



Onset of η -nuclear binding in a pionless EFT approach



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ABSTRACT

η NNN and η NNNN bound states are explored in stochastic variational method (SVM) calculations within a pionless effective field theory (EFT) approach at leading order. The theoretical input consists of regulated NN and NNN contact terms, and a regulated energy dependent η N contact term derived from coupled-channel models of the $N^*(1535)$ nucleon resonance. A self consistency procedure is applied to deal with the energy dependence of the η N subthreshold input, resulting in a weak dependence of the calculated η -nuclear binding energies on the EFT regulator. It is found, in terms of the η N scattering length $a_{\eta N}$, that the onset of binding $\eta^3\text{He}$ requires a minimal value of $\text{Re}a_{\eta N}$ close to 1 fm, yielding then a few MeV η binding in $\eta^4\text{He}$. The onset of binding $\eta^4\text{He}$ requires a lower value of $\text{Re}a_{\eta N}$, but exceeding 0.7 fm.

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1. Introduction

The η N s -wave interaction near threshold, $E_{\text{th}}(\eta N) = 1487$ MeV, is attractive as realized first by coupling to the πN channel [1] and subsequently confirmed, e.g. [2], by coupling the ηN channel to the entire set of meson–baryon channels with 0^- octet mesons and $\frac{1}{2}^+$ octet baryons, thereby generating dynamically the $N^*(1535)$ S_{11} resonance. The size of the resulting η N energy dependent s -wave attraction, however, is strongly model dependent with values of the real part of the η N scattering length as low as 0.2 fm [2] and up to nearly 1 fm in the K -matrix model of Green and Wycech (GW) [3] and in the recent Giessen coupled channels study [4]. Following the work of Ref. [1] it was soon realized that η -nuclear quasibound states might exist [5,6] with widths determined by the scale of the imaginary part of the η N scattering length. This imaginary part, due mostly to $\eta N \rightarrow \pi N$, is small in all models, between 0.2 to 0.3 fm. Nevertheless, no η -nuclear quasibound state has ever been established beyond doubt [7].

Recent optical-model calculations of such bound states [8,9] using several energy-dependent η N model amplitudes are summarized in Refs. [10,11]. Whereas the appearance of η -nuclear bound states is robust in any of these η N interaction models, the value of mass number A at which binding begins is model dependent. Thus, the relatively strong η N attraction in model GW even admits in such calculations a $1s_{\eta}$ bound state in ^4He , with as low bind-

ing energy as 1.2 MeV and width of 2.3 MeV [11] calculated using a static ^4He density. Unfortunately, the η -nucleus optical model approach is not trustworthy for as light nuclei as ^4He , and genuine few-body calculations are required.

Photon- and hadron-induced reactions on nuclear targets provide useful constraints on possible η bound states in very light nuclei, where according to a recent review [12] the most straightforward interpretation of the data is that ηd is unbound, $\eta^3\text{He}$ is nearly or just bound, and $\eta^4\text{He}$ is bound. Our previous few-body η NN and η NNN calculations [13], using the Minnesota [14] and Argonne AV4' [15] NN potentials, agree with this conjecture as far as the ηd and $\eta^3\text{He}$ systems are concerned. A similar conclusion for $\eta^3\text{He}$ has been reached recently by evaluating the $pd \rightarrow \eta^3\text{He}$ near-threshold reaction [16]. And a recent WASA-at-COSY search for a possible $\eta^4\text{He}$ bound state in the $dd \rightarrow ^3\text{He}N\pi$ reaction placed upper limits of a few nb on the production of a near-threshold bound state [17]. On the theoretical side, no precise few-body calculation of η NNNN bound-state has ever been reported for $\eta^4\text{He}$.¹

The present work reports for the first time on precise few-body η NNNN calculations in which the Stochastic Variational Method (SVM) is applied to η plus few-nucleon Hamiltonians constructed in Leading Order (LO) within a Pionless Effective Field Theory. While this $\not\propto$ EFT approach has been applied before to few-nucleon

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¹ A very recent preprint by Fix and Kolesnikov [18] reports on few-body calculations of the $\eta^3\text{He}$ and $\eta^4\text{He}$ scattering lengths, concluding that these systems are unbound.

