

## Partial dynamical symmetry and the vibrational structure of Cd isotopes

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**Abstract.** The recently reported deviations of selected non-yrast states in <sup>110</sup>Cd from the expected spherical-vibrator behaviour, is addressed by means of an Hamiltonian with U(5) partial dynamical symmetry. The latter preserves the U(5) symmetry in a segment of the spectrum and breaks it in other states. The effect of intruder states is treated in the framework of the interacting boson model with configuration mixing.

The Cd isotopes have been traditionally considered to be a prime example of spherical vibrators. Recently, advanced experimental studies have reported significant deviations from this behaviour in selected two- and three-phonon states, along the Cd chain (A=108-126) [1–3]. These observations have led to claims for the “breakdown of the vibrational motion” in these isotopes and the need for a paradigm shift [1, 2]. In the present contribution, we examine an alternative explanation for the structure of the Cd isotopes, in terms of U(5) partial dynamical symmetry (PDS) [4].

A convenient starting point for describing spherical nuclei is the U(5) limit of the interacting boson model (IBM) [5], corresponding to the chain of nested algebras,

$$U(6) \supset U(5) \supset SO(5) \supset SO(3). \quad (1)$$

The basis states  $[[N], n_d, \tau, n_\Delta, L]$  have quantum numbers which are the labels of irreducible representations of the algebras in the chain. Here  $N$  is the total number of monopole ( $s$ ) and quadrupole ( $d$ ) bosons,  $n_d$  and  $\tau$  are the  $d$ -boson number and seniority, respectively, and  $L$  is the angular momentum. The multiplicity label  $n_\Delta$  counts the maximum number of  $d$ -boson triplets coupled to  $L = 0$ . The dynamical symmetry (DS) Hamiltonian has the form

$$\hat{H}_{DS} = t_1 \hat{n}_d + t_2 \hat{n}_d^2 + t_3 \hat{C}_{SO(5)} + t_4 \hat{C}_{SO(3)}, \quad (2)$$

where  $\hat{C}_G$  is the Casimir operator of  $G$ , and  $\hat{n}_d = \sum_m \hat{d}_m^\dagger \hat{d}_m = \hat{C}_{U(5)}$ .  $\hat{H}_{DS}$  is completely solvable with eigenstates  $[[N], n_d, \tau, n_\Delta, L]$  and eigen-energies

$$E_{DS} = t_1 n_d + t_2 n_d^2 + t_3 \tau(\tau + 3) + t_4 L(L + 1). \quad (3)$$

A typical U(5)-DS spectrum exhibits  $n_d$ -multiplets of a spherical vibrator, with a two-phonon ( $n_d = 2$ ) triplet of states ( $L = 4, 2, 0$ ) at an energy  $E(n_d = 2) \approx 2E(n_d = 1)$

above the ground state ( $n_d = L = 0$ ), and a three-phonon ( $n_d = 3$ ) quintuplet of states ( $L = 6, 4, 3, 0, 2$ ) at  $E(n_d = 3) \approx 3E(n_d = 1)$ . A quadrupole operator proportional to

$$\hat{Q} = d^\dagger s + s^\dagger \bar{d}, \quad (4)$$

enforces strong ( $n_d + 1 \rightarrow n_d$ ) E2 transitions with particular ratios, e.g.,  $\frac{B(E2; n_d=2, L=0, 2, 4 \rightarrow n_d=1, L=2)}{B(E2; n_d=1, L=2 \rightarrow n_d=0, L=0)} = 2 \frac{(N-1)}{N}$ .

The empirical spectrum of <sup>110</sup>Cd consists of both normal levels [shown in Fig. 1(a)], and intruder levels [shown in Fig. 2(a)] based on 2p-4h proton excitations across the Z=50 closed shell. The experimental energies and E2 rates in Fig. 1(a), demonstrate that most normal states have good spherical vibrator properties, and conform well with the U(5)-DS calculation shown in Fig. 1(b). However, the measured rates for E2 decays from the non-yrast states,  $0_3^+$ , ( $n_d = 2$ ) and  $[0_4^+, 2_2^+$  ( $n_d = 3$ )], reveal marked deviations from this behaviour. In particular,  $B(E2; 0_3^+ \rightarrow 2_1^+) < 7.9$ ,  $B(E2; 2_2^+ \rightarrow 4_1^+) < 5$ ,  $B(E2; 2_2^+ \rightarrow 2_2^+) < 0.7^{+0.5}_{-0.6}$  W.u., are extremely small compared to the U(5)-DS values: 46.29, 11.02, 19.84 W.u., respectively. Absolute B(E2) values for transitions from the  $0_4^+$  state are not known, but its branching ratio to  $2_2^+$  is small.

