

Eur. Phys. J. A (2016) **52**: 92

DOI 10.1140/epja/i2016-16092-3

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Springer

# The generalized nuclear contact and its application to the photoabsorption cross-section

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Received: 10 November 2015 / Revised: 22 January 2016

Published online: 19 April 2016 – © Società Italiana di Fisica / Springer-Verlag 2016

Communicated by M. Hjorth-Jensen

**Abstract.** Using the zero-range model, it was demonstrated recently that Levinger’s quasi-deuteron model can be utilized to extract the nuclear neutron-proton contact. Going beyond the zero-range approximation and considering the full nuclear contact formalism, we rederive here the quasi-deuteron model for the nuclear photoabsorption cross-section and utilize it to establish relations and constraints for the general contact matrix. We also define and demonstrate the importance of the diagonalized nuclear contacts, which can be also relevant to further applications of the nuclear contacts.

## 1 Introduction

For photons in the energy range above the giant resonance, the nuclear photoabsorption cross-section is dominated by two competing mechanisms: the one-body photomesonic (PM) effect associated mainly with the  $M1$  transition, and the two-body quasi-deuteron (QD) process associated with the  $E1$  transition [1, 2]. The QD process, proposed by Levinger more than 60 years ago [3], is the leading photoabsorption mechanism at energies below the pion threshold, and it has a sizable contribution to the total cross-section up to about 600 MeV [4]. Above the pion threshold, the PM effect takes over and isobaric excitations combined with meson production become the dominant features in the cross-section.

Here we focus on the QD process, and more specifically on the relation between the QD process and the short-range correlations in the nuclear wave function.

In the QD picture, the photonuclear reaction mechanism goes through an absorption of the photon by a correlated proton-neutron ( $pn$ ) pair being close to each other, followed by an emission of the two particles flying back to back. The resulting photonuclear cross-section of a nucleus composed of  $Z$  protons and  $N$  neutrons,  $A = N + Z$ , is therefore expected to be proportional to the deuteron cross-section  $\sigma_d$ ,

$$\sigma_A(\omega) = L \frac{NZ}{A} \sigma_d(\omega). \quad (1)$$

The proportionality constant  $L \approx 6$  is known as the Levinger constant and  $\omega$  is the frequency of the photon. Fol-

lowing Levinger’s seminal work, the importance of short-range correlations in nuclear reactions such as photonuclear reactions [5], hard electron [6–8] and proton scattering, was realized. Moreover, the QD process got a remarkable experimental verification in proton and electron scattering on carbon [9, 10], and other nuclei [11–14], where high momentum, correlated,  $pn$  pairs flying back to back were measured.

When the distance between two particles  $r$ , is much smaller than the average interparticle distance  $d_{av}$ , the pair’s wave function assumes a specific form that depends only on the interaction [15]. This form assumes an universal  $1/r$  shape when the interaction range diminishes  $R \rightarrow 0$ . Consequently the high-momentum tail of the momentum distribution fulfills the relation  $\lim_{k \rightarrow \infty} k^4 n(k) = C$ . Considering a system of two-component fermions interacting via such short-range interaction, Tan [16–19] has established a series of relations between the amplitude of the high-momentum tail of the momentum distribution  $C$ , which he coined the “contact”, and different properties of the system, such as the energy, pair correlations and pressure. These relations, commonly known as the “Tan relations”, were further extended to other properties and systems by different groups, see for example [19] and references therein. Following the theoretical discovery of the Tan relations, they were verified in ultracold atomic systems, both in  $^{40}\text{K}$  [20, 21] and in  $^6\text{Li}$  [22–24] systems. Moreover, the measured value of the contact, as a function of the system’s density, was found to be in accordance with the theoretical predictions of [23].

In essence, Tan’s contact is a measure of the probability of finding a particle pair close to each other.

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It is therefore not surprising that the Levinger constant, that counts the number of quasi-deuterons in a nucleus, is closely related to the contact. This relation was exposed in [25] using the zero-range approximation, and was utilized to evaluate the neutron-proton contact from the experimental value of the Levinger constant.