systems, e.g. [19,20], and more recently in lattice-nuclei calculations [21,22], it is extended here to include constituent pseudoscalar mesons for which pion exchange with nucleons is forbidden by parity conservation of the strong interactions. In particular, the single ηN contact term required in LO is provided by the ηN s -wave scattering amplitude $F_{\eta N}(E_{sc})$ at a subthreshold energy E_{sc} derived self consistently within the few-body calculation, as practised in our previous work [13]. This is demonstrated in two ηN interaction models, GW [3] and CS (Cieplý–Smejkal [23]), which exhibit strong energy dependence of $F_{\eta N}(E)$ arising from the proximity of the $N^*(1535)$ resonance. The results of the few-body calculations reported in the present work suggest that the onset of binding η ^3He requires a value of $\text{Re} a_{\eta N}$ close to 1 fm, whereas the onset of binding η ^4He requires a somewhat weaker ηN interaction with $\text{Re} a_{\eta N}$ exceeding 0.7 fm.

2. Methodology

Here we outline the methodology of the present work, including the few-body SVM used, the choice of NN , NNN and ηN $\not\propto$ EFT regulated contact terms, and the self consistent treatment of the energy-dependent ηN term.

2.1. SVM calculations

SVM calculations were introduced in the mid seventies to few-body nuclear problems [24], and used extensively with correlated Gaussian bases since the mid 1990s [25]. The SVM was benchmarked together with six other few-body methods in calculating the ^4He binding energy [26]. Correlated Gaussian trial wavefunctions in this method are written as

$$\Psi = \sum_k c_k \mathcal{A} \left(\left[\mathcal{Y}_L^k(\hat{\mathbf{x}}) \chi_S^k \right]_{JM} \xi_{TTz}^k \exp\left(-\frac{1}{2} \mathbf{x}^T A_k \mathbf{x}\right) \right) \quad (1)$$

where the summation index k runs with linear variational parameters c_k on all possible values of the total spin S and the total orbital angular momentum L , as well as on all possible intermediate coupling schemes, χ_S and ξ_T stand for spin and isospin functions of the N -particle system, respectively, \mathcal{Y}_L is the orbital part of Ψ formed by coupling successively spherical harmonics in the $(N-1)$ relative coordinates of which the vector \mathbf{x} is made, and \mathcal{A} antisymmetrizes over nucleons. The matrix A_k introduces $N(N-1)/2$ nonlinear variational parameters which are chosen stochastically. For a comprehensive review see Ref. [27].

2.2. Pionless EFT nuclear interactions

Here we follow a $\not\propto$ EFT approach at LO. To this order the nuclear interaction consists of two-body and three-body contact (zero-range) terms,

$$V_2(ij) = \left[c_S^\Lambda \frac{1}{4} (1 - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) + c_T^\Lambda \frac{1}{4} (3 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \right] \delta_\Lambda(r_{ij}), \quad (2)$$

$$V_3(ijk) = d_3^\Lambda \delta_\Lambda(r_{ij}, r_{jk}), \quad (3)$$

where these contact terms are smeared by using normalized-to-one Gaussians with a regulating momentum-space scale (cut-off) parameter Λ :

$$\delta_\Lambda(r_{ij}) = \left(\frac{\Lambda}{2\sqrt{\pi}} \right)^3 \exp\left(-\frac{\Lambda^2}{4} r_{ij}^2\right),$$

$$\delta_\Lambda(r_{ij}, r_{jk}) = \delta_\Lambda(r_{ij}) \delta_\Lambda(r_{jk}), \quad (4)$$

with $\delta_\Lambda(r_{ij})$ in the zero-range limit $\Lambda \rightarrow \infty$ becoming a Dirac $\delta(\mathbf{r}_{ij})$ function. For a given value of the scale parameter Λ , the

Table 1

^4He binding energies $B(^4\text{He})$ (in MeV) in LO $\not\propto$ EFT SVM calculations, with LECs fitted to NN and NNN low-energy data [28]. The $\Lambda \rightarrow \infty$ limit of $B(^4\text{He})$ was evaluated by using higher values of the scale Λ than listed here.

