ADVANCED ASTROPHYSICS (77938) - PROBLEM SET No. 4

Due Date: June 21, 2012

## 1. Very Low Mass Stars

Consider stars on the main sequence, having a very low mass, such that they are completely convective.

a. Using Homology or a polytropic relation (plus a radiative relation for the surface!), find how the luminosity and radii of these objects will change according to the hydrogen mass fraction $X$ in them.

b. What is the $X$ for which the star becomes degenerate?

c. How long will it take the star to reach this state?

## 2. Homologous Stars

a. In class we developed the scaling relations between homologous stars. Extend these relations to the case in which the gravitational constant $G$ is no longer a constant in time.

b. Over its lifetime till the present day, the sun has increased its luminosity by $\sim 25\%$. By how much must we change $G$ in order to explain this? What is the rate of change of $G$ of over the sun's lifetime ($\sim 10^{10}$ yrs)?

## 3. Non-Degenerate Envelope in a White Dwarf

For the case of Kramer's opacity $\kappa = \kappa_0 \rho T^{-7/2}$, one can integrate over the envelope of the White Dwarf to get

$$\rho = \left( \frac{4}{17} \frac{16\pi acG}{3\kappa_0} \frac{\mu m_H}{k} \frac{M}{L} \right)^{\frac{1}{2}} T^{13/4}$$

a. Use the above expression and the expression for Kramer's Opacity in the equation for radiative transport. Integrate from the edge of the White Dwarf $R$ to the point $r_*$ where the electrons become degenerate and show the temperature there is

$$T_* = \frac{4}{17} \frac{\mu m_H}{k} \frac{GM}{R} \left( \frac{R}{r_*} - 1 \right)$$

b. The non-degenerate envelope in White Dwarfs can be shown to be small. Show that for a typical White Dwarf with $T_*$ of order $10^6$ K $r_*$ and $R$ differ by less than $1\%$.

## 4. Pulsar Wind-down by Gravitational Radiation

General Relativity effects are important in neutron stars, and specifically, radiating energy via gravitational waves, if the rotating neutron star has a quadrupole moment which changes with time. For example, if its equator is elliptical. For such a star, with an elliptical equator of eccentricity $\epsilon$, the rate of gravitational energy radiated is

$$\dot{E}_G = -\frac{32}{5}\frac{G}{c^5}I^2\epsilon^2\Omega^6$$

Where $I$ is the moment of inertia and $\Omega$ the orbital frequency.

a. Find an expression for the age of a pulsar $\tau_{GW}$ given the orbital frequency when it first formed $\Omega_i$ and today $\Omega_0$, and assuming the energy is lost *only* through gravitational radiation. Compare to the similar expression in the case of the magnetic dipole model $\tau_{MD} = T/2$, where $T$ is a typical time scale today. Assume $\Omega_0 \ll \Omega_i$.

b. Assume that the Crab nebula pulsar started out with $\Omega_i = 10^4\,\text{sec}^{-1}$. The following values are given:

$$\epsilon = 3 \times 10^{-4} \qquad I = 1.4 \times 10^{45}\,\text{gr cm}^2$$
$$R = 12\,\text{km} \qquad B = 5 \times 10^{12}\,\text{gauss}$$

Compare the rate of energy loss due to magnetic dipole radiation and gravitational radiation and answer the following:

i. Which energy dissipation process was dominant when the pulsar was formed?

ii. How long after the pulsar was formed were the energy dissipation rates equal for the two processes?

iii. What is the ratio between the two rates today?

c. The frequency of the Crab Nebula Pulsar is $\Omega_{Crab} = 188.7\,\text{sec}^{-1}$. Find its age using the formula you derived. Is the result compatible with the known age of the pulsar $950\,\text{yrs}$? What do we learn from this?