Beliaev Damping of Quasiparticles in a Bose-Einstein Condensate

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(Received 23 May 2002; published 6 November 2002)

We report a measurement of the suppression of collisions of quasiparticles with ground state atoms within a Bose-Einstein condensate at low momentum. These collisions correspond to Beliaev damping of the excitations, in the previously unexplored regime of the continuous quasiparticle energy spectrum. We use a hydrodynamic simulation of the expansion dynamics, with the Beliaev damping cross section, in order to confirm the assumptions of our analysis.

DOI: 10.1103/PhysRevLett.89.220401 PACS numbers: 03.75.Fi, 32.80.Pj

In a Bose-Einstein condensate (BEC) collisions between indistinguishable excitations (quasiparticles) and the condensate are an important channel for the dissipation of these excitations. Understanding the decay of excitations gives insight into higher order terms of the interaction Hamiltonian beyond mean field, relevant even in the zero-temperature limit [1]. In addition, these coupling terms are predicted to generate squeezing and entanglement of quasiparticle excitations [2], influencing future works on coherent outcoupling of matter waves (atom laser) from BEC [3].

The case of identical particle collisions has been extensively studied using many-body theory, starting with [1]. Recently, these results have been applied to BEC explicitly [4–7]. In this Letter we present a measurement of collisions between quasiparticles and the BEC, at velocities approaching the superfluid critical velocity v_c. We find a suppression of the collision cross section between the quasiparticles and the BEC in agreement with the theory of Beliaev damping.

The Landau criterion [8] states that v_c cannot be greater than $E_k^B/\hbar k$, for any excitation E_k^B in the spectrum with wave number k. This criterion follows from considering the collision between an impurity (e.g., distinguishable excitation) and the superfluid, under the constraints of momentum and energy conservation. The impurities follow the dispersion relation $E_k^0 = k^2$. The wave number k is in units of $\xi^{-1} = \sqrt{8\pi n a}$, the inverse healing length of the condensate, with a the s-wave scattering length [9] and n the density. We express energy in units of $\mu = gn$, the chemical potential of the BEC, where g is $4\pi\hbar^2 a/m$, and m is the mass of the BEC atoms. The recently measured [10] Bogoliubov dispersion relation [11] of the condensate is given by $E_q^B = \sqrt{q^4 + 2q^2}$. Consequently, the BEC has a superfluid critical velocity $v_c = \sqrt{\mu/m}$, below which collisions between impurities and the condensate are completely suppressed.

In the case of quasiparticle collisions only the Bogoliubov dispersion relation is relevant, and the Landau criterion does not apply, since no impurities are involved. Conservation of energy and momentum requires that ($E_k^B = E_q^B + E_{\mathbf{k}-\mathbf{q}}^B$), where an initial excitation of wave number k collides with the BEC creating two excitations with wave vectors $\mathbf{q}$ and $\mathbf{k} - \mathbf{q}$. We solve this condition and find the angle θ between the initial direction $\mathbf{k}$ and the scattered direction $\mathbf{q}$ to be [see Fig. 1(a)]

$$\cos(\theta) = (2kq)^{-1}[k^2 + q^2 + 1 - \sqrt{1 + (E_k^B - E_q^B)^2}]. \tag{1}$$

Equation (1) has solutions for any finite k; therefore, there is no longer any well defined critical velocity at which collisions are completely suppressed. However, not all angles are allowed. At a given k we find that the maximal allowed angle is $\cos(\theta_{\max}) = \sqrt{(k^2 + 2)/2}/(k^2 + 1)$. At the limit of small k, this angle approaches zero, and collisions are allowed only for $\mathbf{q}$ parallel to $\mathbf{k}$.

The collision rate between a quasiparticle with wave number k and the BEC is proportional to $|A_{q;k}|^2$, where $A_{q;k}$ is the q-dependent, momentum conserving, scattering matrix element, which includes suppression or enhancement of the collision process due to many-body effects [12].

