Parametric Down Conversion

Quantum optics seminar 77740

Winter 2008

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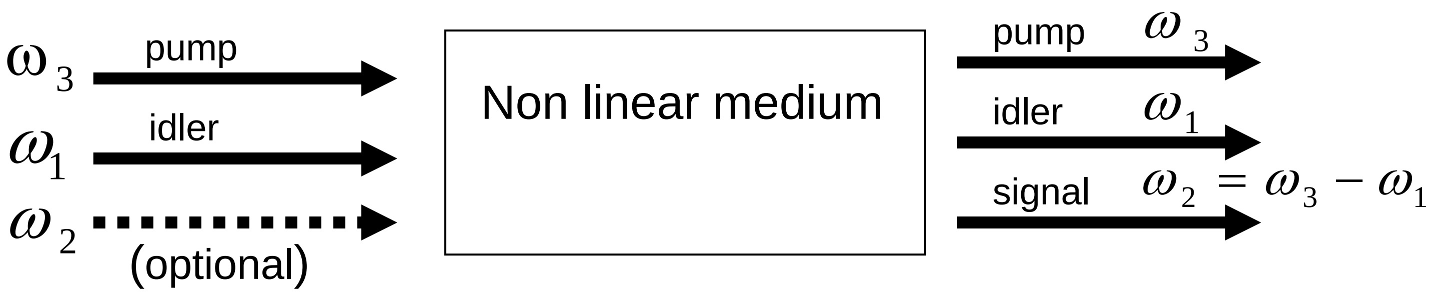
context

- Introduction
- Theory of classic Sum Frequency Generation
- Quantum Hamiltonian for PDC
- Applications

Non linear optics

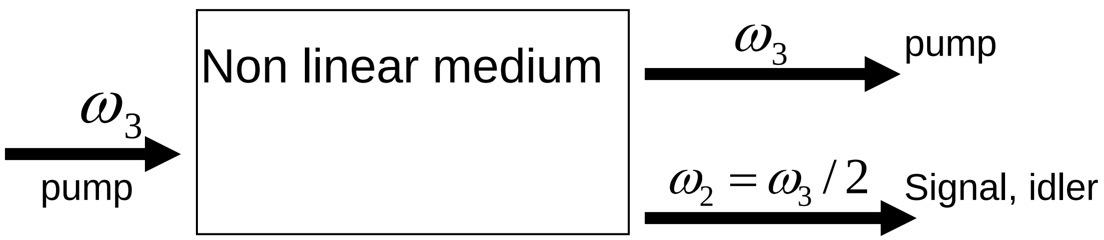
- Creation of new frequencies in the system.
- Usually, the new frequencies have small amplitude relative to the input frequencies.

Harmonic Difference Generation



PDC: the degenerate case

$$\omega_1 = \omega_2 = \omega_3 / 2$$



Nonlinear medium

Non linear dependency of the polarization in the electrical field:

c.g.s

$$\vec{D} = \vec{E} + 4\pi\vec{P}$$

$$\begin{aligned}
\vec{P}(t) &= \overleftrightarrow{\chi}^{(1)}\vec{E}(t) + \overleftrightarrow{\chi}^{(2)}\vec{E}^{2}(t) + \overleftrightarrow{\chi}^{(3)}\vec{E}^{3}(t) + \ldots = \\
&= \vec{P}^{(1)}(t) + \vec{P}^{(2)}(t) + \vec{P}^{(3)}(t) + \ldots = \\
&= \vec{P}^{(1)}(t) + \vec{P}^{NL}(t)
\end{aligned}$$

The susceptibility $\overleftrightarrow{\chi}$ is a tensor.

$$\vec{D} = \vec{E} + 4\pi\left(\vec{P}^{(1)}(t) + \vec{P}^{NL}(t)\right) = \varepsilon^{(1)}\vec{E} + 4\pi\vec{P}^{NL}(t)$$

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{d\vec{B}}{dt}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \frac{1}{c}\frac{d\vec{D}}{dt} + \frac{4\pi}{c}\vec{J}$$

Substituting in Maxwell equation gives:

$$-\nabla^2\vec{E} + \nabla(\nabla \cdot \vec{E}) + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = -\frac{4\pi}{c}\frac{\partial^2 \vec{P}}{\partial t^2}$$

Assuming $\nabla^2\vec{E} >> \nabla(\nabla \cdot \vec{E})$ ($\rho = 0$, Slowly varying Amplitude) and we get the wave equation in a medium:

$$-\nabla^2\vec{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = -\frac{4\pi}{c}\frac{\partial^2 \vec{P}}{\partial t^2}$$

Representing the frequency component:

$$\vec{E}(r,t) = \sum_n \vec{E}_n(r,t) = \sum_n \vec{E}_n(r) \cdot e^{-i\omega_n t} + c.c.$$

$$\vec{P}^{NL}(r,t) = \sum_n \vec{P}_n^{NL}(r,t) = \sum_n \vec{P}_n^{NL}(r) \cdot e^{-i\omega_n t} + c.c.$$

For every n we have:

$$-\nabla^2\vec{E}_n(r) - \frac{\omega_n^{\;2}}{c^2}\varepsilon^{(1)}\vec{E}_n(r) = -\frac{4\pi\omega_n^{\;2}}{c}P_n^{NL}(r)$$

