assume that $\hbar q \ll mc_s$; i.e., the *virtual* impurity atoms move at a subsonic velocity, whereas the detuning of the beams Δ from the Raman resonance ensures a significantly larger momenta (and subsequent energy) for the decay products.

The quantity $\hbar\mathbf{k}\mathbf{q}/m$ can then be neglected compared to ω_k, and Eq. (8) can be approximately expressed as

$$\Gamma(t) = \Gamma_* \int_0^\infty d\delta \frac{\sin[(\delta - \Delta_q)t]}{\pi(\delta - \Delta_q)} G(\delta). \tag{9}$$

Here

$$\Gamma_* = 4\sqrt{\pi}\sqrt{\frac{a}{a_0}}\frac{\hbar\Omega^2}{\mu}\sqrt{na^3} \tag{10}$$

is the characteristic decay rate, $G(\delta) = (\hbar\delta/\mu)[(\hbar\delta/\mu) + 1]^{-5/2}$ is the dimensionless response function of the uniform BEC, and $\Delta_q = \Delta - \hbar q^2/(2m) \approx \Delta$. The proportionality of the rate prefactor Γ_* in Eq. (10) to $\sqrt{na^3}$ clearly indicates that the process under consideration relies on non-mean-field (impurity scattering) effects. For $\mu t/\hbar \gg 1$ Eq. (9) gives the golden rule rate $\Gamma_{\text{GR}} = \Gamma_* G(\Delta_q)$.

In Fig. 2 we display $\Gamma(t)/\Gamma_{\text{GR}}$ for three different detunings. For $\mu t/\hbar \lesssim 4$ the drastic deviations from Γ_{GR} imply that we are within the correlation time t_{corr} of the elementary excitations before irreversibility sets in.

The Raman excitation fraction, namely, the number of correlated pairs created per BEC atom, $P(t) = \int_0^t dt'\Gamma(t')$, is assumed to be small so as to avoid condensate depletion:
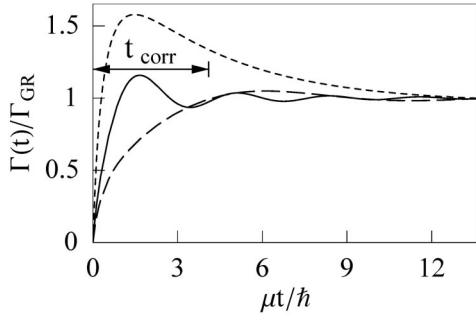
FIG. 2. The creation rate Γ (normalized to Γ_{GR}) of the laser-induced impurity atoms and Bogoliubov excitations as a function of dimensionless time for $\hbar\Delta_q/\mu$ equal to 2.0 (solid line), 0.66 (long-dashed line), and 0.07 (short-dashed line). Typical correlation time is t_{corr}.

$P \ll 1$. Then Eq. (9) yields

$$P(t) = \Gamma_* \int_0^\infty d\delta \frac{2\sin^2[(\delta - \Delta_q)t/2]}{\pi(\delta - \Delta_q)^2} G(\delta). \quad (11)$$

The long correlation time of the BEC can lead to a significant deviation of $P(t)$ from that expected from the golden rule. In principle, this deviation is observable, even at long times, as a shift from the golden rule prediction, $P(t) = \Gamma_{GR}t$. However, this deviation is difficult to observe for realistic experimental parameters, as we show below.

We therefore suggest an alternative scheme, which allows us to observe large deviations from the golden rule rate, even at the limit of large times. Suppose that the difference of the two Raman laser frequencies is randomly modulated [3], so that the well-defined detuning Δ_q is replaced by the spectral distribution $F(\delta)$ normalized to 1. Its spectral rms fluctuation (around the mean frequency Δ_q), τ^{-1}, may be regarded as the inverse dephasing time. Under these conditions, even the long-time limit of the excitation rate $\Gamma_\tau(\infty)$ obtained at $t \gg \hbar/\mu$ may strongly differ from Γ_{GR}. As follows from the universal formulas for the quantum Zeno effect (QZE) [3], Eqs. (9) and (11) take in this case the form