Λ (fm $^{-1}$)	2	4	6	8	$\rightarrow \infty$	exp.
$B(^4\text{He})$	22.4	22.9	24.2	25.1	27.8 \pm 0.2	28.3

two-body low-energy constants (LEC) c_S^Λ and c_T^Λ are fitted to the $S=0$ pn scattering length and to the $S=1$ deuteron binding energy, respectively, and the three-body LEC d_3^Λ is fitted to the ^3H binding energy. Following Ref. [28] a small corrective proton–proton contact term, with LEC c_{pp}^Λ , is introduced together with the Coulomb interaction between protons to reproduce the ^3He binding energy. These two-body and three-body LECs are listed in Ref. [28] where c_{pp}^Λ was found to effectively adjust c_S^Λ by less than 0.1% over the full Λ -range tested in the present work. The ^4He calculated binding energy $B(^4\text{He})$ provides then a check on how reasonable this LO $\not\propto$ EFT version is. This is demonstrated in Table 1 for four representative values of Λ . The calculated values of $B(^4\text{He})$ depend only moderately on the scale parameter Λ , exhibiting renormalization scale invariance by approaching in the limit $\Lambda \rightarrow \infty$ a finite value 27.8 \pm 0.2 MeV which compares well with $B_{\text{exp}}(^4\text{He}) = 28.3$ MeV, despite the fact that only LO contributions are accounted for in this $\not\propto$ EFT version.

2.3. Pionless EFT ηN interactions

Parity conservation forbids pion exchange in the ηN interaction, suggesting thereby that a $\not\propto$ EFT approach may be justified. With spin and isospin zero for the η meson, a single ηN contact term is all one needs at LO. Below we derive the corresponding LEC from the ηN s -wave scattering amplitude $F_{\eta N}(E)$ calculated in two meson–baryon coupled-channel interaction models, GW [3] and CS [23], and shown in Fig. 1. Whereas the GW model used in our previous work [13] is an on-shell K -matrix model that considers $\eta N \leftrightarrow \pi N$ coupling, the CS model is a meson–baryon multi-channel chirally motivated model in which the ηN interaction is extremely short ranged and practically momentum independent below the momentum breakdown scale specified in the next paragraph. Both models capture the main features of the underlying $N^*(1535)$ resonance which peaks about 50 MeV above the ηN threshold energy $E_{\text{th}} = 1487$ MeV and generates considerable energy dependence of $F_{\eta N}$ near threshold. In particular, both real and imaginary parts of $F_{\eta N}(E)$, which at threshold are given by the scattering lengths (in fm)

$$a_{\eta N}^{\text{GW}} = 0.96 + i0.26 \quad a_{\eta N}^{\text{CS}} = 0.67 + i0.20, \quad (5)$$

decrease monotonically in these models upon going into the sub-threshold region while displaying considerable model dependence.

The ηN scattering lengths listed above are of order 1 fm or less, much smaller than the NN scattering lengths whose large size justifies the use of $\not\propto$ EFT in light nuclei. For ηN interactions as weak as implied by this size of $a_{\eta N}$, and with no ηN bound or virtual state expected, the ηN scattering length alone does not provide a meaningful criterion of fitting into an EFT approach. Alternatively, we estimate $p_\eta R \lesssim \frac{\pi}{2}$ for the momentum p_η of a weakly bound η -nuclear state in a square well of radius R . With $R = 2$ fm or a bit larger for the He isotopes, we get $p_\eta \lesssim 150$ MeV/ c (≈ 0.76 fm $^{-1}$). Since the lowest-mass allowed meson exchange in pseudoscalar meson interaction with octet baryons is owing to vector mesons, with a typical mass of $m_\rho = 770$ MeV, this range of η -nuclear momenta can be accommodated comfortably within the nuclear LO $\not\propto$ EFT approach of the preceding subsection. The $\not\propto$ EFT small parameter associated with

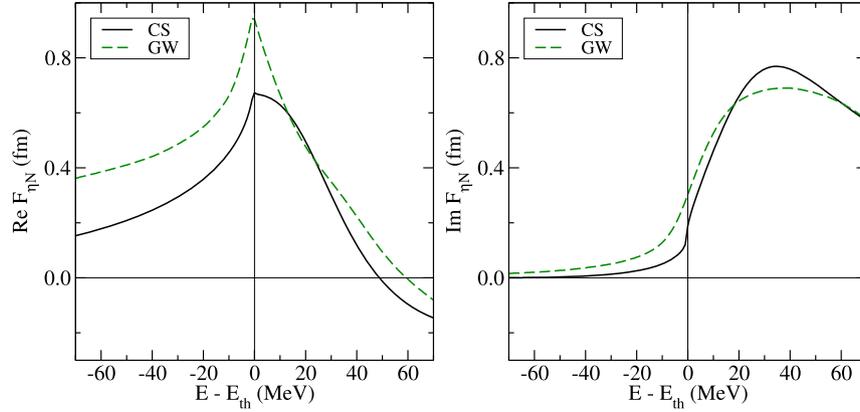


Fig. 1. Real (left panel) and imaginary (right panel) parts of the ηN cm s -wave scattering amplitude $F_{\eta N}(E)$ as a function of $E - E_{\text{th}}$, with $E = \sqrt{s}$ the total ηN cm energy, in the GW [3] and CS [23] meson–baryon coupled-channel interaction models. The vertical line marks the ηN threshold energy E_{th} . Figure adapted from Ref. [11].