Properties of the second order susceptibility

$$P_i(\omega_n + \omega_m) = \sum_{j,k} \sum_{n,m} \chi^{(2)}_{ijk}(\omega_n + \omega_m, \omega_n, \omega_m) E_j(\omega_n) E_k(\omega_m)$$

- Symmetries reduce the number of numbers need to describe $\overleftrightarrow{\chi}^{(2)}$.
- $\overleftrightarrow{\chi}^{(2)}$ is real for Lossless media.
- Kleinman symmetry: (ω_n, ω_m) is far away from $\omega_{resonance}$ of the media.

$$P_i(\omega_n + \omega_m) = \sum_{j,k} \sum_{n,m} \chi^{(2)}_{ijk} E_j(\omega_n) E_k(\omega_m)$$

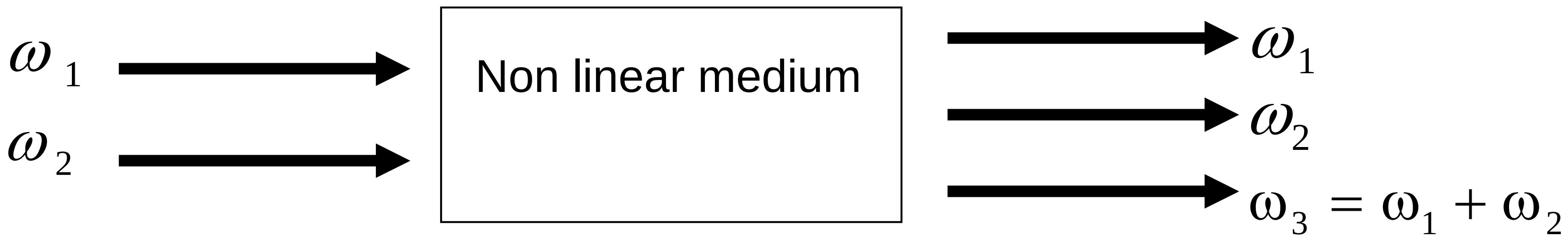
$$P(\omega_3) = 4 d_{eff} E(\omega_1) E(\omega_2)$$

For the degenerate case:

$$P(2\omega) = 2 d_{eff} E^2(\omega)$$

Theory of classic Sum Frequency Generation

Inverse process of Harmonic Difference Generation



We concentrate on the ω_3 wave, defining:

$$\vec{E}_3(z,t) = A_3 e^{i(k_3 z - \omega_3 t)} + c.c. \qquad k_3 = \frac{n_3 \omega_3}{c}, \qquad n_3 = \sqrt{\varepsilon^{(1)}(\omega_3)}$$

$$P_3(z,t) = P_3 e^{-i\omega_3 t} + c.c. = 4d_{eff} E_1 E_2 e^{-i\omega_3 t} + c.c. = 4d_{eff} A_1 A_2 e^{i(k_1+k_2)} e^{-i\omega_3 t}$$

Substituting in $-\nabla^2 \vec{E}_n(r) - \frac{\omega_n^2}{c^2}\varepsilon^{(1)}\vec{E}_n(r) = -\frac{4\pi\omega_n^2}{c} P_n^{NL}(r)$ gives

$$\frac{d^2 A_3}{d^2 z} + 2ik_3 \frac{dA_3}{dz} = -\frac{16\pi d_{eff}\,\omega_3^2}{c^2} A_1 A_2 e^{i(k_1+k_2-k_3)z}$$

Slowly varying Amplitude approximation: $\left|\frac{d^2 A_3}{d^2 z}\right| << \left|k_3 \frac{dA_3}{dz}\right|$

We find the dependency of A_3 in A_1 and A_2

$$\frac{dA_3}{dz} = 8\pi i \frac{d_{eff}\,\omega_3{}^2}{k_3 c^2} A_1 A_2 e^{i\Delta kz} \qquad \left(\Delta k = k_1 + k_2 - k_3\right)$$

By the same way we find also that

$$\begin{cases} \dfrac{dA_1}{dz} = 8\pi i \dfrac{d_{eff}\,\omega_1{}^2}{k_1 c^2} A_3 A_2{}^* e^{-i\Delta kz} \\ \dfrac{dA_2}{dz} = 8\pi i \dfrac{d_{eff}\,\omega_2{}^2}{k_2 c^2} A_3 A_1{}^* e^{-i\Delta kz} \end{cases}$$

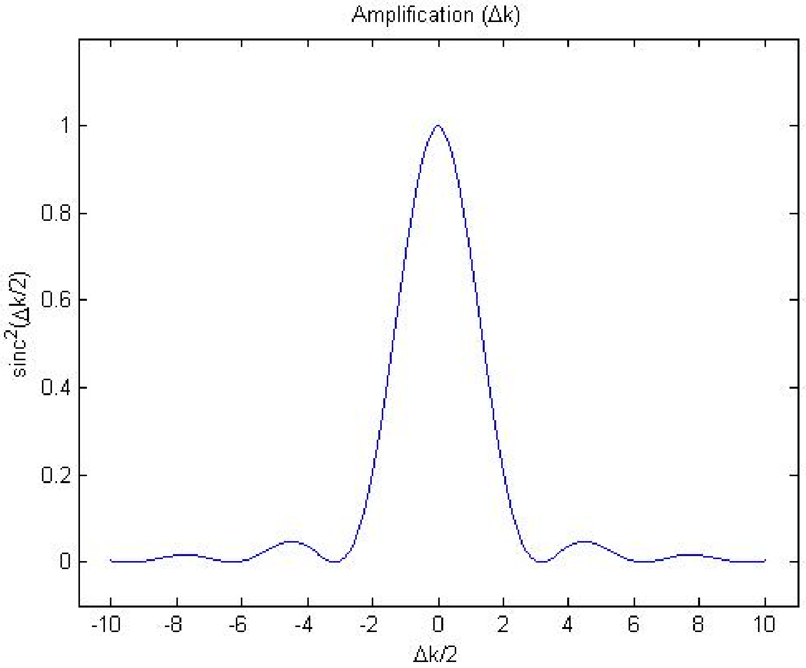
Non depleting pump approximation: $A_3 << A_1, A_2$

