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Decoherence and interferometric sensitivity of boson sampling in superconducting resonator networks

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Multiple bosons undergoing coherent evolution in a coupled network of sites constitute a so-called quantum walk system. The simplest example of such a two-particle interference is the celebrated Hong-Ou-Mandel interference. When scaling to larger boson numbers, simulating the exact distribution of bosons has been shown, under reasonable assumptions, to be exponentially hard. We analyze the feasibility and expected performance of a globally connected superconducting resonator based quantum walk system, using the known characteristics of state-of-the-art components. We simulate the sensitivity of such a system to decay processes and to perturbations and compare with coherent input states.

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Superconducting Josephson devices are a remarkable quantum information processing platform, with single and two qubit coherences close to and even surpassing fault tolerant thresholds [1,2]. In harmonic superconducting resonators, various nonclassical states have been formed on-demand and complex entangled states between such resonators have also been demonstrated, including NOON states of high order [3–5]. It is very important to benchmark the quality of entanglement achieved and to steadily expand the size of the systems under study. As such entanglement grows larger, it becomes a resource for potential sensing applications [6,7] and, more fundamentally, challenges various models of spontaneous collapse [8,9] or correlated error [10].

A relatively simple (experimentally) platform for such explorations is the recently reformulated problem of quantum walks and associated BosonSampling [11]. Indistinguishable bosons are placed in a coupled array of resonators and allowed to interfere via the noninteracting coherent quantum walk of the particles among the resonators. Assuming a closed system (without gauge fields), the boson network then evolves in time with the Hamiltonian

$$H = \hbar \sum_{i,j} J_{ij} \hat{a}_i^\dagger \hat{a}_j \tag{1}$$

where the summation is over all nodes in the graph, J_{ij} is the coupling strength between node i and j, and $\hat{a}_i$ is the ladder operator for resonator i—terms for which $i = j$ can be included to account for varying resonator oscillation energies [12].

The resulting distribution of occupation probabilities in the different resonators is (thought to be) both exponentially hard to compute and verify classically [11]. This means the problem belongs to the complexity class #P, which is considered larger and more difficult than the notorious NP class. Note, however, that BosonSampling is expected to be nonuniversal in the computational sense. Universality with a multiparticle quantum walk hardware was only recently established in the presence of interactions between the bosons [13].

Although initially this form of quantum simulation was thought to have no practical applications, it has recently been shown to be capable of simulating the vibronic spectra of molecules, if a nontrivial initial state can be created [14], in contrast, however, with the simple single photon inputs of BosonSampling.

Experimental implementations of such multiboson interference experiments have been carried out mostly in optical qubit experiments by several groups [15–18]. The main limiting factor in these experiments is the exponential overhead in generating the input few-particle state, due to the lack of a deterministic generation of photons. Low collection and detection efficiencies also limit the rate and scaling. Phonon evolution in ion trap arrays have also been implemented for quantum walks, but the geometry and coupling in this system limit the scaling of what is achieved to only a few particles. A theoretical proposal for gated implementation of BosonSampling for microwave photons in a linear array of superconducting resonators was also recently presented [19], followed by a proof of its supremacy [20].

In this Rapid Communication, we numerically analyze the potential of a superconducting qubit and resonator BosonSampling implementation with current state-of-the-art coherence times and known coupling and readout capabilities. A schematic design and layout of such a system is shown in Fig. 1. We analyze both sensitivity to relaxation and dephasing processes and compare the interferometric sensitivity of the network given either coherent states or single photons as inputs.

In our proposed quantum walk system, single microwave excitations (‘bosons’) are loaded into interface qubits and transferred coherently into the resonator network [23]. The qubits are tuned away from coupling resonance during the quantum walk evolution of the coupled network. The boson occupation numbers in each resonator are subsequently determined by dispersive microwave measurements on the qubits [24] after swapping them back at a predetermined time.