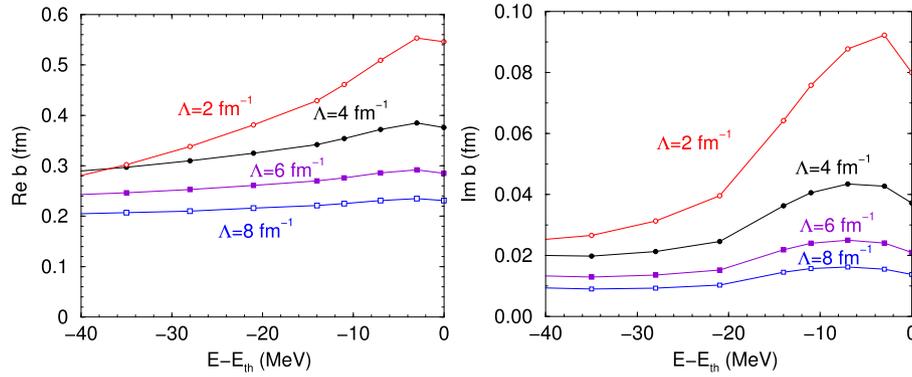


Fig. 2. Real (left panel) and imaginary (right panel) parts of the strength parameter $b_{\eta N}^\Lambda(E)$ of the ηN effective potential (6) at subthreshold energies $E < E_{\text{th}}$ for four values of the scale (cut-off) parameter Λ , all of which result in the same scattering amplitude $F_{\eta N}^{\text{GW}}$ [3] shown in Fig. 1.

this momentum breakdown scale of $Q_{\text{high}}^\rho \approx m_\rho = 3.9 \text{ fm}^{-1}$ is given by $(p_\eta/Q_{\text{high}}^\rho)^2 \approx 0.04$.

As in previous work [13], and in order to account for the energy dependence inherent in the meson–baryon coupled channel dynamical generation of the $N^*(1535)$ resonance, we construct energy-dependent local potentials $v_{\eta N}(E)$ that produce the ηN energy dependent scattering amplitude $F_{\eta N}(E)$ below threshold in models GW and CS. For a given ηN interaction model, the on-shell scattering amplitude $F_{\eta N}(E)$ serves as a single datum to which LO \not{x} EFT regulated contact terms of the form

$$v_{\eta N}(E; r) = c_{\eta N}^\Lambda(E) \delta_\Lambda(r), \quad c_{\eta N}^\Lambda(E) = -\frac{4\pi}{2\mu_{\eta N}} b_{\eta N}^\Lambda(E), \quad (6)$$

are fitted. Here δ_Λ is a regulating normalized-to-one Gaussian with scale parameter Λ , as per Eq. (4), and $c_{\eta N}^\Lambda(E)$ is an energy dependent LEC conveniently related through the ηN reduced mass $\mu_{\eta N}$ to a strength function $b_{\eta N}^\Lambda(E)$ of length dimension. The specific value $c_{\eta N}^\Lambda(E_{\text{sc}})$ of this LEC for a given cut-off Λ is determined self consistently in the η -nuclear SVM calculation as detailed in the next subsection. By using the same value of Λ in all NN , NNN and ηN regulating Gaussians we reach a consistent extension of the nuclear \not{x} EFT to a combined η -nuclear \not{x} EFT approach. In order to study the renormalization scale invariance of our few-body η -nuclear results, as shown for the purely nuclear case of $B(^4\text{He})$ in Table 1, we discuss below \not{x} EFT calculations done for several representative values of the scale parameter, $\Lambda = 2, 4, 6, 8 \text{ fm}^{-1}$. The last two values, clearly, exceed the momentum breakdown

