Partial Dynamical Symmetry and Anharmonicity in Gamma-Soft Nuclei

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Abstract. Partial dynamical symmetry is shown to be relevant for describing the anharmonicity of excited bands in ^{196}Pt while retaining solvability and good $SO(6)$ symmetry for the ground band.

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Gamma-soft nuclei can be described in the interacting boson model (IBM) in its SO(6) dynamical symmetry (DS) limit [1]. The latter limit corresponds to the chain of nested algebras

$$\begin{array}{ccccccccc} \mathrm{U}(6) & \supset & \mathrm{SO}(6) & \supset & \mathrm{SO}(5) & \supset & \mathrm{SO}(3) & \supset & \mathrm{SO}(2) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ [N] & & \langle\Sigma\rangle & & (\tau) & \nu_\Delta & L & & M \end{array} \tag{1}$$

where, below each algebra, its associated labels of irreducible representations (irreps) are given and ν_Δ is a multiplicity label. The eigenstates $|[N]\langle\Sigma\rangle(\tau)\nu_\Delta LM\rangle$ are obtained with a Hamiltonian with SO(6) DS which can be transcribed in the form

$$\hat{H}_{\mathrm{DS}} \;=\; A\hat{P}_+\hat{P}_- + B\hat{C}_{\mathrm{SO(5)}} + C\hat{C}_{\mathrm{SO(3)}}. \tag{2}$$

Here $\hat{C}_G$ denotes the quadratic Casimir operator of G, $\hat{P}_+ \equiv \frac{1}{2}(s^\dagger s^\dagger - d^\dagger\cdot d^\dagger)$, $4\hat{P}_+\hat{P}_- = \hat{N}(\hat{N}+4) - \hat{C}_{\mathrm{SO(6)}}$ and $\hat{P}_- = \hat{P}_+^\dagger$. The monopole ($s$) and quadrupole ($d$) bosons represent valence nucleon pairs whose total number, $\hat{N} = \hat{n}_s + \hat{n}_d$, is conserved. The SO(6)-DS Hamiltonian, $\hat{H}_{\mathrm{DS}}$, is completely solvable with eigenenergies

$$E_{\mathrm{DS}} \;=\; \tfrac{1}{4}A\,(N-\Sigma)(N+\Sigma+4) + B\,\tau(\tau+3) + CL(L+1). \tag{3}$$

The spectrum resembles that of a γ-unstable deformed rotor, where states are arranged in bands with SO(6) quantum number $\Sigma = N-2v$, ($v = 0,1,2,\ldots$). The in-band rotational structure is governed by the SO(5) and SO(3) terms in $\hat{H}_{\mathrm{DS}}$ (2), with characteristic $\tau(\tau+3)$ and $L(L+1)$ splitting. A comparison with the experimental spectrum and E2 rates of ^{196}Pt [2] is shown in Fig. 1 and Table 1. It displays a good description for properties of states in the ground band ($\Sigma = N$). This observation was the basis of the claim [3] that the SO(6)-DS is manifested empirically in ^{196}Pt. However, the resulting fit to energies of excited bands is quite poor. The 0_1^+, 0_3^+, and 0_4^+ levels of ^{196}Pt at excitation energies 0, 1403, 1823 keV, respectively, are identified as the bandhead states of the ground ($v=0$), first- ($v=1$) and second- ($v=2$) excited vibrational bands [3]. Their empirical anharmonicity, defined by the ratio $R = E(v=2)/E(v=1) - 2$, is found

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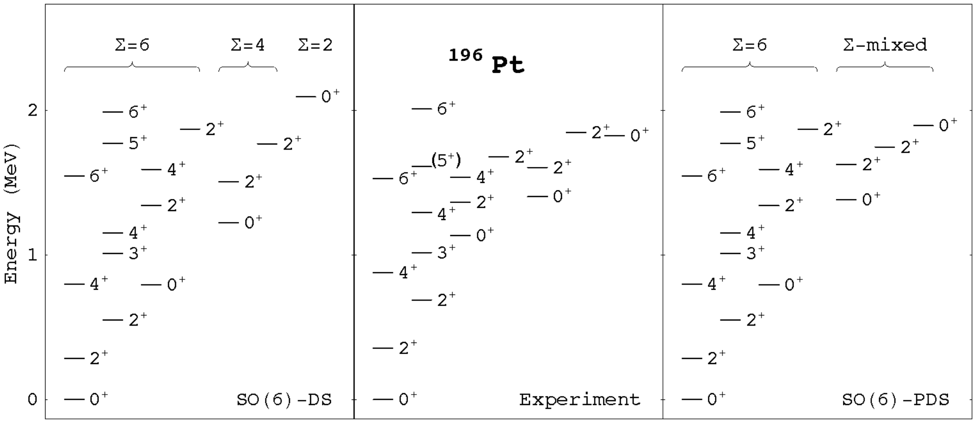