With all its beauty and simplicity, the zero-range model cannot be considered as an accurate description of the nuclear force and the nuclear wave function. Within effective field theory (EFT) the zero-range model is equivalent to the leading order in a pionless theory (see *e.g.* [26,27]), which is known to have a limited range of applicability. Furthermore, realistic nuclear potentials such as  $\chi$ EFT [28,29] or AV18 [30] that include pion exchange forces acquire a natural range associated with the pion mass. This range is smaller, but not much smaller, than the average nuclear interparticle distance.

In view of these limitations of the zero-range model, a more general derivation of the relation between the neutron-proton contact and the Levinger constant is called for, which is the aim of the current paper. To this end we follow the formalism presented in [31] and utilize it to rederive the QD model.

## 2 The contact in nuclear systems

When nucleons  $i$  and  $j$  come close to each other, the nuclear wave function  $\Psi$  takes on the asymptotic form [15]

$$\Psi \xrightarrow{r_{ij} \rightarrow 0} \sum_{\alpha} \varphi_{ij}^{\alpha}(\mathbf{r}_{ij}) A_{ij}^{\alpha}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k \neq i,j}), \quad (2)$$

where  $\mathbf{r}_k$  are the single-particle coordinates,  $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$  is the pair's relative distance and  $\mathbf{R}_{ij} = (\mathbf{r}_i + \mathbf{r}_j)/2$  is the center-of-mass (CM) vector. The functions  $\varphi_{ij}^{\alpha}$  are called the asymptotic pair wave functions. They are universal across the nuclear chart (*i.e.* they are independent of the specific nuclear system), and due to symmetry they only depend on the pair type, *i.e.* whether the pair  $ij$  is a proton-proton ( $pp$ ) pair, a neutron-neutron ( $nn$ ) pair or a neutron-proton ( $np$ ) pair. The sum over  $\alpha$  denotes a sum over the four quantum numbers of the pair ( $s_2, \ell_2, j_2, m_2$ ), which are the pair's total spin  $s_2$ , its spatial angular momentum  $\ell_2$  with respect to  $\mathbf{r}_{ij}$ , and the total angular momentum and its projection  $j_2$  and  $m_2$ ,

$$\varphi_{ij}^{\alpha} \equiv \varphi_{ij}^{(\ell_2 s_2) j_2 m_2} = \left[ \varphi_{ij}^{\{s_2, j_2\} \ell_2} \otimes \chi_{s_2} \right]^{j_2 m_2}, \quad (3)$$

where  $\chi_{s_2 \mu_s}$  is the two-body spin function, and  $\varphi_{ij}^{\{s_2, j_2\} \ell_2 \mu_{\ell}}(\mathbf{r}_{ij}) = \phi_{ij}^{\{\ell_2, s_2, j_2\}}(r_{ij}) Y_{\ell_2 \mu_{\ell}}(\hat{r}_{ij})$ . Assuming that the nucleus has total angular momentum  $J$  and projection  $M$ , the matrices of the two-body nuclear contacts are defined as [31]

$$C_{ij}^{\alpha\beta}(JM) = 16\pi^2 N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle. \quad (4)$$

Here,  $ij$  stands for one of the pairs:  $pp$ ,  $nn$  or  $np$ ,  $N_{ij}$  is the number of  $ij$  pairs, and  $\alpha, \beta$  are the matrix indices. In

many cases we do not know the nuclear magnetic quantum number. It is therefore convenient to introduce the averaged nuclear contacts, defined as

$$C_{ij}^{\alpha\beta} = \frac{1}{2J+1} \sum_M C_{ij}^{\alpha\beta}(JM). \quad (5)$$

The averaged contacts  $C_{ij}^{\alpha\beta}$  do not depend on  $m_{\alpha}$  or  $m_{\beta}$ , but only on  $(s_{\alpha}, \ell_{\alpha}, j_{\alpha})$ , and  $(s_{\beta}, \ell_{\beta}, j_{\beta})$ . The contacts  $C_{ij}^{\alpha\beta}$  are the generalized nuclear analogs of Tan's contact [16–18].