scale $Q_{\text{high}}^\rho \approx 3.9 \text{ fm}^{-1}$ of the underlying $N^*(1535)$ resonance model for the ηN interaction, or even more so the lower momentum breakdown scale $q_{\text{high}}^\rho \approx 3.0 \text{ fm}^{-1}$ set by excitation of vector meson degrees of freedom absent in the underlying $N^*(1535)$ dynamical models considered here, such as the ρ meson produced at threshold in the strong pion exchange reaction $\pi N \rightarrow \rho N$ with $p_{\text{th}}^\rho = 586 \text{ MeV}/c$. Finally, the model dependence of the LO ηN contact term introduced by studying two quite different ηN interaction models, GW and CS, leaves little motivation to go at present beyond LO. Hence, discussion of higher orders in \not{x} EFT is left to future work.

For a given value of Λ , the subthreshold values of the complex strength parameter $b_{\eta N}^\Lambda(E)$ in Eq. (6) were fitted to the complex phase shifts derived from the subthreshold scattering amplitudes $F_{\eta N}(E)$ in models GW and CS. The resulting values of the strength parameter $b_{\eta N}^\Lambda(E)$ for ηN subthreshold energies in model GW, shown in Fig. 2, fall off monotonically for both real and imaginary parts in going deeper below threshold, except for small kinks near threshold that reflect the threshold cusp of $\text{Re } F_{\eta N}(E)$ at E_{th} in Fig. 1. Similar curves for $b_{\eta N}^\Lambda(E)$ are obtained in model CS, with values smaller uniformly for both real and imaginary parts than model GW yields, in accordance with the relative strength of the generating scattering amplitudes $F_{\eta N}(E)$ shown in Fig. 1. We note that increasing Λ leads to weaker strengths $b_{\eta N}^\Lambda(E)$ and also to a weaker energy dependence. Inspecting Fig. 2, one also notes that $\text{Im } b_{\eta N}^\Lambda(E) \ll \text{Re } b_{\eta N}^\Lambda(E)$, which justifies treating $\text{Im } v_{\eta N}$ perturbatively in the present calculations.

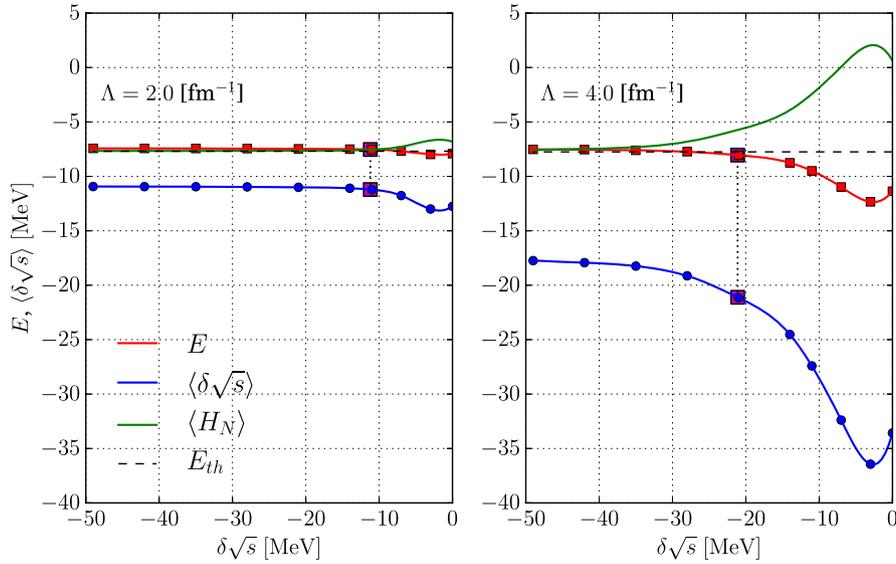


Fig. 3. Calculated $\eta^3\text{He}$ bound-state energies E (squares) and expectation values $\langle \delta\sqrt{s} \rangle$ (circles) from Eq. (7), using LO π EFT NN and NNN potentials, and the ηN potentials $v_{\eta N}^{GW}(E)$ for two values of the scale parameter Λ , as a function of the input energy shift $\delta\sqrt{s}$ used for the energy argument of $v_{\eta N}^{GW}(E)$. The dashed vertical line marks the self consistent values of E and $\langle \delta\sqrt{s} \rangle$. The dashed horizontal line marks the ${}^3\text{He}$ core g.s. energy serving as threshold for a bound η , and the curve above it shows the squeezed core energy $\langle H_N \rangle$.