$$A_3(L) = 8\pi i \frac{d_{eff}\,\omega_3{}^2}{k_3 c^2} A_1 A_2 \int_0^L e^{i\Delta kz} dz = 8\pi i \frac{d_{eff}\,\omega_3{}^2}{k_3 c^2} A_1 A_2 \left(\frac{e^{i\Delta kL} - 1}{i\Delta k}\right)$$

L =non linear medium length

$$I_3 = \frac{n_3 c}{2\pi}\left|A_3\right|^2 = \frac{n_3 c}{2\pi} 64\pi^2 \frac{d_{eff}{}^2 \omega_3{}^4}{k_3{}^2 c^4}\left|A_1\right|^2\left|A_2\right|^2\left|\left(\frac{e^{i\Delta kL} - 1}{i\Delta k}\right)\right| = 32\pi \frac{d_{eff}{}^2 \omega_3{}^4 n_3}{k_3{}^2 c^3}\left|A_1\right|^2\left|A_2\right|^2 L^2 \mathrm{sinc}^2\left(\frac{\Delta kL}{2}\right)$$

$$I_3 = 512\pi^5 \frac{d_{eff}^{\ 2} I_1 I_2}{n_1 n_2 n_3 \lambda_3^{\ 2} c} L^2 \mathrm{sinc}^2\left(\frac{\Delta k L}{2}\right)$$



Implications:

- The intensity is quadratic in L if $\Delta k = 0$.
- $I_3 \propto I_1 \cdot I_2$.
- If $\omega_1 = \omega_2$ (Second Harmonic Generation), then

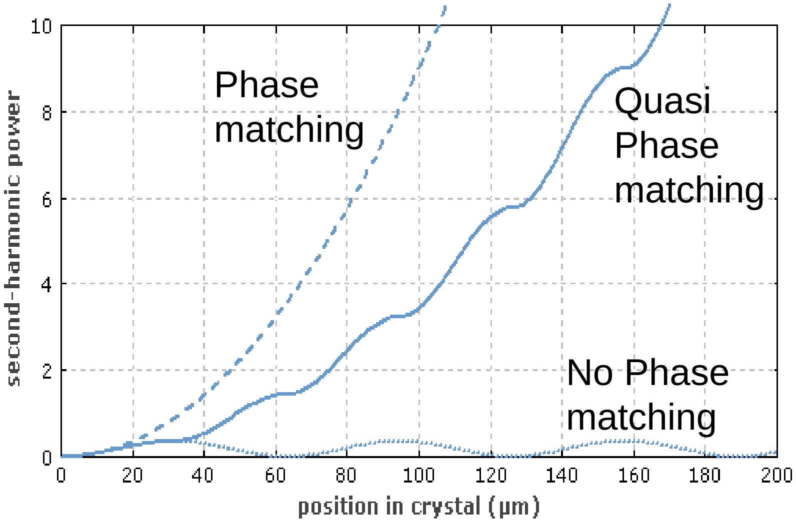
$$I_3(\omega_3 = 2\omega_1) \propto I_1^{\ 2}(\omega_1)$$

Method to achieve phase matching $k_1 + k_2 - k_3 = 0$

We need to control the refraction index n for every

frequency:

- Angle tuning - using birefringence properties of the nonlinear crystal.
  (KDP - $KH2PO_4$, BBO - Beta-BaB_2O_4)
- Temperature tuning
  (lithium niobate $LiNbO_3$)
- Quasi phase matching
  ($LiNbO_3$, $LiTaO_3$,
  KTP - KTiOPO4)



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Birefringence

$$k_1 + k_2 - k_3 = 0$$

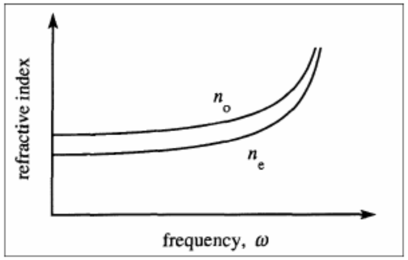
$$\omega_3 = \omega_1 + \omega_2$$

assume $\omega_3 \geq \omega_2 \geq \omega_1$

$$n_1\omega_1 + n_2\omega_2 = n_3\omega_3$$

$$n_3 - n_2 = (n_1 - n_2)\frac{\omega_1}{\omega_3}$$

For normal dispersion, $n(\omega)$ is rising monotonically



Sollution: using birefringence crystal

$$n(\omega_1, j_1)\omega_1 + n(\omega_2, j_2)\omega_2 = n(\omega_3, j_3)\omega_3$$