The appropriate suppression term $|A_{q;k}|^2$ for quasiparticles has been calculated [4,5]. In this work we expect mainly Beliaev processes which involve creation of lower energy excitations. The Landau damping rate is expected to be an order of magnitude slower than the observed Beliaev collision process, since the experimental regime is at sufficiently low temperature to suppress this thermally activated damping process [5].

We start with the atomic interaction Hamiltonian $H' = \frac{g}{2V}\sum_{j,l,m,n} a_j^\dagger a_l^\dagger a_m a_n \delta_{j+l-m-n}$, where V is the volume of the BEC and $a_i^\dagger$ and a_i are, respectively, the atomic creation and annihilation operators at wave number i. We approximate $a_0^\dagger \approx a_0 \approx \sqrt{N_0}$, with N_0 the number of atoms in the condensate. We take the Bogoliubov transform $a_p^\dagger = (u_p b_p^\dagger - v_p b_{-p})$, with u_p and v_p the appropriate quasiparticle amplitudes, which were recently measured [13]. We are interested in terms of the form $b_k b_{\mathbf{k}-\mathbf{q}}^\dagger b_q^\dagger$, which remove a quasiparticle of wave number k and create two in its stead. Calculating the matrix element prefactor of this term in the atomic interaction Hamiltonian, we arrive at $A_{q;k} = \frac{1}{2}(S_q + 3S_qS_kS_{\mathbf{k}-\mathbf{q}} + S_{\mathbf{k}-\mathbf{q}} - S_k)/\sqrt{S_kS_qS_{\mathbf{k}-\mathbf{q}}}$, where S_q is the static structure factor of the BEC (at wave number q) [14]. This result can be viewed as the explicit zero-temperature limit of more general calculations [7].

Applying Eq. (1) and $|A_{q;k}|^2$ to the Fermi golden rule integrated over all allowed scattering modes, and using the Feynman relation [15], $S_q = E_q^0/E_q^B$, we arrive at the rate of excitation-condensate collisions

$$n\sigma_k^B v_k = 8\pi na^2 v_k \times \frac{1}{2k^2}\int_0^k dqq|A_{q;k}|^2 \frac{E_k^B - E_q^B}{\sqrt{1+(E_k^B - E_q^B)^2}}, \quad (2)$$

where $v_k = \hbar k\xi^{-1}/m$ is the free particle velocity of the excitations. The effective cross section σ_k^B for the quasiparticles is shown in Fig. 1(b). For large k, σ_k^B approaches $8\pi a^2$ (compared to $4\pi a^2$ for impurities, due to the boson quantum mechanical exchange term). In Eq. (2), for small k, we verify that the scattering rate indeed scales as k^5, which is the classic result [1,5]. In particular, it remains finite even for $v_k < v_c$, in contrast with impurity scattering.