The advantages of superconducting BosonSampling lie in the on-demand state preparation and high fidelity readout [25] at ∼99%. Another striking strong point of this implementation is the possibility to arbitrarily couple many or even all the resonators to each other and not just along a one-dimensional array [19,26].

In order to numerically analyze this proposal we have simulated the state preparation (boson loading) fidelity and the subsequent evolution fidelity. The Hilbert space of N bosons

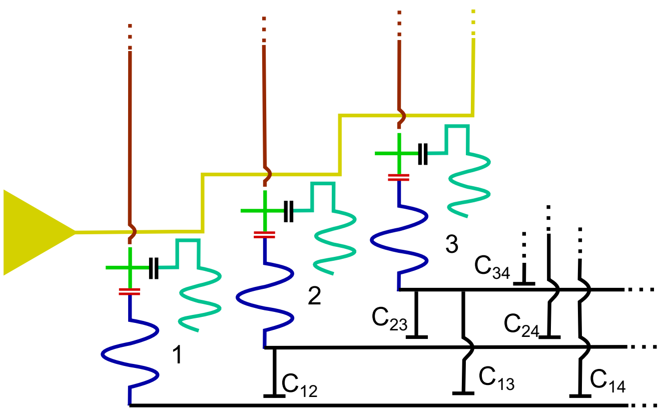

FIG. 1. A scheme for implementing BosonSampling. The resonators are loaded from qubits (here represented as the green schematic xmons [21]), and their final state is dispersively measured from readout resonators (magenta) at the desired time when qubits are brought back into resonance with their respective resonators (blue). The resonators are capacitively coupled to each other (black waveguides) with capacitance C_{ij}, and this graph is made possible by air bridges [22] (arcs on control lines and graph coupling lines) that add a negligible and known capacitive coupling between lines.

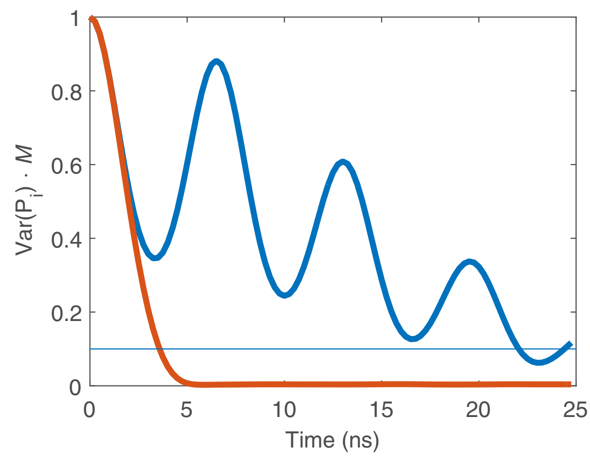

FIG. 2. Normalized variance of the occupation probabilities for a single boson in the network $Var(\langle P_i(t)\rangle)M$ for $M=10$ (red line for $M=250$) randomly and globally connected resonators. The horizontal line ($\frac{1}{M}$ for $M=10$) intersects the blue curve at $T_{\rm rich}$. Note that the normalized variance can rise again over $1/M$ as there is no decay in this simulation. For 250 resonators the richness time is reduced to ~5 ns.

evolving in M resonators (including all decay channels) is of dimension $D=\binom{N+M}{M}$, so $D\approx 3\times 10^7$ for $N=10$ and $M=20$, and as the system evolves in time full matrices must be used with $\approx 10^{15}$ entries per time step. Some features can be approximated by contour integrals methods [27], but not all. Thus, a sufficiently high fidelity BosonSampling machine simulation will quickly grow beyond the simulation capabilities of classical computers. Furthermore, as an open physical system it will also be exposed to errors (in particular decoherence effects and perturbations to network couplings). It is the main purpose of this paper to quantify the errors accumulated, subsequently defining the limits of performance for the proposed BosonSampling device.