$$\Gamma_\tau(t) = \Gamma_* \int_0^\infty d\delta \int_0^\infty d\delta' \frac{\sin[(\delta - \delta')t]}{\pi(\delta - \delta')} G(\delta)F(\delta'),$$

$$P_\tau(t) = \Gamma_* \int_0^\infty d\delta \int_0^\infty d\delta' \frac{2\sin^2[(\delta - \delta')t/2]}{\pi(\delta - \delta')^2} G(\delta)F(\delta').$$

$$(12)$$

Equation (12) implies that the modulation spectrum $F(\delta)$ controls the asymptotic decay rate $\Gamma_\tau(\infty)$ and excitation fraction $P_\tau(\infty)$, so that a suitable choice of the modulation spectrum may yield conspicuous deviations from the golden rule, in contrast to the case of Eq. (11).

Specifically, we choose the modulation spectrum $F(\delta)$ with a sharp low-frequency cutoff in order to suppress undesired transitions from $|0\rangle$ to the state containing only

a dressed impurity with the momentum $\hbar\mathbf{q}$ and no real Bogoliubov excitations. These requirements are satisfied by $F(\delta) = C\delta^{-3/2}$ for $\delta_1 < \delta < \delta_2$ and zero otherwise (see inset of Fig. 3). The coefficient C ensures normalization to 1, and the cutoff frequencies δ_1, δ_2 are chosen so that the mean frequency is Δ_q and the rms fluctuation is τ^{-1}. The resulting values for $\Gamma_\tau(\infty)$ are shown in Fig. 3 as functions of τ for different values of Δ_q.

The convolution of $G(\delta)$ and $F(\delta)$ may give rise to either the QZE or the opposite, anti-Zeno effect (AZE), i.e., decay speedup at short times [2]. As expected [1–3], the QZE always takes place for extremely small times τ. For large detunings, the modulation spreads the accessible final states towards the range where the response $G(\delta)$ *increases* as τ^{-1} grows, and the irreversible transitions are accelerated (AZE takes place) [3]. Alternatively, for small and moderate detunings, the modulation spreads the final states over a range of energies such that G *decreases* as τ^{-1} grows, thereby bringing about the QZE. Comparison of Figs. 2 and 3 indicates that the dephasing time scale τ associated with the QZE can be identified with the correlation time t_{corr} of elementary excitations.

We obtain qualitatively similar results for off-resonant Bragg scattering, although there are some subtle caveats, due to the indistinguishability of the excitations, as opposed to the Raman process. The general expression for the rate $\Gamma_{Bragg}(t)$ of this process is quite cumbersome. Here we present the $t \to \infty$ limit of $\Gamma_{Bragg}(\infty)$ for the particular case $\Delta \gg \mu/\hbar \gg \hbar q^2/(2m)$:

$$\Gamma_{Bragg}(\infty) \approx 8\sqrt{\pi}\left(\frac{\mu}{\hbar\Delta}\right)^{3/2} \frac{\hbar\Omega^2}{\mu}\sqrt{na_0^3}. \quad (13)$$

Remarkably, Γ_{Bragg} does not depend on q, as long as $q\ell \lesssim 1$, ℓ being the length of the trapped BEC sample. The
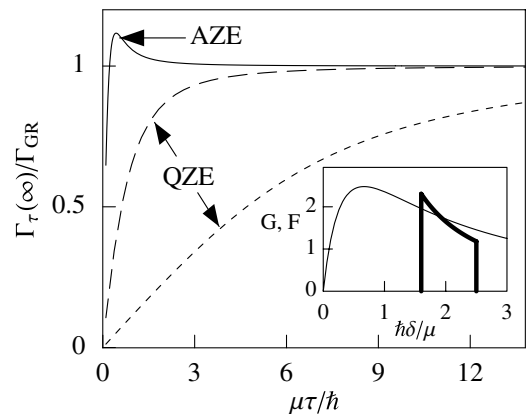