2.4. Energy dependence

To determine the ηN subthreshold energy at which $v_{\eta N}(E)$ is evaluated as input to the η -nuclear few-body calculations reported below, we denote the shift away from threshold by $\delta\sqrt{s} \equiv \sqrt{s} - \sqrt{s_{th}}$, expressing it in terms of output expectation values [13]:

$$\langle \delta\sqrt{s} \rangle = -\frac{B}{A} - \beta_N \frac{1}{A} \langle T_N \rangle + \frac{A-1}{A} E_\eta - \xi_A \beta_\eta \left(\frac{A-1}{A} \right)^2 \langle T_\eta \rangle, \quad (7)$$

where $\beta_{N(\eta)} \equiv m_{N(\eta)}/(m_N + m_\eta)$, $\xi_A \equiv Am_N/(Am_N + m_\eta)$, T_N and T_η are the nuclear and η kinetic energy operators evaluated in terms of internal Jacobi coordinates, with $T = T_N + T_\eta$ the total intrinsic kinetic energy of the system, B is the total binding energy of the η -nuclear few-body system and $E_\eta = \langle \Psi | (H - H_N) | \Psi \rangle$, where H_N is the Hamiltonian of the purely nuclear part in its own cm frame, and the total Hamiltonian H is evaluated in the overall cm frame. The imaginary, absorptive part of the ηN interaction is suppressed in this discussion. Noting that $(A-1)\langle T_{N:N} \rangle$ in Eq. (7) of Ref. [13] equals $\langle T_N \rangle$ here, Eq. (7) coincides with the former equation apart from a kinematical factor ξ_A introduced here to make correspondence with the η -nuclear, last Jacobi coordinate with which T_η is associated. Requiring that the expectation value $\langle \delta\sqrt{s} \rangle$ on the l.h.s. of Eq. (7), as derived from the solution of the Schroedinger equation, agrees with the input value $\delta\sqrt{s}$ for $v_{\eta N}(E)$, this equation defines a self-consistency cycle in our few-body η -nuclear calculations. Since each one of the four terms on the r.h.s. of Eq. (7) is negative, the self consistent energy shift $\delta\sqrt{s_{sc}}$ is also negative, with size exceeding a minimum nonzero value obtained from the first two terms in the limit of vanishing η binding. Eq. (7) in the limit $A \gg 1$ coincides with the nuclear-matter expression used in Refs. [8,9] for calculating η -nuclear quasibound states.

Fig. 3 demonstrates how the self consistency requirement works in actual calculations. The three curves plotted in each panel are obtained by interpolating a sequence of calculated $\eta^3\text{He}$ bound-state energies (squares) and the corresponding expectation values $\langle \delta\sqrt{s} \rangle$ (circles) from Eq. (7) for $A = 3$, as a function of the input

$\delta\sqrt{s}$ to the energy argument $E_{th} + \delta\sqrt{s}$ of $v_{\eta N}^{GW}$, for two choices of the momentum scale parameter $\Lambda = 2, 4 \text{ fm}^{-1}$. The dashed vertical line marks the self consistent value of $\delta\sqrt{s}$ at which the outcome bound-state energy $E(\eta^3\text{He})$ is evaluated, and the dashed horizontal line marks the ${}^3\text{He}$ core energy $E({}^3\text{He})$. Note that the self consistent value $E_{sc}(\eta^3\text{He})$ is higher than $E({}^3\text{He})$ in the left panel for $\Lambda = 2 \text{ fm}^{-1}$, while it is lower than $E({}^3\text{He})$ in the right panel for $\Lambda = 4 \text{ fm}^{-1}$. This means that, correspondingly, $\eta^3\text{He}$ is slightly unbound for $\Lambda = 2 \text{ fm}^{-1}$ while slightly bound for $\Lambda = 4 \text{ fm}^{-1}$. We note furthermore that for threshold values $v_{\eta N}^{GW}(E_{th})$, i.e. $\delta\sqrt{s} = 0$, $\eta^3\text{He}$ is bound in both cases (and also if the often used but unfortunately unfounded self consistency requirement [29] $\delta\sqrt{s} = E_\eta$ is imposed). Finally, the upper curve in Fig. 3 shows the expectation value $\langle H_N \rangle$ of the nuclear core energy, which is bounded from below by the ${}^3\text{He}$ core energy $E({}^3\text{He})$ marked by the dashed horizontal line.