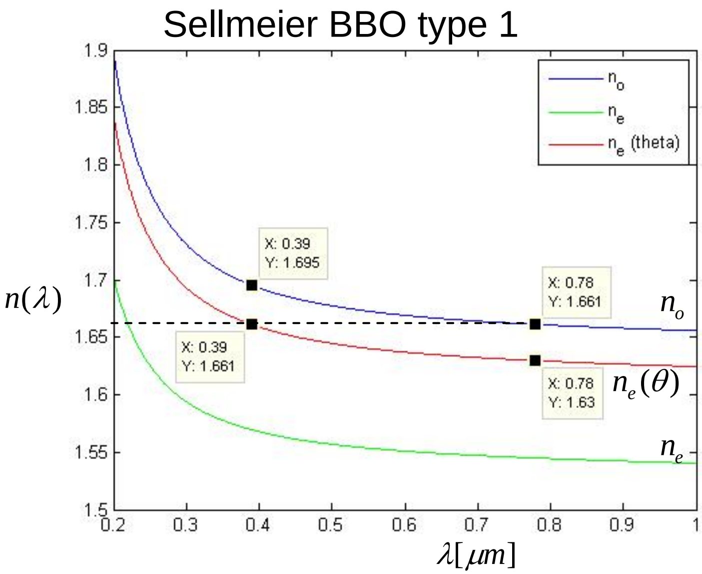
$$j = e, o$$

Example (for type 1 uniaxial)

$$\omega_3 = 2\omega \qquad (\omega_1 = \omega_2 = \omega)$$

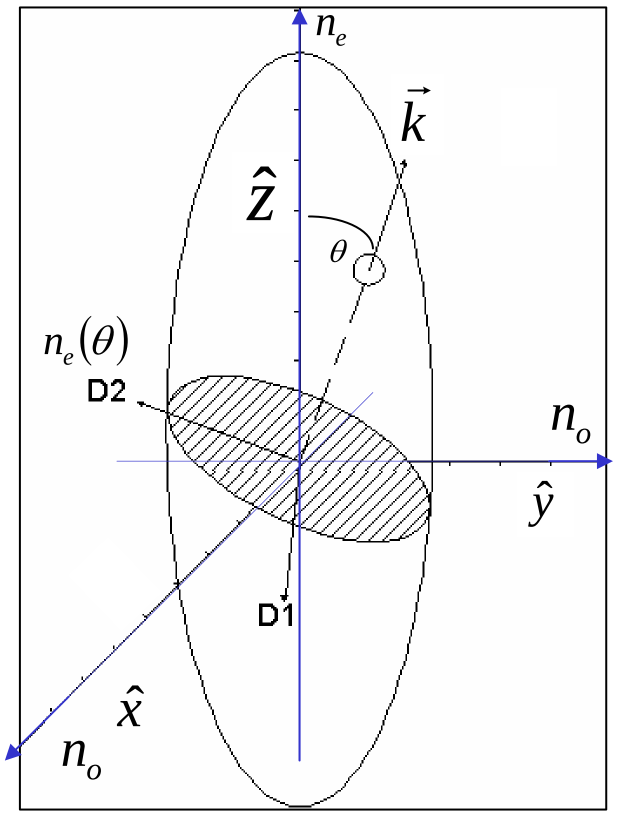
$$j_1 = j_2 = o, \quad j_3 = e$$

$$2 \cdot n(\omega, j_1 = o)\omega = n(\omega_3, j_3 = e)\omega_3 = n(\omega_3, j_3 = e)2 \cdot \omega$$



$$n_{e\theta}(2\omega, \ \lambda = 0.39\mu m) = n_o(\omega, \ \lambda = 0.78\mu m)$$

Index ellipsoid for uniaxial crystals



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- z is the optical axis (extraordinary).
- x, y are ordinary axes.

$$\frac{x^2}{n_o^2}+\frac{y^2}{n_o^2}+\frac{z^2}{n_e^2}=1$$

- Polarization D1 is on the x-y plane and have n_o.
- Polarization D2 have $n_e(\theta)$.

$$\frac{1}{n_e(\theta)^2}=\frac{\sin^2(\theta)}{n_e^2}+\frac{\cos^2(\theta)}{n_o^2}$$

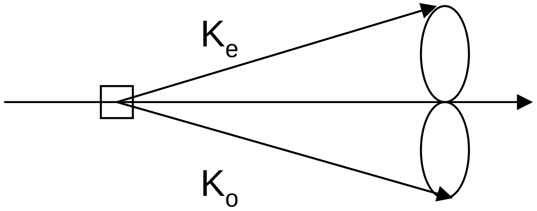
Properties of birefringence crystals

- For uniaxial crystals:

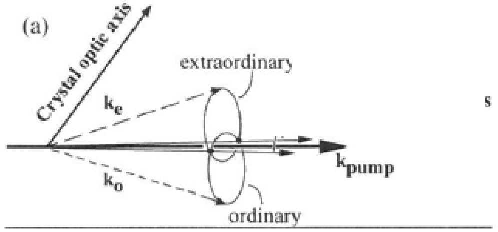
	Positive uniaxial $(n_e > n_o)$	Negative uniaxial $(n_e < n_o)$
Type 1	$n_{o3}\omega_3 = n_{e1}\omega_1 + n_{e2}\omega_2$	$n_{e3}\omega_3 = n_{o1}\omega_1 + n_{o2}\omega_2$
Type 2	$n_{o3}\omega_3 = n_{o1}\omega_1 + n_{e2}\omega_2$	$n_{e3}\omega_3 = n_{e1}\omega_1 + n_{o2}\omega_2$

