ADVANCED ASTROPHYSICS (77938) - PROBLEM SET NO. 3

Due Date: May 31, 2012

1. The Greenhouse Effect

Let us describe the greenhouse effect of an earth like planet (e.g., same size, distance from the sun). We assume for simplicity that the atmosphere is totally transparent to the short waves coming from the star (though a fraction a is immediately reflected back). On the other hand, we assume that at the far infrared wavelengths, the atmosphere can be described by the gray approximation, with an effective total optical depth τ. We also assume for simplicity that the planet can be described by one average flux and one average temperature. Also, Earth's albedo a is around 0.3, and each unit area of the top of the atmosphere receives 1367 W/m^2/4 (4 is the ratio between the cross-section and area of a sphere).

a. What is the equilibrium temperature of the planet as a function of the optical depth τ? Note that the problem is equivalent to the problem of radiative transfer in a star. The surface receives a given flux from the star, which then has to pass through the planetary atmosphere to get out.

b. If we assume that the atmosphere is opaque in the infrared, only from a wavelength of $\lambda_0 = 20000\text{Å}$, then we can assume that for $\lambda < \lambda_0$, the thermal emission from the ground can leave unhindered, while for $\lambda > \lambda_0$ it has to pass through the optically thick atmosphere. (Assume that there is no rethermalization of the radiation field in the atmosphere, as if it is scattering). For the shortwave radiation coming from the sun, we can assume that everything arriving at $\lambda < \lambda_0$, can reach the surface, while the rest returns to space without entering the system. Solve (numerically, with matlab or mathematica) for the equilibrium temperature as a function of the optical depth τ. What is the saturation temperature?

2. Convection

The efficiency of convective energy transport Γ_c may be defined as the ratio of a convective elements excess heat content (relative to its surroundings just before it mixes with its surroundings) to the energy it radiates during its lifetime.

a. Show that, apart from numerical factors,

$$\Gamma_c \simeq \frac{c_p \bar{\kappa}_v \bar{v} l_m \rho}{acT^3},$$

where $\bar{v}$ is a typical velocity and l_m is the typical mixing length. Evaluate Γ_c for the solar core in which the effective extinction is $\bar{\kappa}_v \simeq 10^4\,\mathrm{cm}^{-1}$

b. When convection is present, the energy flux is carried partly by convection and partly by radiative transport $F_c + F_R$. Express the fractional convection flux $F_c/(F_c + F_R)$ in terms of the convective efficiency Γ_c, and the gradients $\frac{dT}{dr}$ and$\left(\frac{dT}{dr}\right)_{ad}$. Discuss the limits $\Gamma_c \to 0$ and $\Gamma_c \to \infty$. Does convection always carry most of the flux when Γ_c is large?

2. Nuclear Reactions

Show that by taking

$$S(E) = S(E_0) + \left(\frac{\partial S}{\partial E}\right)_{E_0} (E - E_0)$$

the ratio of the corrected nuclear reaction rate to the zeroth-order expression is

$$G(T) \equiv \frac{r_1}{r_0} = 1 + \frac{5}{6}\frac{kT}{S(E_0)}\left(\frac{\partial S}{\partial E}\right)_{E_0},$$

where r_0 is the zeroth-order rate and r_1 is the rate including first order corrections. If $\left.\frac{\partial S}{\partial E}\right|_{E_0}$ is of order $\frac{S}{E_0}$, what will be the correction using the Gamow Peak?

3. $p - p$ Chain

The rate equation for the proton number density n_p depleted by the reaction

$$p + p \to d + e^+ + \nu_e$$

is

$$\frac{dn_p}{dt} = -{n_p}^2 \langle \sigma v \rangle_{pp}.$$

Deuterium d is destroyed through the reaction

$$d + p \to {}^3\mathrm{He} + \gamma$$

a. What is the rate equation of the deuterium number density n_d (given $\langle \sigma v \rangle_{pp}$ & $\langle \sigma v \rangle_{dp}$)?

b. In equilibrium, deuterium is produced and destroyed at the same rate. What is n_d in this case (given $\langle \sigma v \rangle_{pp}$ & $\langle \sigma v \rangle_{dp}$)?

c. Find the ratio of n_d to n_p in equilibrium in the center of the sun, given that a d nucleus is lives about $1\,\mathrm{sec}$ before interacting.

d. The next reaction in the $p - p$ chain is:

$$^3\mathrm{He} + {}^3\mathrm{He} \to {}^4\mathrm{He} + 2p$$

What are the additional equations for $dn_{^3\mathrm{He}}/dt$ and $dn_{^4\mathrm{He}}/dt$?

e. The first reaction $p + p$ is *much* slower than the other reactions. Find a simple equation for $dn_{^4\mathrm{He}}/dt$ as a function of n_p and $\langle \sigma v \rangle_{ij}$. What is $n_{^3\mathrm{He}}$ in equilibrium?

4. CNO-cycle

Draw a diagram of the reactions in the CNO-cycle in the number-of-protons vs. number of neutrons plane.