3. Results and discussion

Separation energies $B_\eta \equiv B(\eta^A\text{He}) - B({}^A\text{He})$ (often called η binding energies) of the $\eta^A\text{He}$ isotopes with $A = 3, 4$ were calculated self consistently in the SVM using LO π EFT NN and NNN regulated contact terms introduced in Eqs. (2)–(4), and a regulated ηN energy dependent contact term specified by Eq. (6), with scale parameters $\Lambda = 2, 4, 6, 8 \text{ fm}^{-1}$. Two ηN coupled-channels models were used, GW [3] and CS [23]. The CS ηN interaction was found by far too weak to bind $\eta^3\text{He}$, and only by a fraction of MeV short of binding $\eta^4\text{He}$. The binding energies B_η were evaluated using real Hamiltonians in which $\text{Im } v_{\eta N}$ was disregarded. Restoring $\text{Im } v_{\eta N}$ in optical model calculations was found particularly important for near-threshold bound states, lowering their calculated B_η by 0.2 ± 0.1 MeV. The η -nuclear widths Γ_η were calculated with wavefunctions $\Psi_{g.s.}$ generated by these real Hamiltonians:

$$\Gamma_\eta = -2 \langle \Psi_{g.s.} | \text{Im } V_\eta | \Psi_{g.s.} \rangle. \quad (8)$$

Here, V_η sums over all pairwise ηN interactions. Since $|\text{Im } V_\eta| \ll |\text{Re } V_\eta|$, this is a reasonable approximation.

Binding energies B_η and widths Γ_η resulting from these self consistent calculations are listed in Table 2 for the ηN poten-

Table 2

η binding energies and widths (MeV) in the He isotopes from SVM calculations using ηN potentials $v_{\eta N}^{\text{GW}}(E)$ with scale parameters $\Lambda = 2, 4 \text{ fm}^{-1}$, together with the corresponding self consistent values of the downward energy shift (in MeV) $\delta\sqrt{s_{\text{sc}}}$. The values of $\Gamma(\eta^3\text{He})$ shown here outdate the erroneous, too large widths listed in Ref. [13].

$\Lambda \text{ (fm}^{-1}\text{)}$	$\eta^3\text{He}$			$\eta^4\text{He}$		
	$\delta\sqrt{s_{\text{sc}}}$	B_η	Γ_η	$\delta\sqrt{s_{\text{sc}}}$	B_η	Γ_η
2	-11.2	-0.16	0.24	-16.5	-0.15	0.34
4	-21.1	+0.30	1.46	-32.2	+1.54	2.82

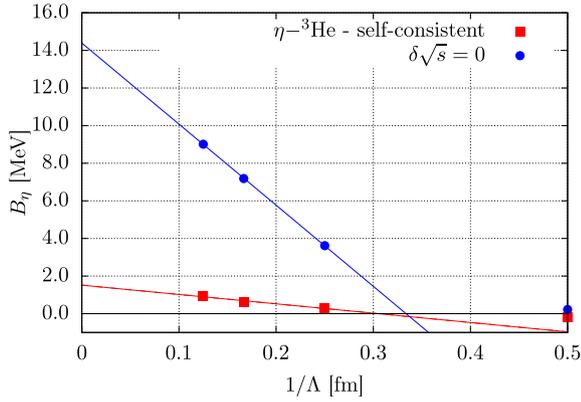


Fig. 4. $B_\eta(\eta^3\text{He})$ as a function of Λ^{-1} , calculated using ηN potentials $v_{\eta N}^{\text{GW}}(E)$ with scale parameters (from right to left) $\Lambda = 2, 4, 6, 8 \text{ fm}^{-1}$. Squares (red) denote self consistent calculations, circles (blue) denote calculations with threshold values of the ηN interaction. Linear extrapolation to a point-like interaction, $\Lambda \rightarrow \infty$, is marked by straight lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

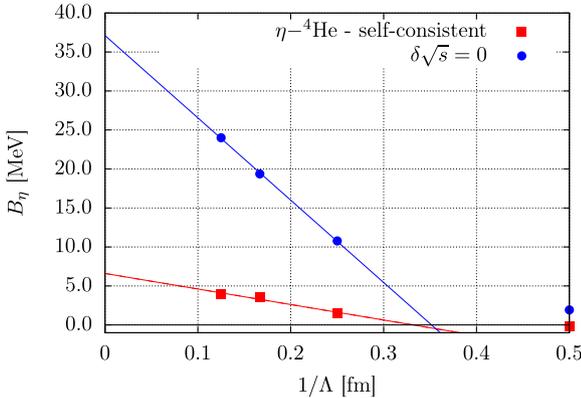


Fig. 5. Same as in Fig. 4, but for $\eta^4\text{He}$ instead of $\eta^3\text{He}$.