The factorized asymptotic form given in eq. (2) should be satisfied in the limit  $r_{ij} \rightarrow 0$  but its exact range of validity is not fully understood in the available studies. Such a factorization is the basis for many of the Tan relations [16–19]. The relevant length scales in nuclear systems are the average distance between two nucleons, and the scattering lengths. It is reasonable to assume that  $r_{ij}$  should be smaller than these length scales for the factorization to be valid. Furthermore, using the variational Monte Carlo (VMC) results of Wiringa *et al.* [8], one can estimate that this asymptotic form is valid for  $r_{ij}$  smaller than about 1 to 2 fm. These VMC results were calculated using the Argonne v18 two-nucleon and Urbana X three-nucleon potentials for  $A \leq 12$  nuclei. As the inner parts of these nuclei already possess the nuclear saturation density we expect that the above estimate will hold for heavier nuclei.

The above asymptotic factorization does not take into account three-body correlations. In the zero-range model, the asymptotic form for the case where three particles approach each other is given in ref. [32]. The contribution of three-body correlations is expected to be less significant than the contribution of two-body correlations [33]. Thus, we will not consider here three-body correlations, and it is left for future studies.

As will be clear later, it will be useful to work in a basis for which the contact matrices are diagonal.  $C_{ij}(JM)$  is an Hermitian matrix and therefore can be diagonalized. So, there exists a unitary matrix  $U$  (generally,  $U$  depends on the type of the pair  $ij$ , on the nucleus and its quantum numbers  $J$  and  $M$ ) and a diagonal matrix  $D_{ij}(JM)$  such that

$$D_{ij}(JM) = U C_{ij}(JM) U^{-1}. \quad (6)$$

We can also define

$$\tilde{\varphi}_{ij}^{\alpha} = \sum_{\beta} U_{\alpha\beta} \varphi_{ij}^{\beta} \quad (7)$$

and

$$\tilde{A}_{ij}^{\alpha} = \sum_{\beta} (U^{-1})_{\beta\alpha} A_{ij}^{\beta} = \sum_{\beta} U_{\alpha\beta}^* A_{ij}^{\beta}. \quad (8)$$

It is now simple to prove that

$$\sum_{\alpha} \varphi_{ij}^{\alpha} A_{ij}^{\alpha} = \sum_{\alpha} \tilde{\varphi}_{ij}^{\alpha} \tilde{A}_{ij}^{\alpha} \quad (9)$$

and

$$16\pi^2 N_{ij} \langle \tilde{A}_{ij}^{\alpha} | \tilde{A}_{ij}^{\beta} \rangle = \delta_{\alpha\beta} D_{ij}^{\alpha\alpha}(JM). \quad (10)$$

This way we have defined here a new basis for which the contact matrices are diagonal. In some sense it is not the natural basis to work with, because the  $\tilde{\varphi}_{ij}^\alpha$  are not universal as they depend on the specific nucleus (because of  $U$ ). Nevertheless, this basis will be very useful to our purpose of rederiving the QD model.

The averaged diagonal contacts can also be defined:

$$D_{ij}^{\alpha\alpha} = \frac{1}{2J+1} \sum_M D_{ij}^{\alpha\alpha}(JM). \quad (11)$$

### 3 The quasi-deuteron model

In the leading  $E1$  dipole approximation, the total photo absorption cross-section of a nucleus is given by

$$\sigma_A(\omega) = 4\pi^2 \alpha \hbar \omega R(\omega), \quad (12)$$

where  $\alpha$  is the fine structure constant, and

$$R(\omega) = \sum_i \sum_f |\langle \Psi_f | \epsilon \cdot \hat{D} | \Psi_0 \rangle|^2 \delta(E_f - E_0 - \hbar\omega) \quad (13)$$

is the response function.  $\hat{D}$  is the unretarded dipole operator  $\hat{D} = \sum_{i=1}^A \frac{1+\tau_i^3}{2} \mathbf{r}_i$ , and  $\epsilon$  is the photon's polarization vector. The initial (ground) state and the final state wave functions are denoted by  $|\Psi_{0/f}\rangle$  and the energies by  $E_{0/f}$ , respectively. The operator  $\tau_i^3$  is the third component of the  $i$ -th nucleon isospin operator. The sum  $\sum_f$  in the response function is a sum over the final states that becomes an integration in the limit of infinite volume. The response function also contains an average over the initial states which amounts to an average over the magnetic projection of the ground state,  $\sum_i = 1/(2J_0 + 1) \sum_{M_0}$ . We note that the different final states must be orthogonal eigenstates of the nuclear Hamiltonian. For that reason it will be important to work with the diagonalized contact matrices.