- Propagating direction examples:

Collinear,



non collinear



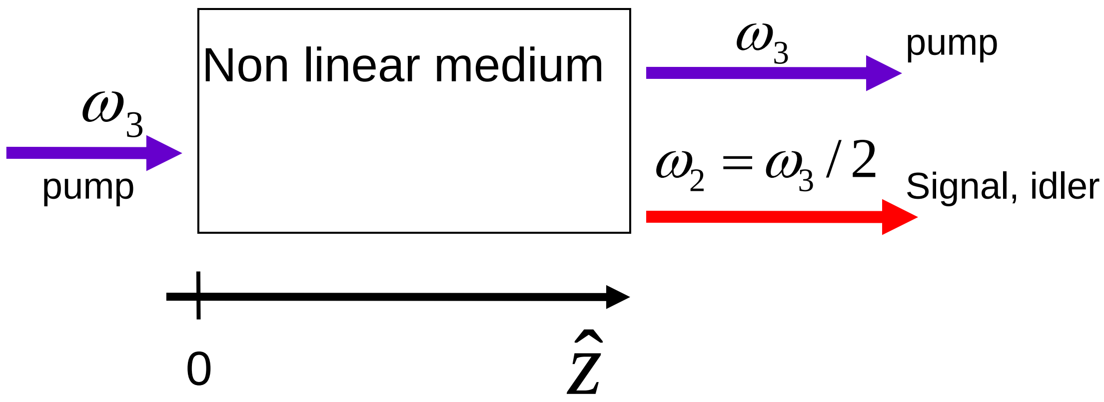
- Walk off: spatial, temporal.

classical parametric down conversion is impossible

$$\omega_1 = \omega_3 = \omega_2 / 2$$

$$I_3 = 512\pi^5 \frac{d_{eff}{}^2 I_1 I_2}{n_1 n_2 n_3 \lambda_3{}^2 c} L^2 \mathrm{sinc}^2\left(\frac{\Delta k L}{2}\right)$$

If $I_1(z=0) = I_{idler}(z=0) = 0$ there will be no PDC according to the classical theory.



=> We have to use Quantum mechanics

Quantum Hamiltonian for Spontaneous Parametric Down Conversion

Classical electrical energy in non linear media:

$$H = \frac{1}{8\pi}\int B^2(r,t)d^3r + \frac{1}{8\pi}\int d^3r \int_0^{D(r,t)} E(r,t)\cdot dD(r,t) =$$

$$= \frac{1}{8\pi}\int B^2(r,t)d^3r + \frac{1}{8\pi}\int d^3r \int_0^{D(r,t)} E(r,t)\cdot d(\varepsilon^{(1)}E + 4\pi P^{NL})$$

$$P_i = \chi_{ij}^{(1)}(\omega_1;\omega_1)E_j(\omega_1) + \chi_{ijk}^{(2)}(\omega_1;\omega_1-\omega_2,\omega_2)E_j(\omega_1-\omega_2)E_k(\omega_2) +$$

$$+ \chi_{ijkl}^{(3)}(\omega_1;\omega_1-\omega_2-\omega_3,\omega_2,\omega_3)E_j(\omega_1-\omega_2-\omega_3)E_k(\omega_2)E_l(\omega_3) + \ldots.$$

$$H = \frac{1}{8\pi}\int B^2(r,t)d^3r + \frac{1}{8\pi}\int d^3r(\varepsilon^{(1)}E^2 + X_1(r) + X_2(r) + \ldots)$$

$$X_1(r) = 4\pi\frac{1}{2}\iint d\omega d\omega' \chi_{ij}^{(1)}(\omega,\omega')E_i(r,\omega')E_j(r,\omega)$$

$$X_2(r) = 4\pi\frac{1}{3}\iiint d\omega d\omega' d\omega'' \chi_{ijk}^{(2)}(\omega'';\omega-\omega',\omega')E_i(r,\omega'')E_j(r,\omega'-\omega)E_k(r,\omega)$$

Second quantization for the $X_2(r)$ term:

$$X_2(r) = 4\pi \frac{1}{3} \iiint d\omega d\omega' d\omega'' \chi_{ijk}^{(2)}(\omega''; \omega - \omega', \omega') E_i(r, \omega'') E_j(r, \omega' - \omega) E_k(r, \omega)$$

Define:

$$E_j^{(+)}(\omega) = \sum_k E_{jk} \hat{a}_{jk} e^{i(kr - \omega_{jk} t)}$$

$$E_{jk} = \sqrt{\frac{\hbar \omega_{jk}}{n_{jk} V}} \qquad j = 1,2 \qquad V = quatization\ volume$$

$\hat{a}_{jk}$ is the annihilation operator for j,k mode

$$H_{X_2} \propto \int d^3 r \left(\chi E^{(+)}(\omega) E^{(-)}(\omega - \omega') E^{(-)}(\omega'') + \chi E^{(-)}(\omega) E^{(+)}(\omega - \omega') E^{(+)}(\omega'') \right)$$