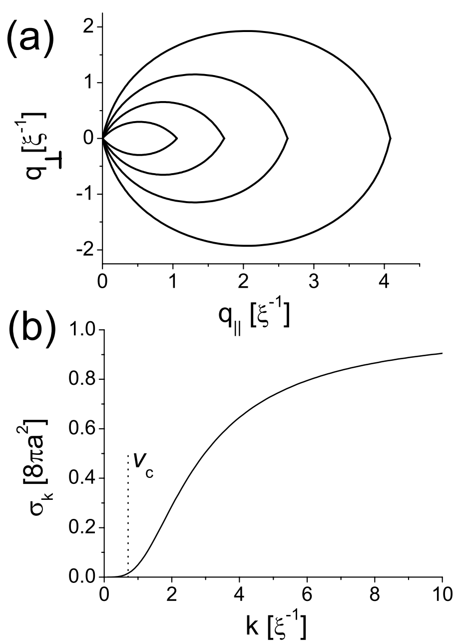

FIG. 1. (a) The allowed momentum manifold for quasiparticle collisions due to conservation of energy and momentum. The $q_\parallel$ and $q_\perp$ axes correspond to the parallel and orthogonal components to $\mathbf{k}$ of the scattered momentum, respectively, where $\tan(\theta) = q_\perp/q_\parallel$. The manifolds represent the experimental k's 4.09 (outermost), 2.63, 1.73, and 1.06 (innermost). The momentum is in units of the inverse healing length, $\xi^{-1} = \sqrt{8\pi na}$. (b) Cross section for collisions between quasiparticles and a homogeneous condensate, taken from Eq. (2). The cross section is in units of the free particle scattering cross section for identical particles $8\pi a^2$. V_c marks the superfluid critical velocity.

In [16], the identical particle collision cross section for large k was measured to be $2.1(\pm 0.3) \times 4\pi a^2$. Scattering rates in four-wave mixing experiments in BEC [17] were also shown to agree with the high-k limit of Eq. (2) [18].

In the opposite regime of extremely low wave number, where the energy levels are discrete, Beliaev damping was observed for the scissors mode of a BEC [19]. The discrete energy levels were tuned so that Beliaev damping of the initial mode to exactly one mode of half the energy was achieved. Equation (2) did not apply, since there was no need to integrate over various scattering modes.

Our experimental apparatus is described in [10]. Briefly, a nearly pure ($>95\%$) BEC of 10^5 ^{87}Rb atoms in the $|F, m_f\rangle = |2, 2\rangle$ ground state is formed in a quadrupole-Ioffe-configuration-type magnetic trap [20]. The trap is cylindrically symmetric, with radial ($\hat{r}$) and axial ($\hat{z}$) trapping frequencies of $2\pi \times 220$ Hz and $2\pi \times 25$ Hz, respectively. Thus $\xi = 0.24$ μm via averaging in the local density approximation (LDA) [10].

We excite quasiparticles at a well defined wave number using two-photon Bragg transitions [21]. The two Bragg beams are detuned 6.5 GHz from the $5S_{1/2}, F = 2 \rightarrow 5P_{3/2}, F' = 3$ transition. The frequency difference $\Delta\omega$ between the two lasers is controlled via two acousto-optical modulators. Bragg pulses of 1 msec duration are applied to the condensate. The angle between the beams is varied to produce excitations of various k along the z axis, thus preserving the cylindrical symmetry of the unperturbed BEC. The beam intensities are chosen to excite no more than 20% of the total number of atoms in the condensate.

After the Bragg pulse, the magnetic trap is rapidly turned off, and after a short acceleration period the interaction energy between the atoms is converted into ballistic kinetic energy [22]. After 38 msec of time-of-flight (TOF) expansion the atomic cloud is imaged by an on-resonance absorption beam, perpendicular to the z axis. Figure 2(a) shows the resulting absorption image for $k = 2.63$, with the large cloud at the origin corresponding to the BEC. A halo of scattered atoms is visible between the BEC and the cloud of unscattered outcoupled excitations. No excitations with energy greater than that of the unscattered excitations are observed, confirming our low estimate of the Landau damping rate. Figure 2(b) shows the absorption image for $k = 1.06$. For this k value the distinction between scattered and unscattered excitations is not clear in the image, since both types of excitations occupy the same region in space.

At a given k the number of excitations is varied by scanning $\Delta\omega$ around the resonance frequency ω_k^B. The number of excitations that was formed initially n_i is measured by determining the total momentum (in units of the recoil momentum $\hbar k\xi^{-1}$) contained in the outcoupled region outside the unperturbed BEC. This region