For inverse photon wave number  $q^{-1}$  somewhat shorter than the average interparticle distance ( $qd_{av} > 1$ ), the photon is absorbed by a QD pair. Consider a reaction mechanism where the photon is absorbed by a proton  $p$  that is emitted with large momentum  $\mathbf{k}_p$ . For high photon energies this process is fast enough so we can use the Born approximation, which means that any interaction between the emitted proton and the rest of the nucleus can be neglected. Hence, due to momentum conservation, another particle must be emitted. As pointed out by Levinger [3], because of the  $E1$  nature of the process, this other particle must be a neutron  $n$ , since proton-proton pair posses no dipole moment. The emitted neutron's momentum  $\mathbf{k}_n$  is such that  $\mathbf{k}_n \approx -\mathbf{k}_p \equiv \mathbf{k}$ , thus the relative momentum of the emitted pair is  $\frac{\mathbf{k}_n - \mathbf{k}_p}{2} = \frac{2\mathbf{k}_n}{2} = \mathbf{k}$ . This point can be further reinforced, under the assumption that the center of mass obeys  $\sum_{i=1}^A \mathbf{r}_i = 0$ , through the algebraic relation,

$$\hat{D} = \frac{1}{A} \sum_{i,j=1}^A \frac{\tau_i^3 - \tau_j^3}{4} (\mathbf{r}_i - \mathbf{r}_j), \quad (14)$$

explicitly writing the dipole operator as a two-body operator that vanishes for all but  $np$  pairs. Nevertheless, it will be more convenient to work with the one-body representation of the dipole operator in the following derivations. The relative motion of the emitted pair contains most of the photon energy, whereas the photon's momentum is translated into the CM motion. The energy fraction associated with the CM coordinate  $\mathbf{R}_{pn}$  is  $\hbar\omega/4Mc^2$ , where  $M$  is the mass of the nucleons, which is only few percents for the photon energies under consideration. We can therefore safely neglect the pair's recoil.

Assuming that the residual  $A - 2$  particles wave function is frozen throughout this process, we can write the final state wave functions for the channel  $\alpha$ , normalized in a box of volume  $\Omega$ , in the following way:

$$\Psi_f^{\alpha s \mu_s} = \mathcal{N}_\alpha \hat{A} \left\{ \frac{1}{\sqrt{\Omega}} e^{-i\mathbf{k} \cdot \mathbf{r}_{pn}} \chi_{s, \mu_s} \tilde{A}_{pn}^\alpha(\mathbf{R}_{pn}, \{\mathbf{r}_j\}_{j \neq p, n}) \right\}. \quad (15)$$

Here  $\mathcal{N}_\alpha$  is a normalization factor, and  $\hat{A} = (1 - \sum_{p' \neq p} (p, p')) (1 - \sum_{n' \neq n} (n, n'))$  is the proton-neutron antisymmetrization operator with  $(i, j)$  being the transposition operator. The sums over  $p'$ ,  $n'$  extends over all protons and neutrons in the system but  $p$ ,  $n$ . As  $A_{pn}^\alpha(\mathbf{R}_{pn}, \{\mathbf{r}_j\}_{j \neq p, n})$  is antisymmetric under permutation of all identical particles but the pair  $pn$ ,  $\Psi_f^{\alpha s \mu_s}$  is antisymmetric under proton permutations and under neutron permutations. Note that the different final state functions are indeed orthogonal as required. Here the importance of the "diagonal basis", the  $\tilde{A}$  functions, becomes clear. If we were to use the original functions  $A$  in the definition of the final states, then they would not be orthogonal and the whole derivation would be wrong. The normalization factor is given by

$$\mathcal{N}_\alpha = \frac{1}{\sqrt{NZ}} \frac{1}{\sqrt{\langle \tilde{A}_{pn}^\alpha | \tilde{A}_{pn}^\alpha \rangle}} = \frac{4\pi}{\sqrt{D_{pn}^{\alpha\alpha}(J_0 M_0)}}. \quad (16)$$