In order to reach this goal, we must first determine the necessary simulation duration in order to produce nontrivial results. This is the time period needed for a single boson evolving in the array to establish significant density matrix components throughout (~25 ns in this case), and we term this value the "Richness Time." Hence, a sufficiently complex boson pattern is expected since all the particles have a significant chance of meeting throughout the array, with subsequent multiparticle interference. We define the richness time in terms of the variance of the expected occupations of all possible states accessible to the system by the requirement

$$Var(P_i(T_{\rm rich})) = 1/M^2, \tag{2}$$

where $P_i(t)$ is the occupation probability of each state as a function of time t.

In Fig. 2 we plot the evolution of the normalized variance for a single boson coherently diffusing in the network for two different network sizes ($M=10$ and $M=250$). We have found $T_{\rm rich}$ to decrease slowly with the number of resonators. This can be intuitively understood—as the number of resonators coupled to the initially populated resonator grows, we expect the initial mode to be depleted more quickly.

Having established the evolution time necessary for sufficient mixing of the multiparticle walk, we now turn to the main results of this work. In all numerical experiments, we calculate only a moderate exponential increase in the distribution distance $\Delta(\rho(t),\sigma(t))$ over time, where $\rho(t)$ and $\sigma(t)$ are the density matrices after coherent and decohered time evolutions, respectively. The distribution distance is defined $\Delta(\rho,\sigma)=\|D_\rho - D_\sigma\|_1$, i.e., the 1-norm distance between the vectors D_ρ and D_σ, which represent probability distributions in ρ and σ (the matrix diagonals). We find this quantity to fit the phenomenological formula:

$$\Delta(\rho(t),\sigma(t)) = 2\big(1-e^{-\frac{Nt}{3}(\frac{5}{2}\frac{1}{T_1}+\frac{1}{T_\phi})}\big), \tag{3}$$

where T_1 and T_ϕ are the energy relaxation and pure dephasing times of the resonators. High quality, planar, state-of-the-art superconducting resonators and qubits have demonstrated energy decay times T_1 of ≈ 50 μs and T_ϕ, the dephasing time, is here conservatively assumed to be ≈ 50 μs [28].

An alternative quantitative estimator of the effects of decoherence is the trace distance, $\mathcal{D}(\rho,\sigma)$. Experimentally, the trace distance is not as easily measured as $\Delta(\rho,\sigma)$ but allows some deeper insight into the evolution of the quantum phases than $\Delta(\rho,\sigma)$. This and other features of the trace distance are discussed in the Supplemental Material [29].

We observe in our simulations that the number of resonators does not appear directly to affect the decay and only enters implicitly through the requirement of achieving the richness time of fully propagating individual bosons throughout the array. The reliability of this empirical approximation can be seen in Fig. 3, as we scan the different parameters of our simulation and plot Eq. (3). For all panels (unless the parameter is explicitly scanned in the x axis), we set $T_1 = 50$ μs, $T_\phi = 50$ μs, three excitations, ten resonators, and random global couplings with a uniform distribution between 20 and 40 MHz. The same randomly generated coupling graph was used in