FIGURE 1. Observed spectrum of ^{196}Pt [2] compared with the calculated spectra of $\hat{H}_{\rm DS}$ (2), with SO(6) dynamical symmetry (DS), and of $\hat{H}_{\rm PDS}$ (6) with partial dynamical symmetry (PDS). The parameters in $\hat{H}_{\rm DS}$ ($\hat{H}_{\rm PDS}$) are $A = 174.2\,(122.9)$, $B = 44.0\,(44.0)$, $C = 17.9\,(17.9)$, and $\eta = 0\,(34.9)$ keV. The boson number is $N = 6$ and Σ is an SO(6) label. From [15].

to be $R = -0.70$. In the SO(6)-DS limit these bandhead states have $\tau = L = 0$ and $\Sigma = N, N-2, N-4$, respectively. The anharmonicity $R = -2/(N+1)$, as calculated from Eq. (3), is fixed by N. For $N = 6$, which is the appropriate boson number for ^{196}Pt, the SO(6)-DS value is $R = -0.29$, which is in marked disagreement with the empirical value. A detailed study of double-phonon excitations within the IBM, has concluded that large anharmonicities can be incorporated only by the inclusion of at least cubic terms in the Hamiltonian [4]. In the IBM there are 17 possible three-body interactions. One is thus confronted with the need to select suitable higher-order terms that can break the DS in excited bands but preserve it in the ground band. These are precisely the defining properties of a partial dynamical symmetry (PDS). The essential idea is to relax the stringent conditions of *complete* solvability, so that only part of the eigenspectrum retains all the DS quantum numbers. Various types of PDS are known to

TABLE 1. Observed [2] and calculated B(E2) values (in e^2b^2) for ^{196}Pt. For both the exact (DS) and partial (PDS) SO(6) dynamical symmetry calculations, the E2 operator is $e_{\rm b}[(s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s})^{(2)} + \chi(d^\dagger \times \tilde{d})^{(2)}]$ with $e_{\rm b} = 0.151$ eb and $\chi = 0.29$. From [15].

Transition	Experiment	DS	PDS	Transition	Experiment	DS	PDS
$2^+_1 \to 0^+_1$	0.274 (1)	0.274	0.274	$2^+_3 \to 0^+_2$	0.034 (34)	0.119	0.119
$2^+_2 \to 2^+_1$	0.368 (9)	0.358	0.358	$2^+_3 \to 4^+_1$	0.0009 (8)	0.0004	0.0004
$2^+_2 \to 0^+_1$	3.10^{-8}(3)	0.0018	0.0018	$2^+_3 \to 2^+_2$	0.0018 (16)	0.0013	0.0013
$4^+_1 \to 2^+_1$	0.405 (6)	0.358	0.358	$2^+_3 \to 0^+_1$	0.00002 (2)	0	0
$0^+_2 \to 2^+_2$	0.121 (67)	0.365	0.365	$6^+_2 \to 6^+_1$	0.108 (34)	0.103	0.103
$0^+_2 \to 2^+_1$	0.019 (10)	0.003	0.003	$6^+_2 \to 4^+_2$	0.331 (88)	0.221	0.221
$4^+_2 \to 4^+_1$	0.115 (40)	0.174	0.174	$6^+_2 \to 4^+_1$	0.0032 (9)	0.0008	0.0008
$4^+_2 \to 2^+_2$	0.196 (42)	0.191	0.191	$0^+_3 \to 2^+_2$	< 0.0028	0.0037	0.0028
$4^+_2 \to 2^+_1$	0.004 (1)	0.001	0.001	$0^+_3 \to 2^+_1$	< 0.034	0	0
$6^+_1 \to 4^+_1$	0.493 (32)	0.365	0.365				

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TABLE 2. $SO(6)$ decomposition of eigenstates of $\hat{H}_{\rm PDS}$ (6), corresponding to bandhead states in ^{196}Pt.

Bandhead	$\Sigma = 6$	$\Sigma = 4$	$\Sigma = 2$	$\Sigma = 0$
$0^+(v=0)$	100 %			
$0^+(v=1)$		76.5 %	16.1 %	7.4 %
$0^+(v=2)$		19.6 %	18.4 %	62.0 %

be relevant to nuclear spectroscopy [5-11], to systems with mixed chaotic and regular dynamics [12, 13] and to quantum phase transitions [14]. In the present contribution we demonstrate the relevance of PDS to the anharmonicity of excited bands in ^{196}Pt [15].

Hamiltonians with SO(6) PDS preserve the analyticity of only a *subset* of the states (1). The construction of interactions with this property requires n-boson creation and annihilation operators, $\hat{B}^{\dagger}_{[n]\langle\sigma\rangle(\tau)\ell m}$ and $\tilde{B}_{[n^5]\langle\sigma\rangle(\tau)\ell m}$, with definite tensor character in the basis (1). Of particular interest are n-boson annihilation operators which satisfy

$$\tilde{B}_{[n^5]\langle\sigma\rangle(\tau)\ell m}|[N]\langle N\rangle(\tau)\nu_\Delta LM\rangle = 0, \tag{4}$$

for all possible values of τ, L contained in the SO(6) irrep $\langle N\rangle$. The annihilation condition (4) is satisfied for tensor operators with $\sigma < n$. This is so because the action of $\tilde{B}_{[n^5]\langle\sigma\rangle(\tau)\ell m}$ leads to an $(N-n)$-boson state that contains the SO(6) irreps $\langle\Sigma\rangle = \langle N-n-2i\rangle$, $i = 0,1,\ldots$, which cannot be coupled with $\langle\sigma\rangle$ to yield $\langle\Sigma\rangle = \langle N\rangle$, since $\sigma < n$. Number-conserving normal-ordered interactions that are constructed out of such tensors (and their Hermitian conjugates) thus have $|[N]\langle N\rangle(\tau)\nu_\Delta LM\rangle$ as eigenstates with zero eigenvalue.