Considering now the transition matrix element we see that

$$\begin{aligned} \langle \Psi_f^{\alpha s \mu_s} | \epsilon \cdot \hat{D} | \Psi_0 \rangle &= NZ \mathcal{N}_\alpha \int \prod_k d\mathbf{r}_k \frac{1}{\sqrt{\Omega}} e^{i\mathbf{k} \cdot \mathbf{r}_{pn}} \chi_{s, \mu_s}^\dagger \\ &\times \tilde{A}_{pn}^{\alpha\dagger}(\mathbf{R}_{pn}, \{\mathbf{r}_j\}_{j \neq pn}) (\epsilon \cdot \hat{D}) \Psi_0, \end{aligned} \quad (17)$$

where we have used the fact that  $\hat{A}\Psi_0 = NZ\Psi_0$ . Due to the orthogonality of the initial and final states, the transition matrix element vanishes unless the photon acts on the outgoing  $pn$  pair. The above integral contains the oscillatory function  $e^{i\mathbf{k} \cdot \mathbf{r}_{pn}}$ , so since the momentum  $\mathbf{k}$  is large only the asymptotic part  $r_{pn} \rightarrow 0$  will contribute to the integral. Therefore the integration over  $r_{pn}$  hereafter can be limited to a small neighborhood of the origin  $\Omega_0$ , where the asymptotic form of the wave function (2) is valid (for more details see ref. [25] and especially its supplemental

materials). Hence,

$$\begin{aligned} \langle \Psi_f^{\alpha s \mu_s} | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}} | \Psi_0 \rangle &= NZ \mathcal{N}_\alpha \sum_\beta \langle \tilde{A}_{pn}^\alpha | \tilde{A}_{pn}^\beta \rangle \\ &\times \langle \mathbf{k} s \mu_s | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}}_{pn} | \tilde{\beta} \rangle, \end{aligned} \quad (18)$$

where

$$\langle \mathbf{k} s \mu_s | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}}_{pn} | \tilde{\beta} \rangle = \int_{\Omega_0} d\mathbf{r} \frac{1}{\sqrt{\Omega}} e^{i\mathbf{k} \cdot \mathbf{r}} \chi_{s, \mu_s}^\dagger \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}}_{pn} \tilde{\varphi}_{pn}^\beta(\mathbf{r}), \quad (19)$$

and  $\hat{\mathbf{D}}_{pn} = \frac{\mathbf{r}_{pn}}{2}$ . Working in the ‘‘diagonal basis’’ only  $\beta = \alpha$  contributes to the sum in (18). Substituting now the normalization factor (16) and utilizing the relation (10),

$$\langle \Psi_f^{\alpha s \mu_s} | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}} | \Psi_0 \rangle = \frac{\sqrt{D_{pn}^{\alpha\alpha}(J_0 M_0)}}{4\pi} \langle \mathbf{k} s \mu_s | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}}_{pn} | \tilde{\alpha} \rangle. \quad (20)$$

If we now express  $\tilde{\varphi}_{pn}^\alpha(\mathbf{r})$  through the asymptotic pair wave functions we get

$$\begin{aligned} \langle \Psi_f^{\alpha s \mu_s} | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}} | \Psi_0 \rangle &= \frac{\sqrt{D_{pn}^{\alpha\alpha}(J_0 M_0)}}{4\pi} \\ &\times \sum_\beta U_{\alpha\beta} \langle \mathbf{k} s \mu_s | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}}_{pn} | \beta \rangle, \end{aligned} \quad (21)$$

where

$$\langle \mathbf{k} s \mu_s | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}}_{pn} | \alpha \rangle = \int_{\Omega_0} d\mathbf{r} \frac{1}{\sqrt{\Omega}} e^{i\mathbf{k} \cdot \mathbf{r}} \chi_{s, \mu_s}^\dagger \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}}_{pn} \varphi_{pn}^\alpha(\mathbf{r}). \quad (22)$$

We can now take the square absolute value of this matrix element, average over the initial states and sum over final states. The sum over the final states is a sum over  $\alpha$ ,  $s$  and  $\mu_s$ , and integration over  $\hat{\mathbf{k}}$ . Starting with the sum over  $\alpha$ ,  $s$  and  $\mu_s$  we get