In the degenerate case, when $\omega = \omega'$:

$$H_{X_2} \propto \hbar \chi [\hat{a}_0(2\omega) \hat{a}_1^{+}(\omega) \hat{a}_1^{+}(\omega) + \hat{a}_0^{+}(2\omega) \hat{a}_1(\omega) \hat{a}_1(\omega)]$$

Normal ordering, and assuming classical pump

$$a_0 = \nu_0 e^{-2\omega t}, \qquad \langle \hat{n}_1(t) \rangle = \langle \hat{n}_2(t) \rangle << |\nu_0|^2$$

χ =Coupling constant

$$H_{X_2} \propto \hbar \chi [\hat{a}_1^{+}(\omega) \hat{a}_1^{+}(\omega) \nu_0 e^{-i2\omega t} + c.c.]$$

Equations of motion

Slowly complex mode amplitudes

$$\hat{A}_1(t) = \hat{a}_1(t)e^{i\omega_1 t}$$

$$\hat{A}_2(t) = \hat{a}_2(t)e^{i\omega_2 t}$$

$$\frac{d\hat{A}_1(t)}{dt} = \frac{1}{i\hbar}[\hat{A}_1(t), \hat{H}_{\chi_2}] = -i\chi\nu_0\hat{A}_2^{+}(t)e^{i(\omega_1+\omega_2-\omega_0)t} = -i\chi\nu_0\hat{A}_2^{+}(t)$$

$$\frac{d\hat{A}_2(t)}{dt} = \frac{1}{i\hbar}[\hat{A}_2(t), \hat{H}_{\chi_2}] = -i\chi\nu_0\hat{A}_1^{+}(t)e^{i(\omega_1+\omega_2-\omega_0)t} = -i\chi\nu_0\hat{A}_1^{+}(t)$$

$$\frac{d^2\hat{A}_1(t)}{dt^2} = \chi^2|\nu_0|^2\hat{A}_1(t)$$

$$\frac{d^2\hat{A}_2(t)}{dt^2} = \chi^2|\nu_0|^2\hat{A}_2(t)$$

$$\hat{A}_1(t) = \hat{A}_1(0)\cosh(\chi|\nu_0|t) - ie^{i\theta}\hat{A}_2^{+}(0)\sinh(\chi|\nu_0|t)$$

$$\hat{A}_2(t) = \hat{A}_2(0)\cosh(\chi|\nu_0|t) - ie^{i\theta}\hat{A}_1^{+}(0)\sinh(\chi|\nu_0|t)$$

$$\nu_0 = |\nu_0|e^{i\theta}$$

Photon statistics

The r'th moment of $\hat{n}_1^{\,r}(t)$:

$$\left\langle \hat{n}_1^{\,r}(t) \right\rangle = < \Psi \,|\, \hat{A}_1^{+r}(t)\hat{A}_1^{\,r}(t) \,|\, \Psi > = {}_{1,2}< 0 \,|\, \hat{A}_1^{+r}(t)\hat{A}_1^{\,r}(t) \,|\, 0 >_{1,2} =$$

$$= {}_{1,2}< 0 \,|\, \left[\hat{A}_1^{+}(0)\cosh(\chi|\nu_0|t) + ie^{-i\theta}\hat{A}_2(0)\sinh(\chi|\nu_0|t)\right]^r \left[\hat{A}_1(0)\cosh(\chi|\nu_0|t) - ie^{i\theta}\hat{A}_2^{+}(0)\sinh(\chi|\nu_0|t)\right]^r \,|\, 0 >_{1,2} =$$

$$= \left| \left(-ie^{i\theta}\right)^r \sinh^r(\chi\,|\,\nu_0\,|\,t) A_2^{+r}(0) \,|\, 0 >_{1,2} \right|^2 =$$

$$= \sinh^{2r}(\chi\,|\,\nu_0\,|\,t)\, {}_{1,2} < 0 \,|\, \hat{A}_2^{\,r}(0)\hat{A}_2^{+r}(0) \,|\, 0 >_{1,2} =$$

$$= \sinh^{2r}(\chi\,|\,\nu_0\,|\,t)\, {}_{1,2} < 0 \,|\, (\hat{n}_2 + 1)(\hat{n}_2 + 2) \ldots\ldots (\hat{n}_2 + r) \,|\, 0 >_{1,2} =$$

$$= r!\sinh^{2r}(\chi\,|\,\nu_0\,|\,t)$$

$$\hat{n}_2^{\,r} = (\hat{n}_2 + 1)(\hat{n}_2 + 2)\ldots(\hat{n}_2 + r)$$

For $\hat{n}_2^{\,r}(t)$ we get the same result: $\left\langle \hat{n}_2^{\,r}(t) \right\rangle = r!\sinh^{2r}(\chi\,|\,\nu_0\,|\,t)$

Result are similar to thermal photon statistics:

$$\hat{\rho} = \frac{\exp(-\hat{H} / K_B T)}{tr\left[\exp(-\hat{H} / K_B T)\right]}$$

$$\left\langle \hat{n}_{k,s}^{\,(r)} \right\rangle = \frac{r!}{\left(e^{\beta} - 1\right)^r}$$

k = momentum

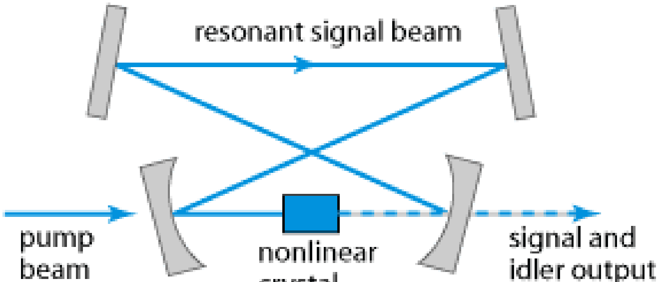
s = polarization

Optical Parametric Oscilator

- Gain is asymptotically if exponential if $\Delta k = 0$:

$$A_1(z) \Rightarrow \frac{1}{2} A_1(0) e^{gz}$$
$$g = 64\pi^2 \omega_1^2 d_{eff}^2 A_3 / k_i^2 c^4$$