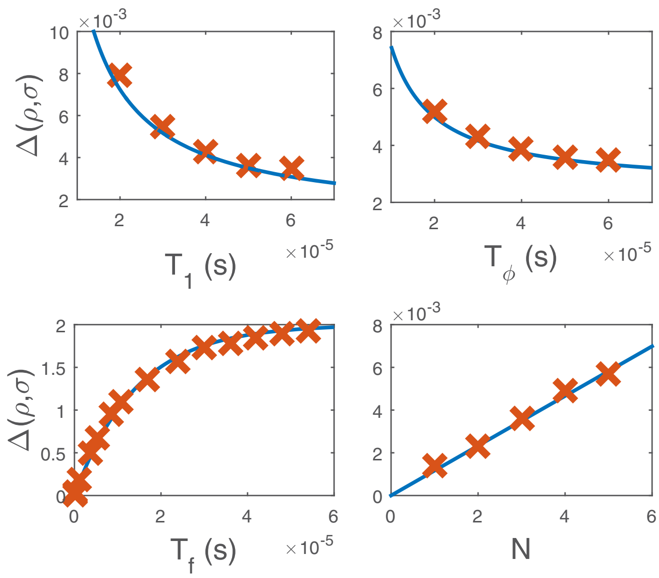

FIG. 3. Distribution distance, $\Delta(\rho,\sigma)$ for a decohering (with relaxation $T_1 = 50\,\mu s$ and dephasing $T_\phi = 50\,\mu s$ processes) quantum walk of $N = 3$ bosons in a 10 site resonator network, simulated up to $T_f = 25$ ns. In each panel plot we simulate the outcome of varying one of the above default values and compare to the phenomenological Eq. (3). Top left: varying T_1, top right: varying T_ϕ, bottom left: varying T_f, bottom right: varying N.

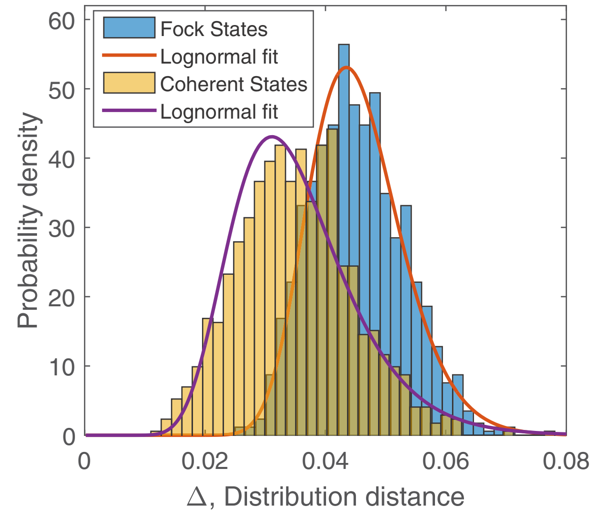

FIG. 4. Histogram of the distribution distances, Δ, for initial single photons in resonators and coherent states. The simulation was carried out in a 10 site network, with an ensemble of perturbations, and $N = 3$ bosons. The same perturbations were applied for both kinds of initial states. For the coherent state a classical "amplitude simulation" was performed, with $\alpha = 1$ in three resonators in the initial state.

all simulations (iterations performed using other randomly generated graphs with similar energies did not reveal any remarkable difference). The exponential growth of the required Hilbert space required extensive runs (over a few hours per run) on a powerful desktop computer to complete simulations for $N > 4$.

Extrapolating to larger resonator arrays and boson numbers, we observe that the mild effects of dephasing allow realistically extending to N of order 20 with currently available coherence times, clearly growing beyond the capabilities of modern classical supercomputing. We note that the fidelity will not be limited by decoherence and decay, rather by loading time for the resonators, initial state fidelity and/or readout fidelity, which are still being developed.

The mild scaling with N is a somewhat surprising result, as the dephasing Kraus operators cause an N^2 faster loss rate of fidelity for a superposition of N bosons and the vacuum state in a single resonator. In this model, however, the initial state diffuses on a very fast time scale between all the modes and the overall state seems to be resistant to dephasing. This is reminiscent of the recent result of Motes *et al.* where BosonSampling was investigated in the context of metrology [30]. In addition, the lack of interactions obviously limits the exploited phase space and degree of entanglement in such a system.

We test the sensitivity of our system by randomly perturbing the couplings between resonators. An ensemble of perturbation graphs is generated (again with random couplings uniformly chosen between 20 and 40 MHz) and added to the reference graph with a pre-factor of 10^{-3}. This procedure establishes a slightly altered Hamiltonian by which the system evolves—here without decoherence operators. The subsequent distribution distance between a perturbed and reference evolution after a simulation of $2T_{\rm rich}$ is calculated. Specifically, we take a random-coupling network of 10 resonators and generate 1000 perturbed graphs. The distribution distances yielded by perturbations in the graphs are now computed for both initial Fock states (one photon in each of three different resonators) and initial coherent states (with an average occupation of one photon in three of the resonators) to distinguish classical interferometric sensitivity vs many-body effects.