$$\begin{aligned} &\sum_{\alpha s \mu_s} \left| \langle \Psi_f^{\alpha s \mu_s} | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}} | \Psi_0 \rangle \right|^2 = \\ &\sum_\alpha \frac{D_{pn}^{\alpha\alpha}(J_0 M_0)}{16\pi^2} \sum_{s, \mu_s} \left| \sum_\beta U_{\alpha\beta} \langle \mathbf{k} s \mu_s | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}}_{pn} | \beta \rangle \right|^2 = \\ &\sum_{\beta, \beta'} \frac{\sum_\alpha U_{\alpha\beta}^* D_{pn}^{\alpha\alpha}(J_0 M_0) U_{\alpha\beta'}}{16\pi^2} R_{\beta\beta'}(\mathbf{k}) = \\ &\sum_{\beta, \beta'} \frac{C_{pn}^{\beta\beta'}(J_0 M_0)}{16\pi^2} R_{\beta\beta'}(\mathbf{k}), \end{aligned} \quad (23)$$

where

$$R_{\beta\beta'}(\mathbf{k}) = \sum_{s, \mu_s} \langle \mathbf{k} s \mu_s | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}}_{pn} | \beta \rangle^* \langle \mathbf{k} s \mu_s | \boldsymbol{\epsilon} \cdot \hat{\mathbf{D}}_{pn} | \beta' \rangle. \quad (24)$$

Integrating now over the momentum  $\hat{\mathbf{k}}$  and averaging over the initial states we get

$$R(\omega) = \sum_{\beta, \beta'} \frac{C_{pn}^{\beta\beta'}}{16\pi^2} R_{\beta\beta'}(\omega), \quad (25)$$

where

$$R_{\beta\beta'}(\omega) = \int \frac{d\hat{\mathbf{k}}}{(2\pi)^3} R_{\beta\beta'}(\mathbf{k}). \quad (26)$$

Deriving (25), we have utilized the sum over  $M_0$  and replaced the contact matrix  $C_{pn}(J_0 M_0)$ , with the averaged contacts  $C_{pn}$ . This step could not have been done before, as in general the matrix  $U$  depends on the specific nucleus and its quantum numbers  $J_0$  and  $M_0$ .

The response function (25) is a general result valid for all nuclei. It is composed of a particular part, that depends on the specific nucleus through the values of the contacts, and an universal part  $R_{\beta\beta'}(\omega)$  that doesn't change along the nuclear chart. It is universal since it is written using the original and physical  $\varphi$  functions and does not include the  $\tilde{\varphi}$  functions nor the matrix  $U$ . As explained before, it was necessary to use the diagonal basis in the derivation, but it is more useful to write the final result using the physical basis. We notice that only  $\beta$  and  $\beta'$  with  $s_\beta = s_{\beta'}$  can contribute to the response (even though generally the contacts are not diagonal in  $s$ ) because  $\hat{\mathbf{D}}_{pn}$  is a spin scalar, and therefore if  $s_\beta \neq s_{\beta'}$  then all terms in the sum over  $s, \mu_s$  must vanish.

Equation (25) should hold when  $\omega \rightarrow \infty$  and its exact range of validity is directly connected to the validity range of the asymptotic form (2). If eq. (2) holds for  $r$  smaller than some distance  $d_a$  then we would expect that eq. (25) holds for  $q d_a > 1$ , where  $q$  is the photon wave number. As mentioned before,  $d_a \approx 1-2$  fm according to the VMC data of Wiringa *et al.* [8]. Thus we expect eq. (25) to hold for

$$\hbar\omega = \hbar qc > \frac{\hbar c}{d_a} \approx 100-200 \text{ MeV}. \quad (27)$$

We note that the  $E1$  transition considered here is the leading effect up to about 140 MeV and the extraction of the Levinger constant was usually done from experiments with photon energies between 40 MeV and 140 MeV [1,2]. In order to use experimental data with higher energies, a separation of the  $E1$  transition from the total photoabsorption cross-section is needed.

## 4 The relation to the QD model

The result (25) is also valid for the deuteron. The deuteron is a bound proton-neutron pair with angular momentum  $J = 1$ ,  $M = 0, \pm 1$ , positive parity, and total spin  $S = 1$ . Since it is only a two-body system, the quantum numbers of the full state determine that many of the deuteron contacts are zero.  $A_{pn}^\alpha$  can be different from zero only for  $\alpha = (l = 0, s = 1, j = 1, m = M)$  and  $\alpha = (l = 2, s = 1, j = 1, m = M)$ . So, given a projection  $M$ , we have only one  $2 \times 2$  block of non-zero contacts. Each  $M$  defines a different block, however these blocks must be identical. The deuteron averaged contact is therefore composed of three identical  $2 \times 2$  blocks. The deuteron contact  $C_{pn}^{\alpha\beta}$  is defined by the values of  $\ell_\alpha$  and  $\ell_\beta$  (being zero or two).