FIG. 3. The creation rate $\Gamma_\tau(\infty)$ of laser-induced impurity atoms and Bogoliubov excitations versus the dephasing time τ for the same $\hbar\Delta_q/\mu$ as in Fig. 2. AZE and QZE regions are indicated. Inset: the medium response function $G(\delta)$ (thin line) and typical modulation spectrum $F(\delta)$ (thick line), both in arbitrary units.
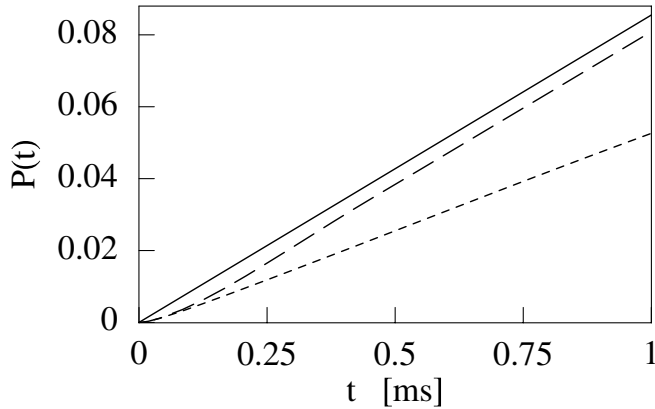
P H Y S I C A L   R E V I E W   L E T T E R S

FIG. 4. The Raman excitation fraction (dimensionless), as a function of time for a ^{87}Rb BEC with $n = 4 \times 10^{14}$ cm^{-3} and $\Omega = \Delta_q \approx 1.3 \times 10^4$ s^{-1}. The solid line is the golden rule result. The long-dashed line is the prediction for a cw Raman experiment, and the short-dashed line is the prediction for a frequency modulated Raman experiment, with the laser-beam correlation time $\tau = 1.0\hbar/\mu$. The modulation leads to a clear asymptotic deviation from the golden rule result that is easily observed experimentally, in contrast to the case of a cw Raman excitation.

Bragg process is characterized by a correlation time, which is similar to that of the Raman process and, hence, can also be revealed via the QZE and AZE.

We used for our plots the response function $G(\delta)$ of a uniform BEC. It can be generalized to the local density approach if the length of the trapped BEC sample satisfies $q\ell > 1$ (usually $\ell \sim 10^{-3}$ cm). This inequality is compatible with the subsonic limit for $\hbar q/m$ under typical experimental conditions [12,14].

As an example, we consider a ^{87}Rb BEC with $n = 4 \times 10^{14}$ cm^{-3} [8] and $\Omega = \Delta_q \approx 1.3 \times 10^4$ s^{-1}, for the Raman process. We expect, after the excitation, to observe by the time-of-flight technique [14] unambiguous momentum shells of Raman and phonon excitations, as shown in Fig. 1(b). Although the probability of direct excitation of dressed impurities with momentum $\hbar\mathbf{q}$ may be non-negligible because of broadening of various kinds (inhomogeneous, Fourier, etc.), impurity atoms produced via this channel are well separated by energy and, therefore, distinguishable by a time-of-flight measurement from those produced by the off-resonant Raman process, as indicated by the double arrow in Fig. 1(b).

In Fig. 4 we show the total excitation fraction $P(t)$ (dashed line), which should be contrasted with the golden rule production of excitations (solid line), indicating the presence of the QZE. Experimentally, measuring the population in the excitation shells may allow us to resolve t_{corr} by observing a shift in the asymptotic increase of excitations. However, this is quite challenging. Therefore, we propose to use the frequency modulation scheme described above. This will lead to an easily measured deviation of the excitation fraction from its golden rule counterpart for all times, as shown in the dotted line, calculated for a laser-beam correlation time $\tau = 1.0\hbar/\mu$. Specifically, after 1 ms this deviation is nearly 40%, and the corresponding excitation fraction $P_\tau \approx 0.05$ is easily detectable experimentally [8,12].

To summarize, we have presented a theory of off-resonant stimulated Raman and Bragg scattering in BECs, followed by the hitherto unexplored creation of correlated pairs of matter-wave quanta—the decay of a composite polaron. These processes are found to be highly suitable for experimental observation of the quantum Zeno and anti-Zeno effects. These effects may be used as unique probes of correlation times of the BEC response to fast (impulsive) perturbations, and the onset of irreversibility in the quasiparticle formation.

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