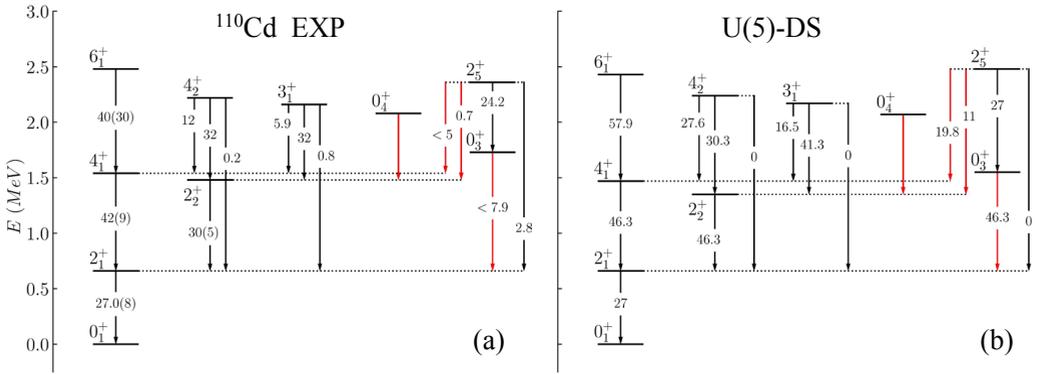
Attempts to explain the above deviations in terms of strong mixing between the normal spherical [U(5)-like] states and intruder deformed [SO(6)-like] states have been shown to be unsatisfactory [1, 2]. The reasons are two-fold. (i) The strong mixing required for an adequate description of the two-phonon  $0_3^+$  state, results in serious disagreements with the observed decay pattern of three-phonon yrast states. (ii) The discrepancy in the decays of the non-yrast two- and three-phonon states persists also in the heavier <sup>A</sup>Cd isotopes (A=110-126), even though the energy of intruder states rises away from neutron mid-shell, and the mixing is reduced. These observations have led to the conclusion that the normal-intruder strong-mixing scenario needs to be rejected, and have raised serious questions on the validity of the multi-phonon interpretation [1, 2]. In what follows, we consider a possible

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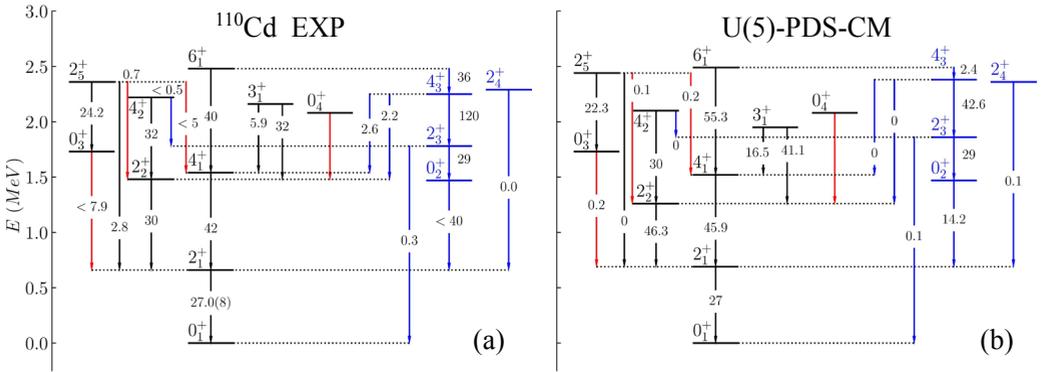
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**Figure 1.** (a) Experimental spectrum and representative E2 rates [2, 6] (in W.u.) of normal levels in  $^{110}\text{Cd}$ . (b) U(5)-DS spectrum obtained from  $\hat{H}_{\text{DS}}$  (2) with parameters  $t_1 = 641.5$ ,  $t_2 = 67.9$ ,  $t_3 = -26.1$ ,  $t_4 = 8.7$  keV and  $N = 7$ . The E2 operator is  $e_B \hat{Q}$ , Eq. (4), with  $e_B = 1.96$  (W.u.) $^{1/2}$ . For additional experimental error bars on B(E2) values, not shown in Figs. 1(a) and 2 (a), see [2, 6].