A systematic enumeration of all interactions with this property is a simple matter of SO(6) coupling. For example, SO(6) tensors, $\hat{B}^{\dagger}_{[n]\langle\sigma\rangle(\tau)\ell m}$, with $\sigma < n = 2$ or $\sigma < n = 3$ are found to be

$$\hat{B}^{\dagger}_{[2]\langle 0\rangle(0)00} \propto \hat{P}_+, \quad \hat{B}^{\dagger}_{[3]\langle 1\rangle(1)2m} \propto \hat{P}_+ d^{\dagger}_m, \quad \hat{B}^{\dagger}_{[3]\langle 1\rangle(0)00} \propto \hat{P}_+ s^{\dagger}\ . \tag{5}$$

The two-boson SO(6) tensor gives rise to a two-body SO(6)-invariant interaction, $\hat{P}_+\hat{P}_-$, which is simply the completely solvable SO(6) term in $\hat{H}_{\rm DS}$, Eq. (2). From the three-boson SO(6) tensors one can construct three-body interactions with an SO(6) PDS, namely, $\hat{P}_+\hat{n}_s\hat{P}_-$ and $\hat{P}_+\hat{n}_d\hat{P}_-$. Since the combination $\hat{P}_+(\hat{n}_s+\hat{n}_d)\hat{P}_- = (\hat{N}-2)\hat{P}_+\hat{P}_-$ is completely solvable in SO(6), there is only one genuine partially solvable three-body interaction which can be chosen as $\hat{P}_+\hat{n}_s\hat{P}_-$, with tensorial components $\sigma = 0,2$.

On the basis of the preceding discussion we propose to use the following Hamiltonian with SO(6)-PDS

$$\hat{H}_{\rm PDS} = \hat{H}_{\rm DS} + \eta\hat{P}_+\hat{n}_s\hat{P}_-, \tag{6}$$

where the terms are defined in Eqs. (2) and (5). The spectrum of $\hat{H}_{\rm PDS}$ is shown in Fig. 1. The states belonging to the $\Sigma = N = 6$ multiplet remain solvable with energies given by the same DS expression, Eq. (3). As shown in Table 2, states with $\Sigma < 6$ are generally admixed but agree better with the data than in the DS calculation. Thus, although the ground band is pure, the excited bands exhibit strong SO(6) breaking. The calculated

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SO(6)-PDS anharmonicity for these bands is $R=-0.63$, much closer to the empirical value, $R=-0.70$. We emphasize that not only the energies but also the wave functions of the $\Sigma=N$ states remain unchanged when the Hamiltonian is generalized from DS to PDS. Consequently, the E2 rates for transitions among this class of states are the same in the DS and PDS calculations. This is evident in Table 1 where most of the E2 data concern transitions between $\Sigma=N=6$ states. Only transitions involving states from excited bands (*e.g.*, the 0_3^+ state in Table 1) can distinguish between DS and PDS.

A similar procedure can be implemented on a general dynamical symmetry chain

$$\begin{array}{ccccccc} G_{\rm dyn} & \supset & G & \supset & \cdots & \supset & G_{\rm sym} \\ \downarrow & & \downarrow & & & & \downarrow \\ [h_N] & & \langle\Sigma\rangle & & & & \Lambda \end{array} \tag{7}$$

where $G_{\rm dyn}$ and $G_{\rm sym}$ are, respectively, the dynamical and symmetry algebras of the system. For N identical particles the irrep $[h_N]$ is either symmetric $[N]$ (bosons) or antisymmetric $[1^N]$ (fermions). Hamiltonians which preserve the solvability of states with $\langle\Sigma\rangle=\langle\Sigma_0\rangle$, involve n-particle annihilation tensor operators satisfying

$$\hat{T}_{[h_n]\langle\sigma\rangle\lambda}\,|[h_N]\langle\Sigma_0\rangle\Lambda\rangle=0, \tag{8}$$

for all possible values of Λ contained in the given G-irrep $\langle\Sigma_0\rangle$. The solution of condition (8) amounts to carrying out a G Kronecker product $\langle\sigma\rangle\times\langle\Sigma_0\rangle$. This establishes a generic and systematic procedure for identifying and selecting interactions, of a given order, with PDS. The resulting Hamiltonians break the DS but retain selected subsets of solvable eigenstates with good symmetry. As demonstrated in the present contribution, the advantage of using higher-order interactions with PDS is that they can be introduced without destroying results previously obtained with a DS for a segment of the spectrum.

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