tials $v_{\eta N}^{\text{GW}}(E)$ with scale parameters $\Lambda = 2, 4 \text{ fm}^{-1}$, values for which our self consistency procedure was demonstrated in Fig. 3. Higher values of Λ exceed by far the momentum breakdown scale q_{high}^ρ introduced in our previous work [13] for $N^*(1535)$ resonance meson-baryon models in which the ηN scattering amplitude $F_{\eta N}$ is determined. Taken literally, this would mean that the GW ηN interaction hardly binds $\eta^3\text{He}$, if at all, and is likely to bind slightly $\eta^4\text{He}$, with B_η of order 1 MeV.

Sequences of calculated values of η binding energy B_η using ηN potentials $v_{\eta N}^{\text{GW}}(E)$ are shown for $\eta^3\text{He}$ and $\eta^4\text{He}$ in Figs. 4 and 5, respectively, as a function of Λ^{-1} . The figures demonstrate that the larger Λ , the larger is the resulting η binding energy B_η in spite of a similar increase in the value of $-\delta\sqrt{s_{\text{sc}}}$ in self consistent calculations which implies a weaker ηN potential strength $b(E_{\text{sc}})$. The dependence of B_η on Λ is weak for $\eta^3\text{He}$ and moderate for

$\eta^4\text{He}$ in these calculations. For $\Lambda \geq 4 \text{ fm}^{-1}$, B_η varies linearly in Λ^{-1} , with an average error of 50 keV for $\eta^3\text{He}$ and 300 keV for $\eta^4\text{He}$, and with twice these errors upon extrapolating $\Lambda \rightarrow \infty$. In contrast, for calculations done at the ηN threshold, i.e. $\delta\sqrt{s} = 0$, the resulting values of B_η shown in the figures depend strongly on Λ with almost perfect linear dependence on Λ^{-1} for $\Lambda \geq 4 \text{ fm}^{-1}$.

Interestingly, Figs. 4 and 5 also suggest that B_η assumes a finite value $B_\eta^{\Lambda \rightarrow \infty}$ in the limit of point ηN interaction, with no need to introduce a stabilizing three-body ηNN repulsive contact term analogous to the NNN contact term d_3^2 of Eq. (3). This might be related to the weakness of the ηN interaction, with no ηN and ηNN bound states, compared to the much stronger NN interaction which generates NN and NNN bound states. In the latter case, a Thomas collapse of the NNN bound state is averted in \not{x} EFT by promoting a non-derivative NNN contact term from $N^2\text{LO}$ to LO.

4. Conclusion

To summarize, we have presented genuine few-body SVM calculations of ηNNN ($\eta^3\text{He}$) bound states and, for the first time, also $\eta NNNN$ ($\eta^4\text{He}$) bound states, using LO \not{x} EFT interactions where the ηN interaction contact term was derived in coupled channels studies of the $N^*(1535)$ nucleon resonance. Special care was taken of the energy dependence of the input ηN subthreshold scattering amplitude by using a self consistency procedure. The present results exhibit renormalization scale invariance of the calculated η binding energies, without having to introduce a repulsive ηNN contact interaction. For physically motivated values of Λ , the onset of $\eta^3\text{He}$ binding occurs for $\text{Re } a_{\eta N}$ close to 1 fm, as in model GW [3], consistently with our previous hyperspherical-basis ηNNN calculations [13]. The onset of $\eta^4\text{He}$ binding requires a lower value of $\text{Re } a_{\eta N}$, exceeding however 0.7 fm; it is comfortably satisfied in model GW but not in model CS [23]. Further dedicated experimental searches for $\eta^4\text{He}$ bound states are desirable in order to confirm the recent negative report from WASA-at-COSY [17] which, taken at face value, implies that $\text{Re } a_{\eta N} \lesssim 0.7 \text{ fm}$. Similar results and conclusions hold valid in SVM calculations using non-EFT realistic nuclear models [14,15] augmented by the same ηN interaction models used here, and will be reported elsewhere.

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