**Figure 2.** (a) Experimental spectrum and representative E2 rates [2, 6] (in W.u.) of normal [as in Fig. 1(a)] and intruder levels ( $0_2^+$ ,  $2_3^+$ ,  $4_3^+$ ,  $2_4^+$ ) in  $^{110}\text{Cd}$ . (b) U(5)-PDS-CM spectrum, obtained from  $\hat{H}$  (11) with parameters  $t_1 = 767.8$ ,  $t_2 = -t_3 = 73.6$ ,  $t_4 = 18.5$ ,  $r_0 = 2.1$ ,  $e_0 = -6.9$ ,  $\kappa = -72.7$ ,  $\Delta/2 = 4989.5$ ,  $\alpha = -42.8$  keV and  $N = 7$ . The E2 operator (12) used has  $e_B^{(N)} = 1.96$ ,  $e_B^{(N+2)} = 1.19$  (W.u.) $^{1/2}$ .

explanation for the ‘‘Cd problem’’, based on a partial dynamical symmetry (PDS). The latter corresponds to a situation in which a given symmetry is obeyed by only a subset of states and is broken in other states. Such a notion has been previously employed in nuclear spectroscopy in conjunction with the SU(3)-DS [7–9] and SO(6)-DS [10–12] chains of the IBM. In the present contribution, we show the relevance of U(5)-PDS to the Cd problem.

The lowest normal levels comprise three classes of states,

$$\text{Class A : } n_d = \tau = 0, 1, 2, 3 \quad (n_\Delta = 0), \quad (5a)$$

$$\text{Class B : } n_d = \tau + 2 = 2, 3 \quad (n_\Delta = 0), \quad (5b)$$

$$\text{Class C : } n_d = \tau = 3 \quad (n_\Delta = 1). \quad (5c)$$

In the U(5)-DS calculation of Fig 1(b), the ‘‘problematic’’ states  $[0_3^+ (n_d = 2)$  and  $2_5^+ (n_d = 3)]$  belong to class B, and  $0_4^+ (n_d = 3)$  belongs to Class C. The remaining ‘‘good’’ spherical-vibrator states  $[0_1^+ (n_d = 0); 2_1^+ (n_d =$

1);  $4_1^+, 2_2^+ (n_d = 2); 6_1^+, 4_2^+, 3_1^+ (n_d = 3)]$  belong to Class A. As mentioned, the spherical-vibrator interpretation is valid for most states in Fig. 1(a), but not all. We are thus confronted with a situation in which some states in the spectrum (assigned to Class A) obey the predictions of U(5)-DS, while other states (assigned to Classes B and C) do not. These empirical findings signal the presence of a partial dynamical symmetry, U(5)-PDS.

The construction of Hamiltonians with U(5)-PDS follows the general algorithm [12, 13], by identifying operators which annihilate particular sets of U(5) basis states. In the present case, we consider the following interaction,

$$\hat{V}_0 = r_0 G_0^\dagger G_0 + e_0 (G_0^\dagger K_0 + K_0^\dagger G_0), \quad (6)$$

where  $G_0^\dagger = [(d^\dagger d^\dagger)^{(2)} d^\dagger]^{(0)}$ ,  $K_0^\dagger = s^\dagger (d^\dagger d^\dagger)^{(0)}$  and standard notation of angular momentum coupling is used.  $\hat{V}_0$  of Eq. (6) is in normal-ordered form and satisfies

$$\hat{V}_0 |[N, n_d = \tau, \tau, n_\Delta = 0, L] = 0, \quad (7)$$

with  $L = \tau, \tau + 1, \dots, 2\tau - 2, 2\tau$ . Eq. (7) follows from the fact that the indicated states have  $n_d = \tau$  and  $n_\Delta = 0$ , hence do not contain a pair or a triplet of  $d$ -bosons coupled to  $L = 0$  and, as such, are annihilated by  $K_0$  [5] and  $G_0$  [14].

The states of Eq. (7), which include those of Class A, form a subset of U(5) basis states, hence remain solvable eigenstates of the following Hamiltonian

$$\hat{H}_{\text{PDS}} = \hat{H}_{\text{DS}} + \hat{V}_0, \quad (8)$$

with good U(5) symmetry and energies given in Eq. (3) with  $n_d = \tau$ . It should be noted that while  $\hat{H}_{\text{DS}}$  (2) is diagonal in the U(5)-DS chain (1), the  $r_0$ -term ( $e_0$ -term) in  $\hat{V}_0$  connects states with  $\Delta n_d = 0$  and  $\Delta \tau = 0, \pm 2, \pm 4, \pm 6$  ( $\Delta n_d = \pm 1$  and  $\Delta \tau = \pm 1, \pm 3$ ). Accordingly, the remaining eigenstates of  $\hat{H}_{\text{PDS}}$  (8), in particular those of classes B and C, are mixed with respect to U(5) and SO(5). The U(5)-DS is therefore preserved in a subset of eigenstates but is broken in other states. By definition,  $\hat{H}_{\text{PDS}}$  exhibits U(5)-PDS. Cubic terms of the type present in  $\hat{V}_0$ , Eq. (6), are frequently encountered in PDS Hamiltonians, *e.g.*, in conjunction with signature splitting [9], band structure [10–12], and shape-coexistence [15, 16] in nuclei.