- Tunable by changing the phase matching condition, or changing the cavity's length.
- Better Finesse



$\omega_p = \omega_3$

$\omega_s = \omega_1$

$\omega_i = \omega_2$

From Ref. pic [1]•

Generating of squeezing state using OPO

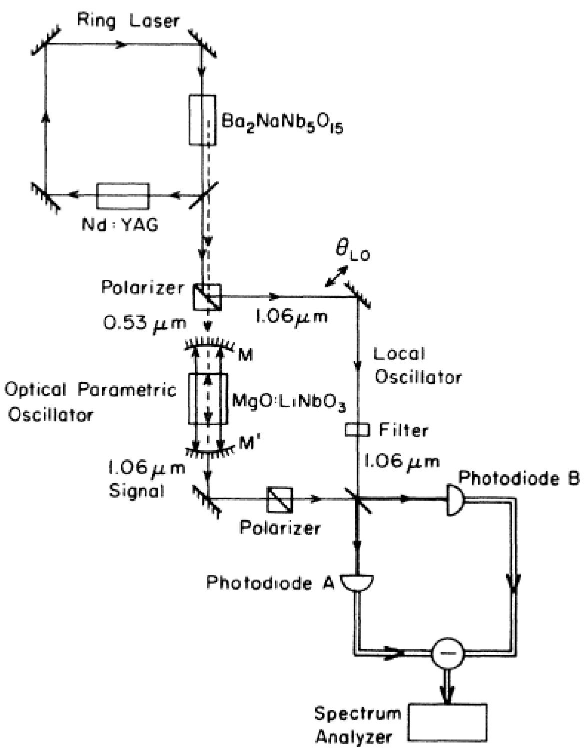


FIG. 2. Diagram of the principal elements of the apparatus for squeezed-state generation by degenerate parametric down conversion.

Signal to noise

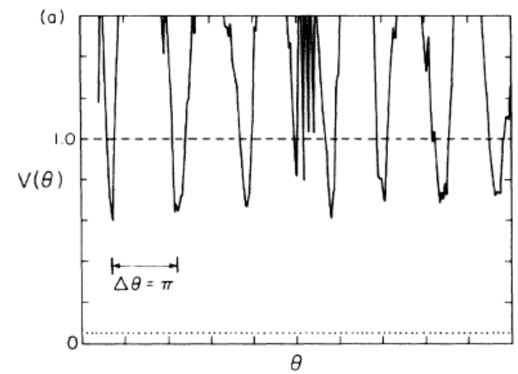
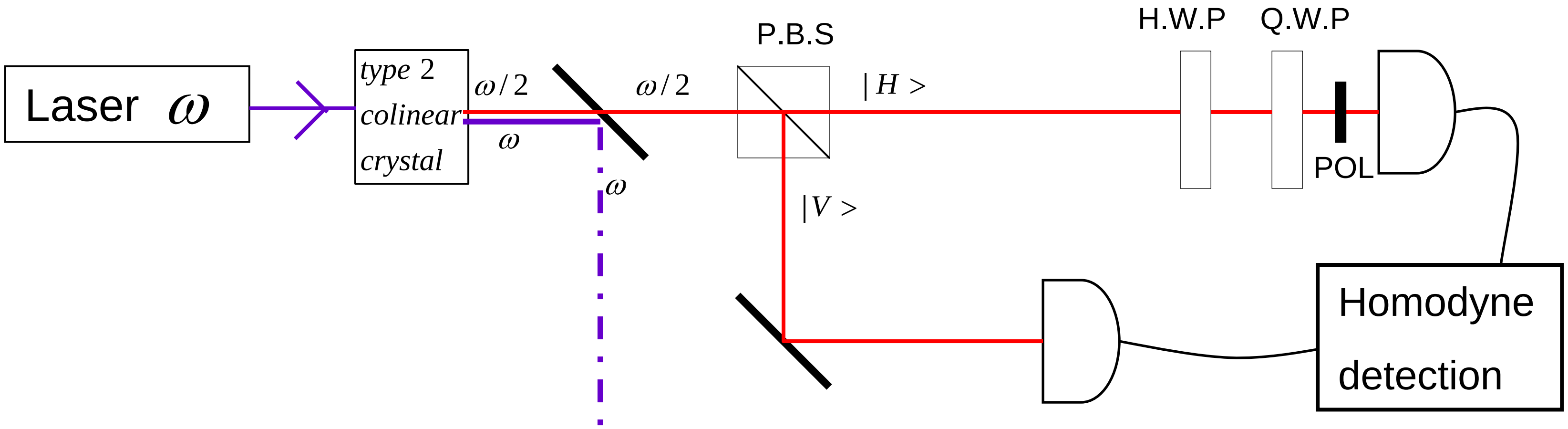


FIG. 3. Dependence of noise voltage $V(\theta)$ on local oscillator phase θ for the signal field produced by the subthreshold OPO. The scale has been expanded relative to Fig. 1(b) to display the deviations below the vacuum noise level (dashed line) more clearly. Otherwise the notation is as in Fig. 1(b). $V_0 = 250\ \mu V$; detection bandwidth = 100 kHz; $\nu/2\pi = 1.2$ MHz.