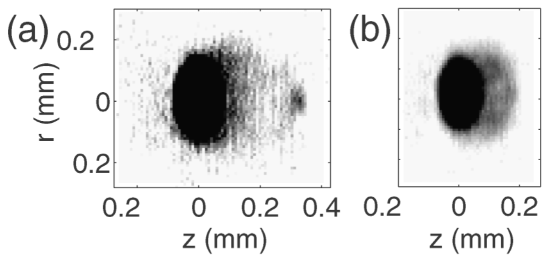

FIG. 2. Absorption TOF images of excited Bose-Einstein condensates. (a) Absorption image for $k = 2.63$, with the large cloud at the origin corresponding to the unperturbed BEC. A clear halo of scattered atoms is visible between the BEC and the cloud of unscattered outcoupled excitations. (b) Absorption image for $k = 1.06$. For this value of k the distinction between scattered and unscattered excitations is not clear, since both types of excitations occupy the same region in space.

includes all the scattered and unscattered excitations, in the direction of $\mathbf{k}$. Thermal effects are removed by subtracting the result of an identical analysis in the direction opposite to $\mathbf{k}$. The results are shown in Fig. 3(a).

In order to quantify the amount of collisions, despite the lack of separation between scattered and unscattered excitations, we take the ratio between n_i and the counted number of atoms n_f, in the same region. The resulting n_i/n_f, as a function of $\Delta\omega$, are shown in Fig. 3(b). The ratio is seen to be independent of the number of excitations and appears to be an intrinsic property of a single excitation. Each collision between an excitation and the condensate creates an additional excitation that is counted in the outcoupled region, increasing n_f, while the momentum (n_i) in this interaction is conserved. Thus the ratio, n_i/n_f, is a good quantifier of the amount of the collisions, even at low k [23]. Bosonic amplification of the collision rate [12], which would appear as minima in Fig. 3(b), is not observed.

At a given k we define the overall probability for an excitation to undergo the first collision p_k. If we ignore secondary collisions the result is $n_i/n_f = 1/(1 + p_k)$, since each collision outcouples, after TOF, an additional particle [24]. Using this relation we can infer p_k for the various measured k's.

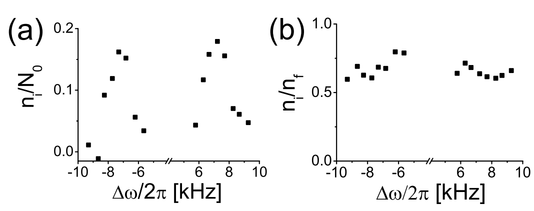

FIG. 3. Quantifying the amount of collisions measured for $k = 2.63$. (a) Measured momentum of the outcoupled atoms vs $\Delta\omega$. The momentum is in units of the recoil momentum and is normalized by the total number of atoms. The peaks represent a spectroscopic measurement of the resonance excitation energy at this k. (b) The measured ratio between n_i and n_f (both defined in the text). Every collision outcouples more atoms, increasing n_f but leaving n_i almost unchanged.

We expect the scattering probability p_k to be equal to $\tilde{n}\sigma_k^B v_k t_{\mathrm{eff}}$, where t_{eff} is the effective interaction time of the excitation with the condensate and $\tilde{n}$ is the average density.

We assume t_{eff} to be k independent, divide p_k by v_k, and arrive at a value that is proportional to the scattering cross section (since the $\tilde{n}$ is constant for all k). These assumptions will be tested below, but must be valid for sufficiently low k, for which the TOF expansion lowers the density rapidly, turning off collisions before the excitations move significantly.

The ratio, $p_k/(\tilde{n} v_k t_{\mathrm{eff}})$, is shown in Fig. 4 (■) and is seen to agree with the theoretical suppression (solid line) calculated as an LDA average [14] of Eq. (2) [25]. Since t_{eff} is not known, absolute calibration is not possible. Therefore, the results are shown with arbitrary units.

