Experimental evaluation of the nuclear neutron-proton contact - Supplemental Material

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In our letter we have argued that in the large k limit the photodisintegration matrix element is sensitive only to the most diverging part of the wave function, namely its behavior in short interparticle distances, and therefore we limit our integrals, for example Eq. (11), to a small neighborhood of the origin Ω_0. Here we present in details the calculation of such integrals.

In the zero range approximation, the deuteron's photodisintegration cross section is proportional to the integral

$$\int d\boldsymbol{r} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \boldsymbol{\epsilon}\cdot\boldsymbol{r}\frac{e^{-r/a}}{r}. \tag{1}$$

For $k \longrightarrow \infty$, the main contribution to this integral comes from the neighborhood of the origin $r=0$, because the fast-oscillating $e^{i\mathbf{k}\cdot\mathbf{r}}$ washes out anything but the most diverging part of the function. In this neighborhood $e^{-r/a} \approx 1$, and the integral can be approximated by

$$\int_{\Omega_0} d\boldsymbol{r} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \boldsymbol{\epsilon}\cdot\boldsymbol{r}\frac{1}{r}, \tag{2}$$

where the integration is limited to a small neighborhood of the origin Ω_0 by some smooth cutoff function $f_R(r)$,

$$\int_{\Omega_0} d\boldsymbol{r} g(r) \equiv \int d\boldsymbol{r} g(r) f_R(r), \tag{3}$$

where $f_R(0) = 1$.

First we note that similar approximation is used to prove the fundamental Tan relation [1], namely

$$\lim_{k\to\infty} n_\sigma(k) = \frac{C}{k^4}. \tag{4}$$

In the two body case [2], one has to Fourier transform the dimer wave function,

$$\int d\boldsymbol{r} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \frac{e^{-r/a}}{r} = \frac{4\pi a^2}{a^2k^2+1} \xrightarrow[ka\gg 1]{} \frac{4\pi}{k^2}. \tag{5}$$

To show that this integral is dominated, in the large k limit, by its short-range behavior, let's approximate it by

$$\int_{\Omega_0} d\boldsymbol{r} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \frac{1}{r} \tag{6}$$

and use $f_R(r) = e^{-r/R}$ as a cutoff function. It is clear that the resulting integral is equivalent to (5), and therefore reproduce the right limit. Using a Gaussian cutoff function $f_R(r) = e^{-(r/R)^2}$ one gets

$$\int d\boldsymbol{r} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \frac{1}{r} e^{-(r/R)^2} = \frac{4\pi R}{k} F\left(\frac{kR}{2}\right), \tag{7}$$

where $F(x) = e^{-x^2}\int_0^x dy e^{y^2}$ is the Dawson integral. For large x, $F(x) = (2x)^{-1} + (4x)^{-3} + O(x^{-4})$ and in the limit $kR \gg 1$ we obtain again $4\pi/k^2$. In fact, we may conclude that any smooth cutoff function such as $\exp(-r/R)$, $exp(-(r/R)^2)$, or $(1-\tanh((r-R)/h))/(1+\tanh(R/h))$ will reproduce this result in the high momentum limit. In contrast, a sharp cutoff such as $f_R(r) = \Theta(R-r)$ will not work, because of the Gibbs phenomenon.

Note that here the same result can be achieved with $f_R(r) = 1$, utilizing the relation $\Delta(1/r) = -4\pi\delta(\boldsymbol{r})$.

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Now we go back to Eq. (1) and show that the same approximation works there. First, operating with $\boldsymbol{\epsilon}\cdot\nabla_k$ on Eq. (5), we get

$$\int d\boldsymbol{r}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\boldsymbol{\epsilon}\cdot\boldsymbol{r}\frac{e^{-r/a}}{r} = \frac{8\pi i a^4 k}{\left(a^2k^2+1\right)^2}\boldsymbol{\epsilon}\cdot\hat{k} \xrightarrow[ka\gg 1]{} \frac{8\pi i\boldsymbol{\epsilon}\cdot\hat{k}}{k^3}. \tag{8}$$

Once again a smooth cutoff function $f_R(r) = e^{-r/R}$ can be added to limit the integral in Eq. (2) to a small neighborhood of the origin, yielding the same result for large k. We can also check the use of a Gaussian cutoff,

$$\int d\boldsymbol{r}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\boldsymbol{\epsilon}\cdot\boldsymbol{r}\frac{1}{r}e^{-(r/R)^2} = 2\pi i\frac{R}{k^2}\boldsymbol{\epsilon}\cdot\hat{k}\left(-kR+\left((kR)^2+2\right)F\left(\frac{kR}{2}\right)\right) \xrightarrow[kR\gg 1]{} \frac{8\pi i\boldsymbol{\epsilon}\cdot\hat{k}}{k^3}. \tag{9}$$

We may conclude that also in this case any smooth cutoff function will reproduce the right high momentum limit.

In conclusion, we have shown here that indeed for $k\longrightarrow\infty$ the main contribution to the integrals in Eqs. (1) and (5) comes from a small neighborhood of the origin. We have explained that it is important to limit the integrals by a smooth cutoff function, and we have checked it explicitly for exponential, Gaussian, and tanh cutoffs.

[1] S. Tan, Ann. Phys. (N.Y.) **323**, 2952 (2008); **323**, 2971 (2008); **323**, 2987 (2008).
[2] R. Combescot, F. Alzetto,and X. Leyronas, Phys. Rev. A **79**, 053640 (2009).