Comparing the Levinger model (1) with the response function (25), we see that the latter is complex and involve contact channels that are missing in the deuteron. As a result, we must conclude that the QD model cannot be completely accurate, since heavy nuclei include two-body channels that does not exist in the deuteron. Nevertheless, the QD model does describe nicely the available experimental data. As a result, we can obtain approximated constrains on the different nuclear contacts within the same accuracy. In the following we shall explore these implications, and analyze under what conditions the nuclear photoabsorption cross-section becomes proportional to the deuteron one.

The  $pn$  contact  $C_{pn}^{\alpha\beta}$  measures the probability to find a neutron close to a proton in the specific  $\alpha\beta$  channel. As nuclei behave as an incompressible liquid having almost constant density, it seems reasonable that the way these probabilities scale with the number of protons,  $Z$ , and neutrons,  $N = A - Z$ , does not depend on the specific channel. It means that if in nucleus  $Y$  the probability to find an  $np$  pair in channel A is twice the probability to find it in nucleus  $X$ , then also in channel B the probability is twice in nucleus  $Y$  than in nucleus  $X$ . If this is the case, then it means that the  $pn$  contacts scale with the number of proton and neutrons regardless of  $\alpha$  or  $\beta$ . It means that there exist  $\eta_{pn}(N, Z)$ , independent of  $\alpha$  or  $\beta$ , such that

$$C_{pn}^{\alpha\beta}(A, X) = \eta_{pn}(N, Z) C_{pn}^{\alpha\beta}(d), \quad (28)$$

where  $X$  is a nucleus in its ground state and  $d$  is the deuteron. This should only be an approximate relation because we don't expect that all the contacts that are exactly zero for the deuteron will also be exactly zero in heavier nuclei. This relation also includes the case where there is only one significant contact for all the nuclei (one significant channel for a neutron to be close to a proton). So, if we assume this relation given in eq. (28), we get directly from eq. (25) that

$$\sigma_X(\omega) = \eta_{pn}(N, Z) \sigma_d(\omega), \quad (29)$$

where  $Z$  and  $N$  are the number of protons and neutrons in the nucleus  $X$ . We can compare it to eq. (1) and get

$$\eta_{pn} = L \frac{NZ}{A}. \quad (30)$$

There might also be a different scenario that yields a constant ratio between the cross-sections. As can be seen in eq. (25), each contact is multiplied by some universal function of  $\omega$ . If these functions are proportional to each other, or at least that is the case for the dominant ones, there will be a constant ratio between the photoabsorption cross-sections. In this case we will get a constant ratio regardless of the scaling of the contacts with  $Z$  and  $N$  because the  $\omega$ -dependence is canceled. This is actually the situation in the zero-range model assumed in [25]. From the zero-range model it follows that the only non-zero deuteron contact is the  $s$ -wave spin-triplet contact. For heavier nuclei, both  $s$ -wave spin-singlet and spin-triplet might be significant, but are assumed to come with the

same asymptotic pair wave function  $\propto 1/r$ . In this scenario we get that

$$\frac{\sum'_{\alpha,\beta} C_{pn}^{\alpha,\beta}(A, X)}{\sum'_{\alpha',\beta'} C_{pn}^{\alpha',\beta'}(d)} = L \frac{NZ}{A}, \quad (31)$$

where the notation  $\sum'$  indicates that the sum is restricted to the dominant channels. In general, we expect these channels to be purely or partially  $s$ -wave channels.

When further details regarding the values of the different nuclear contacts become available, it will be possible to use eq. (25) to deduce the corrections to the QD model.

## 5 Conclusions

Summing up, we have rederived here the QD model using the full nuclear contact formalism. Assuming that Levinger's model is accurate we have obtained few constrains on the  $np$  contact matrix, and on the scaling of the  $np$  contacts along the nuclear chart. We have also defined here the diagonalized nuclear contacts and emphasized their importance in the derivation presented in this manuscript. The diagonalized contacts might turn out to be an important tool in future derivations of the nuclear contact relations.

This work was supported by the Pazy Foundation.

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