Figure 4 shows the distribution distances for many realizations of perturbed couplings, and we observe a definite difference between the coherent state inputs and single photons. Both histograms of distribution distances are fit well by log-normal distributions, but the input Fock state is less robust to perturbations than the coherent states. This is promising as it indicates higher interferometric sensitivity for the quantum walk interferometry vs classical (coherent state) probing. Surprisingly, the result is the opposite for the trace distance metric described above [29].

The higher sensitivity of the Fock states is enhanced as the number of bosons increases. To quantify this effect, we define the "distribution overlap:"

$$\mathcal{S} = \int dx \sqrt{f_{\rm Fock} f_{\rm coh}}, \tag{4}$$

where $f_{\rm Fock}$ and $f_{\rm coh}$ are the probability distributions emerging from iterations with initial Fock states and coherent states, respectively, as those in Fig. 4. $\mathcal{S}$ equals unity, when the two distributions are identical. Figure 5 shows the decrease of $\mathcal{S}$ with growing N.

Another important aspect of the analysis is to address the problem of BosonSampling verification. Obviously the classical computer simulation can serve for verifying the proper distribution and correlations in small boson number

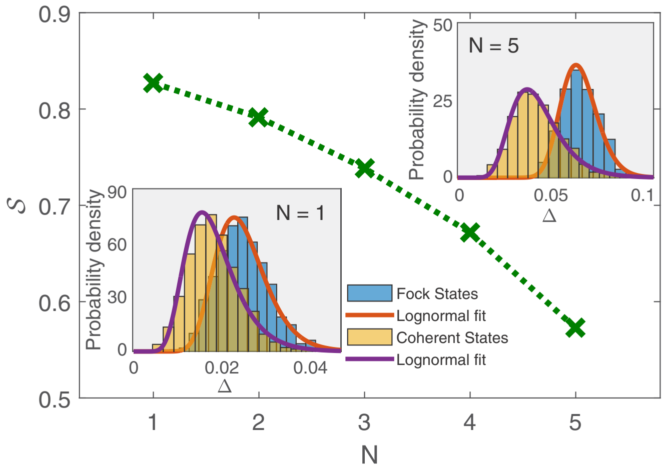

FIG. 5. The distribution overlap, $\mathcal{S}$ as defined in Eq. 4 vs N, the number of bosons, decreases, as N grows. Inserts visualize $\mathcal{S}$ and this effect as the shaded region for $N = 1$ and $N = 5$.

implementations. For large boson numbers, the system may be treated classically [31]. However, in the intermediate regime quantum effects dominate, and a classical computer will fail.

We note also connections to the extensively investigated Anderson localization phenomena; disorder causes a transition from ballistic propagation to localization of particles traveling in a lattice. This has been demonstrated with photons traveling in a two-dimensional coupled lattice of waveguides, a system closely related to our proposal [32,33].

For our fully connected graph, symmetry and interference of the propagating excitations create the exact opposite effect. When disorder is removed from the system and all couplings are made equal, the excitations become confined to their initial resonators since they are detuned from the dressed reservoir of other sites—it is the disorder in interaction strengths that releases them. This effect becomes more pronounced as the number of resonators is increased, starting with simple sloshing between two modes. For the same reasons, when initially placing an equal amount of excitations in each resonator, there is no change in the occupation of the resonators. This is of course true even with disorder.

Our results bode well for a continuous quantum walk implementation of BosonSampling on superconducting Josephson devices with currently achievable lifetimes. From the above analysis, it seems that building a BosonSampling device capable of calculations beyond the abilities of classical computers is within reach.

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[29] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.95.020502 for the alternative formalism of trace distances, revealing deeper insight into the decoherence processes, alongside with a bound on the distribution distance.

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