The effect of intruder levels can be studied in the framework of the interacting boson model with configuration mixing (IBM-CM) [17]. The latter involves the space of normal states described by a system of  $N$  bosons representing valence nucleon pairs, and the space of intruder states described by a system of  $N+2$  bosons, accounting for particle-hole shell model excitations. This procedure has been used extensively in describing coexistence phenomena in nuclei [18–21]. In the present study of  $^{110}\text{Cd}$ , the Hamiltonian in the normal sector is taken to be  $\hat{H}_{\text{PDS}}$  of Eq. (8), acting in a space of  $N = 7$  bosons. The Hamiltonian in the intruder sector is taken to be of SO(6)-type [18],

$$\hat{H}_{\text{intrud}} = \kappa \hat{Q} \cdot \hat{Q} + \Delta, \quad (9)$$

acting in a space of  $N = 9$  bosons, with  $\hat{Q}$  given in Eq. (4). A mixing term between the  $[N]$  and  $[N+2]$  boson spaces is defined as [18–21],

$$\hat{V}_{\text{mix}} = \alpha \left[ (s^\dagger)^2 + (d^\dagger d^\dagger)^{(0)} \right] + \text{H.c.}, \quad (10)$$

where H.c. means Hermitian conjugate. The combined Hamiltonian for the two configurations has the form

$$\hat{H} = \hat{H}_{\text{PDS}}^{(N)} + \hat{H}_{\text{intrud}}^{(N+2)} + \hat{V}_{\text{mix}}^{(N,N+2)}. \quad (11)$$

Here  $\hat{O}^{(N)} = \hat{P}_N^\dagger \hat{O} \hat{P}_N$  and  $\hat{O}^{(N,N')} = \hat{P}_N^\dagger \hat{O} \hat{P}_{N'}$  for an operator  $\hat{O}$ , with  $\hat{P}_N$  a projection operator onto the  $[N]$  boson space. Similarly, the E2 operator is defined as,

$$\hat{T}(E2) = e_B^{(N)} \hat{Q}^{(N)} + e_B^{(N+2)} \hat{Q}^{(N+2)}, \quad (12)$$

with boson effective charges,  $e_B^{(N)}$  and  $e_B^{(N+2)}$ .

The experimental energies and E2 rates for both normal and intruder levels in  $^{110}\text{Cd}$ , are shown in Fig. 2(a). They are well reproduced by an IBM-PDS-CM calculation, shown in Fig. 2(b), employing the Hamiltonian of Eq. (11) and the E2 operator of Eq. (12). The mixing between the intruder states and normal states of class A is weak. The latter states retain a high degree of purity and

good U(5) quantum numbers. This is reflected in their E2 decay properties, which are essentially the same as those of the U(5)-DS shown in Fig. 1(b). In contrast, the states in classes B and C, whose decay properties show marked deviations from the U(5)-DS limit, are mixed with other normal and intruder states. The resulting calculated values:  $B(E2; 0_3^+ \rightarrow 2_1^+) = 0.25$ ,  $B(E2; 2_5^+ \rightarrow 4_1^+) = 0.19$ ,  $B(E2; 2_5^+ \rightarrow 2_2^+) = 0.12$  W.u., are consistent with the measured upper limits: 7.9, 5,  $0.7^{+0.5}_{-0.6}$  W.u., respectively.

In summary, we have considered the vibrational structure of  $^{110}\text{Cd}$ , by means of U(5)-PDS. The PDS Hamiltonian retains good U(5) symmetry for yrast states, but breaks it in particular non-yrast states. The mixing with the intruder levels is weak, and affects mainly the broken U(5)-DS states. Most low-lying normal states maintain the vibrational character and only specific states exhibit a departure from this behaviour, in line with the empirical data. Calculations are underway to see if this approach can be implemented in other neutron-rich Cd isotopes.

This work is supported in part (A.L. and N.G.) by the Israel Science Foundation (Grant 586/16).

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