In order to verify the validity of the simplifying assumptions, 3D simulations are performed, using the hydrodynamic Gross-Pitaevskii equations [26]. The simulations [22] include the TOF dynamics. The expansion of the main BEC cloud is taken from expressions in [27], computed for an elongated condensate by the hydrodynamic equations. The excitations travel within the time varying mean-field potential created by the expanding unperturbed condensate. The excitations collide with the expanding condensate with the correct local scattering cross section, including the angular dependence, taken from Eq. (2). The collisional process involves interaction terms beyond mean field and therefore must be added explicitly to the simulation. The resulting distribution of

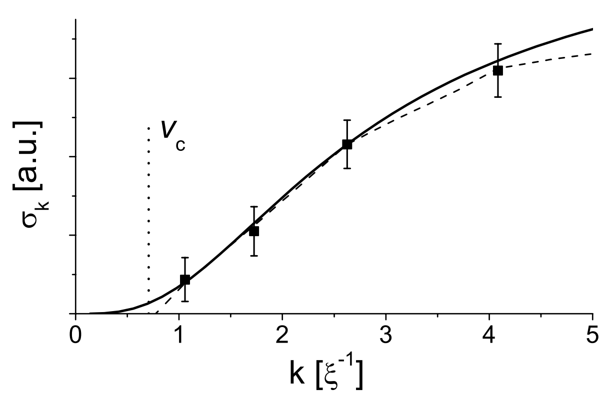

FIG. 4. Suppression of identical particle collisions. Scattering cross section is in arbitrary units. The momentum is in units of the inverse healing length after LDA averaging, $\xi = 0.24\ \mu$m. The error bars represent the statistical uncertainty of the experimental data. The theoretical curve (solid line) is an LDA average of Eq. (2) [25]. The assumptions of our analysis are tested using hydrodynamic simulations and are found to agree with Beliaev damping theory in the experimental regime (dashed line).

simulated outcoupled atoms is analyzed by the same method as the experimental data.

We plot the simulated p_k/v_k in Fig. 4 (dashed line, in the same units as theory). The simulation and Beliaev damping theory agree in the regime of experimentation, to better than 4%, indicating a window of validity for the assumptions used in analyzing the experimental data. At large k (above $k = 4.09$), t_{eff} is made shorter by the rapid transit of the condensate by the excitations. At low k (less than $k = 1.06$) many of the collisional products remain inside the condensate volume and are not counted, preventing analysis in this regime.

The arbitrary unit of suppression used in Fig. 4 was set at $k = 2.63$. The total collision probabilities p_k obtained by comparing the experimental data to the hydrodynamical simulations were higher by a k-independent overall factor of 2.36 ± 0.08, which is not understood. This factor may be caused by various inaccuracies in the TOF parameters of the simulation. However, the trend in the experimental analysis is robust and does not depend on absolute calibration.

We also set the collision rate artificially to zero in the simulation and find n_i/n_f to be unity within 2%, for all k. This implies that there are no significant other mean-field repulsion effects along the z axis [22], confirming our assumption of momentum conservation.

In conclusion, we report a measurement of the suppression of the collision cross section for identical particles within a Bose-Einstein condensate. We find the suppressions in our experiment in agreement with a calculation of Beliaev damping rates, within an overall factor. We use a hydrodynamic simulation of the expansion dynamics, in order to verify our analysis of the experiment. This represents the first measurement of this effect in the quasiparticle continuous spectrum regime.

This work was supported in part by the Israel Science Foundation.

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[23] For $k = 1.06$, there are atoms in front and behind the BEC that should be counted in our integration. We use computerized tomography [22] to find the correct atomic distribution. The systematic error caused by naively integrating the absorption image can be as large as 10%, which is unacceptable for our purposes. At higher k, these systematic integration effects are verified to be negligible.

[24] Hydrodynamical simulations (described below) indicate that there are few higher order multiple collisions for the experimental parameters. We also find that nearly all the collision products are indeed located in the counting region after TOF, confirming our momentum and atom number counting procedures.

[25] The LDA average of the suppression of collisions for a nonhomogeneous BEC is nearly indistinguishable from the suppression of a homogeneous condensate with the LDA average healing length as defined in [10].

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