Source for single photons



P.B.S: Polarizing Beam Spliter
H.W.P: Half Wave Plate
Q.W.P: Quarter Wave Plate
POL: polarizer

Source for entanglement photons

Non collinear type 2 SPDC

$$\Psi = \frac{1}{\sqrt{2}}\left(|H_1, V_2> + e^{i\varphi}|V_1, H_2>\right)$$

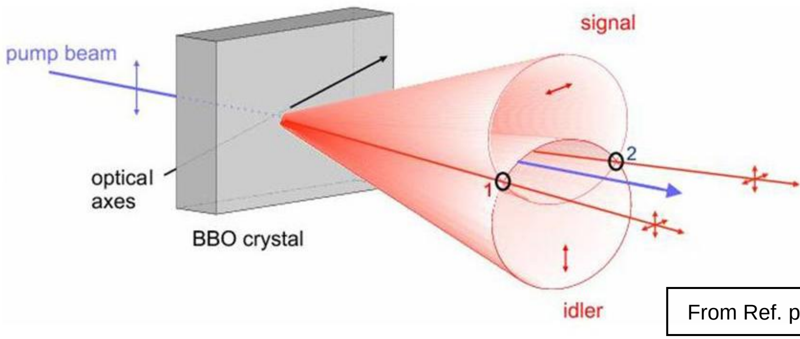


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Figure 1: Creation of entangled photons via SPDC

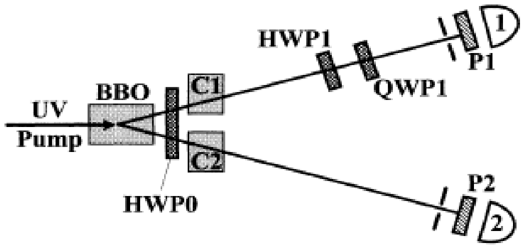

FIG. 2. Schematic of one method to produce and select the polarization-entangled state from the down-conversion crystal. The extra birefringent crystals C1 and C2, along with the half wave plate HWP0, are used to compensate the birefringent walk-off effects from the production crystal. By appropriately setting half wave plate HWP1 and quarter wave plate QWP1, one can produce all four of the orthogonal EPR-Bell states. Each polarizer P1 and P2 consisted of two stacked polarizing beam splitters preceded by a rotatable half wave plate.

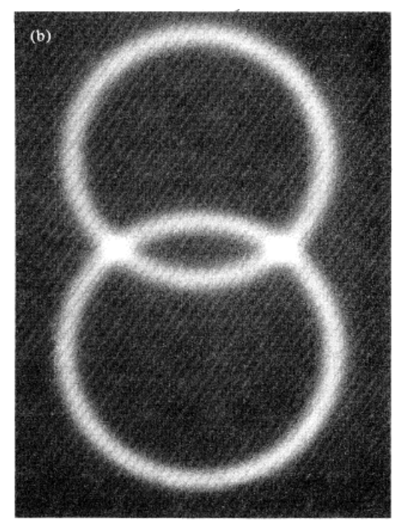

FIG. 1. (a) Spontaneous down-conversion cones present with type-II phase matching. Correlated photons lie on opposite sides of the pump beam. (b) A photograph of the down-conversion photons, through an interference filter at 702 nm (5 nm FWHM). The infrared film was located 11 cm from the crystal, with no imaging lens. (Photograph by M. Reck.)

references

- R.W. Boyd, Nonlinear optics, 2nd edition, Academic Press, New York, 2003.
- L. Mandel, E Wolf, Optical coherence and Quantum optics, Cambridge university press, New York, 1995.
- Wu L.A, Kimble H.j., Hall J.L., Wu Huifa, Generation of squeezed states by parameteric down conversion, Phys rev Lett 57, 2520 – 2523, 1986.
- Breitenbach G, Schiller S, Mlynek J, Measurement of the quantum states of squeezed light, Nature, 387, 471-475, 1997.
- Rubin M.H., Klyshko D.N., Shin Y.H., Sergienko A.V., Theory of two photon entanglement in type-II optical parametric down-conversion , Phys. Rev. A 50, 5122-5133 ,1994.
- Kwiat P. G., Mattle K., Weinfurter H., Zeilinger A., Sergienko A. V., Shih Y., New High-Intensity source of Polarization-entangled photon pairs, Phys. Rev. Lett. 75, 4337-4341, 1995.
- Halevi A., Thesis Work, The Hebrew University of Jerusalem, 2008.

Pictures from:

- [1] RP Photonics: http://www.rp-photonics.com/img/qpm.png
- [2] http://www.cooper.edu/engineering/projects/gateway/ee/solidmat/modlec3/node8.html
- [3] Geiger Mark S. , Keller S. James, Creation and Characterization of entangled photons Via second Harmonic Generation of Infrared Diodes, Kenyon College, Gambier OH. 2006: http://biology.kenyon.edu/HHMI/posters_2005/GeigerM.jpg